

Near-Lower-Bound Approximate Quantum State Preparation with Hardware-Efficient Circuits

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Highlights

- We investigate approximate Haar-random quantum state preparation using variational hardware-efficient circuits.
- Approximation fidelity exhibits a sharp transition near the degree-of-freedom CNOT lower bound.
- Brickwork entangling topology consistently outperforms chain, ring, and dilated-ring circuits.
- Two-parameter local rotations ($R_Y R_Z$) achieve nearly identical performance to full Euler rotations for brickwork circuits.

Take-Home Message:

Approximate quantum state preparation becomes practical with nearly the minimum possible number of CNOT gates.

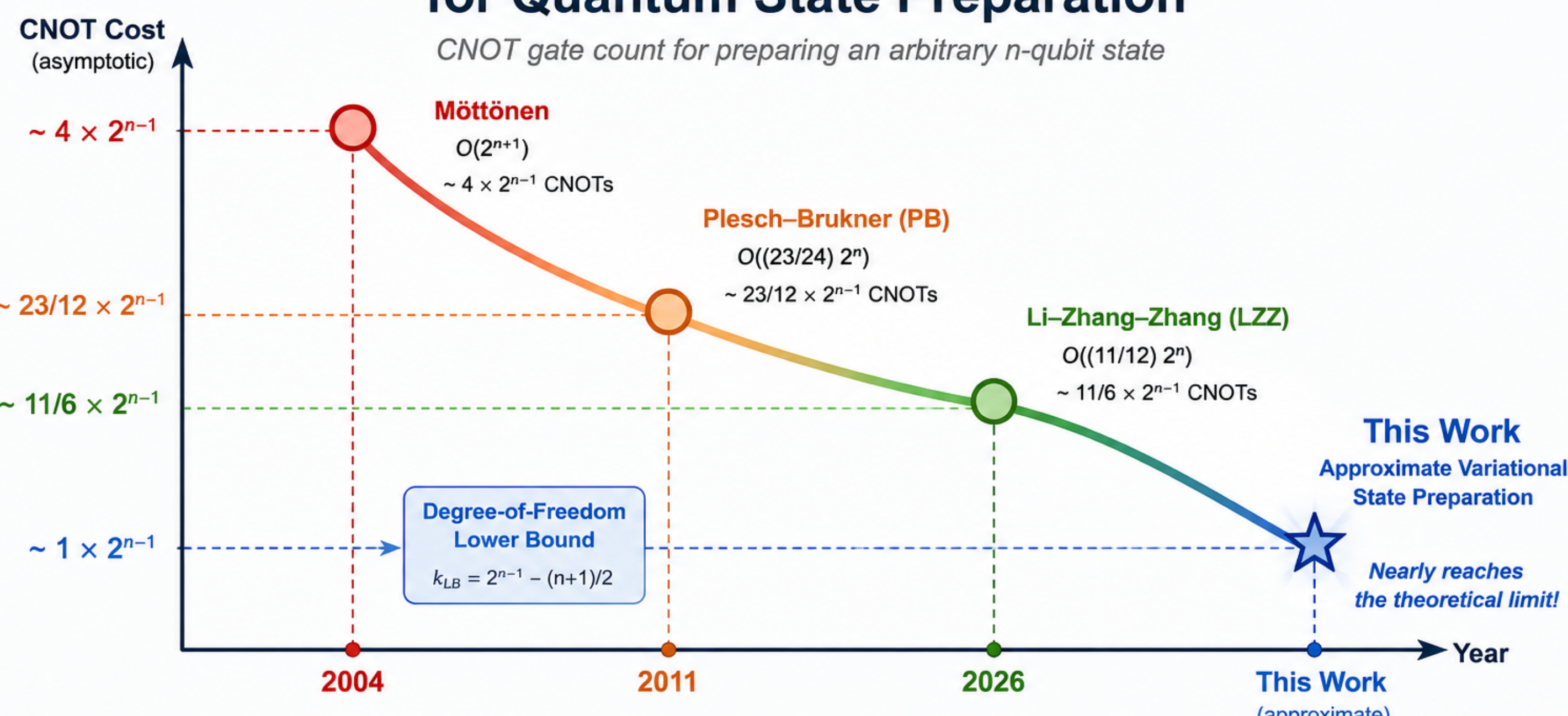
Background

- Arbitrary quantum state preparation requires $\sim 2^{n-1}$ CNOTs:

$$k_{LB} = \lceil 2^{n-1} - (n+1)/2 \rceil. \quad (1)$$

- Möttönen (2004): $\sim 2^{n+1}$
- Plesch–Brukner (2011): $\sim 23/24 \times 2^n$
- Li–Zhang–Zhang (2026): $\sim 11/12 \times 2^n$
- Exact synthesis is still about 2-times more than lower bound.

Approaching the Information-Theoretic Limit for Quantum State Preparation

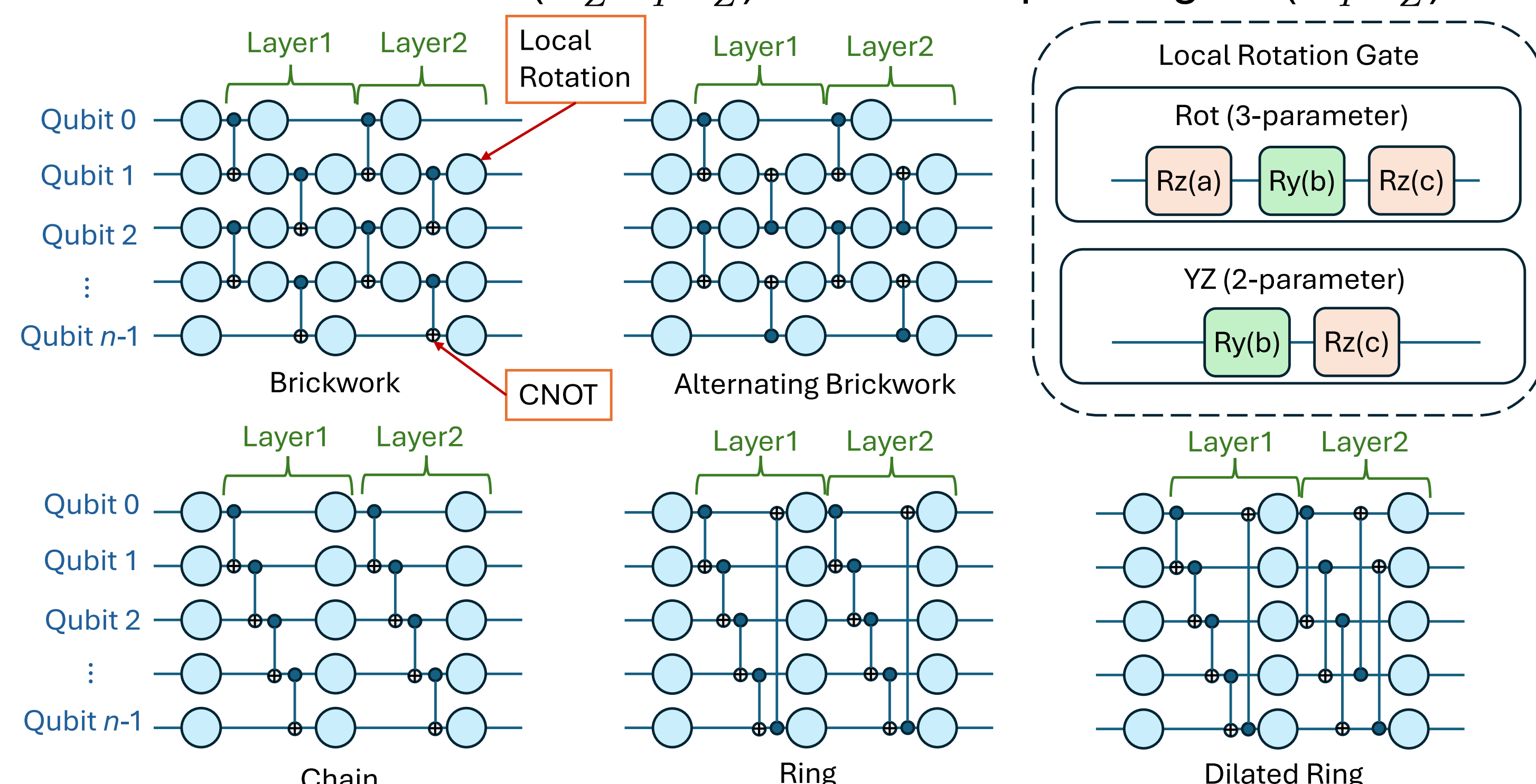


Question:

Can approximate state preparation approach the lower bound?

Hardware-Efficient Quantum Ansatz

- Entangling: alternating brickwork, brickwork, chain, ring, dilated ring
- Rotations: full Euler ($R_Z R_Y R_Z$) or reduced-param gate ($R_Y R_Z$).

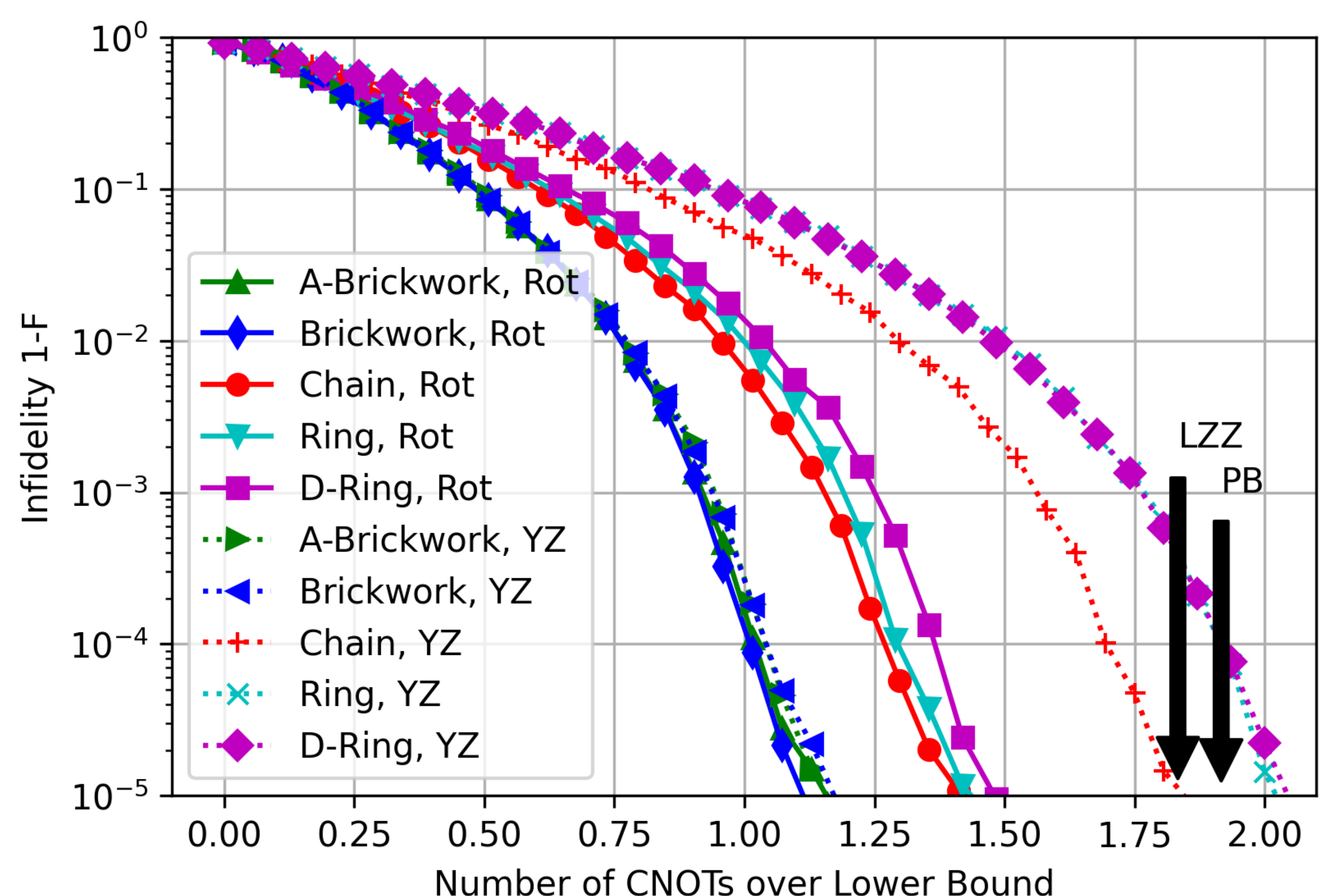


Variational Parameter Optimization

- Given Haar-random target states: $|\phi\rangle$
- Forward ansatz operation: $|\psi\rangle = U(\theta)|0\rangle^{\otimes n}$
- AdamW optimizes rotation angles θ to maximize fidelity $F = |\langle\phi|\psi\rangle|^2$
- 10,000 epochs; One-cycle learning rate scheduler
- Median over multiple random targets and initializations
- Compare topology and rotation parameterization

Expressibility Transition

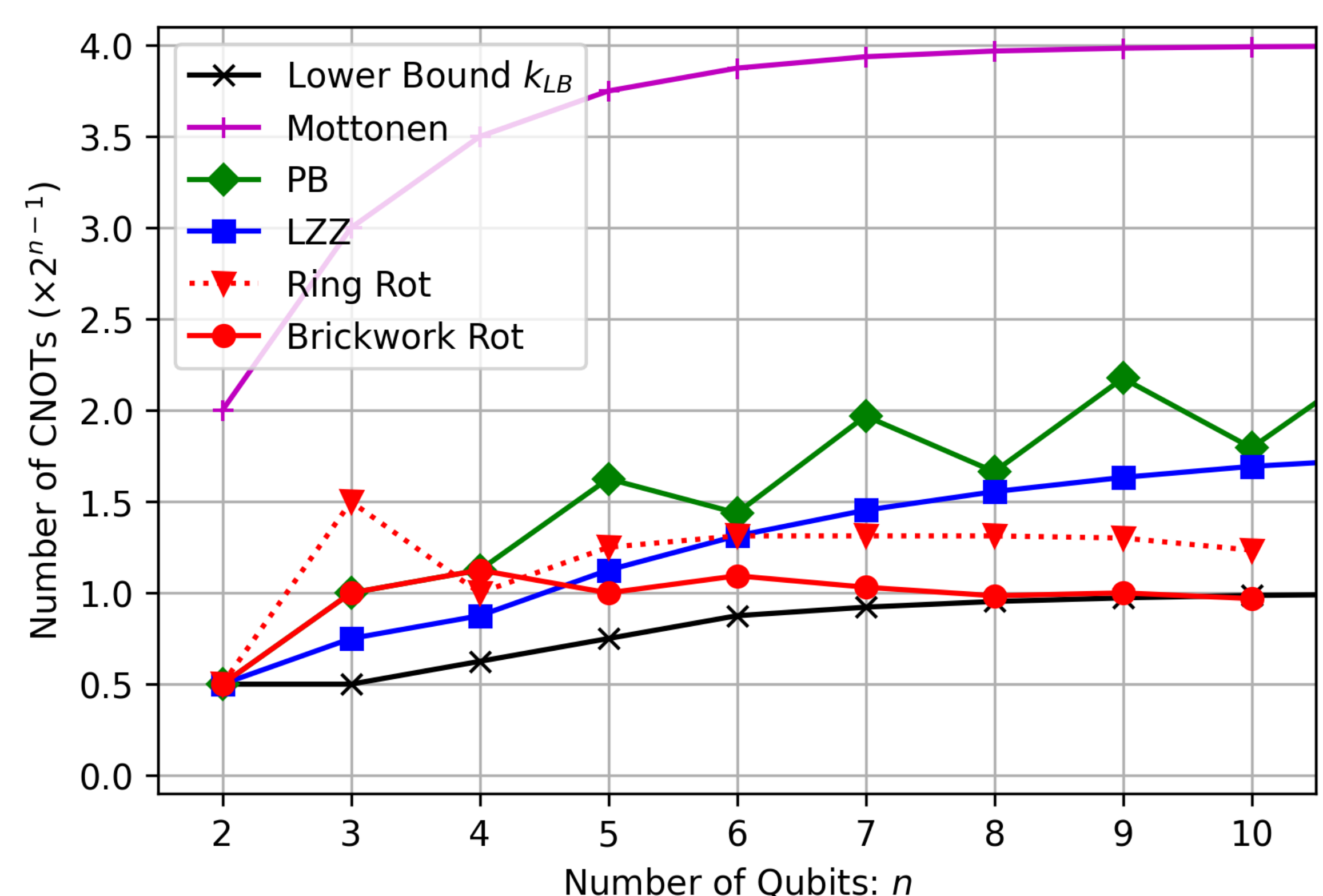
- Sharp transition near $k/k_{LB} \approx 1$.
- Nearly 4-fold reduction over Möttönen.
- Approximately 2-fold reduction over PB/LZZ.



- Brickwork consistently achieves highest fidelity.
- 2-param YZ rotation degrades chain/ring, but not for brickwork.

Scaling Across Qubits

- CNOT cost to meet $F \geq 1 - 10^{-4}$ approaches near-lower-bound.



Summary

- Approximate state preparation exhibits an empirical transition near the degree-of-freedom lower bound.
- Brickwork is the most CNOT-efficient topology among common hardware-efficient circuits.
- Two-parameter local rotations are sufficient for brickwork circuits.
- Approximate variational state preparation can nearly attain the minimum entangling complexity.