MITSUBISHI ELECTRIC RESEARCH LABORATORIES
https://www.merl.com

Robust Controllable Set Computation using Constrained Convex Generators

Silvestre, Daniel; Vinod, Abraham P.

TR2026-063 May 28, 2026

Abstract

Robust Controllable (RC) sets enable safe control of dynamical systems under constraints and uncertainty. Existing approaches typically rely on polytopic representations for the computation of these sets, which suffer from conser- vativeness and scalability issues. Recently, Constrained Convex Generators (CCGs) were proposed to allow set-based control and analysis in presence of both ellipsoidal and polytopic state- input constraints. However, the computation of RC sets using CCGs is currently hindered because the Pontryagin difference set operation has not been developed for CCGs. In this paper, we provide theory and algorithms to address this challenge, and enable safe control under uncertainty using RC sets and CCGs. Specifically, we propose an inner approximation for the RC set using a CCG description. We show in simulations that the proposed approach improves accuracy and memory usage when compared to computations with polytopic approximations.

American Control Conference (ACC) 2026

Robust Controllable Set Computation using Constrained Convex Generators

Daniel Silvestre* and Abraham P. Vinod

***Abstract*— Robust Controllable (RC) sets enable safe control of dynamical systems under constraints and uncertainty. Existing approaches typically rely on polytopic representations for the computation of these sets, which suffer from conservativeness and scalability issues. Recently, Constrained Convex Generators (CCGs) were proposed to allow set-based control and analysis in presence of both ellipsoidal and polytopic state-input constraints. However, the computation of RC sets using CCGs is currently hindered because the Pontryagin difference set operation has not been developed for CCGs. In this paper, we provide theory and algorithms to address this challenge, and enable safe control under uncertainty using RC sets and CCGs. Specifically, we propose an inner approximation for the RC set using a CCG description. We show in simulations that the proposed approach improves accuracy and memory usage when compared to computations with polytopic approximations.**

***Index Terms*— Robust Controllable sets; Pontryagin Difference; Constrained Convex Generators (CCGs).**

I. Introduction

Robust controllable (RC) sets represent the set of states from which a controlled state trajectory can satisfy a collection of possibly time-varying state constraints, despite bounded control authority and uncertainty. These sets are critical for robust model predictive control [1]–[4], fault-tolerant control [5]–[8], and verification [9]–[12]. RC sets have also been applied across various domains, including space [13], transportation [14], and robotics [15], [16].

RC sets are known to be convex and compact for discrete-time linear systems with additive uncertainty and convex and compact state and input constraints. For polytopic constraints, RC sets are polytopes and can be computed using polytope-based set operations. However, the method involves projections, which are computationally expensive and prone to numerical issues for high-dimensional systems or long horizons [3], [16]–[18]. Additionally, these approaches may suffer from additional conservativeness, when state and input constraints involve *nonlinear* convex functions, like Euclidean norm constraints appearing in motion planning problems under conic constraints [10], [19]. Tractable approaches to compute a minimal Robust Positively Invariant (RPI) sets for linear closed-loop system using *constrained convex generator (CCG)* set have been proposed in [20]. However, they may be conservative for safe control design as they represent the minimum set satisfying invariance and require a fixed control law. This paper proposes *novel theory and algorithms for efficiently computing an inner-approximation of RC sets using CCGs.*

Constrained Convex Generators (CCG) is a recently introduced representation for a wide class of convex sets [21]. This representation extends *constrained zonotopes* [5], [6], [9], [22] to include general ℓ_p-norm constraints beyond the ∞-norm in the latent space. The resulting representation can accurately represent polytopes, ellipsoids, and their intersections, and have closed-form expressions for several set operations including affine transformations, Minkowski sums, convex hull of union of sets, and intersections [23]. Similarly to constrained zonotopes [5], CCGs provide a closed-form, exact implementation of the set recursion required for the *controllable set* in the disturbance-free setting. However, the exact computation of RC sets using CCG is limited by the lack of efficient methods to compute the Pontryagin difference when the minuend is a CCG.

Building on our recent work on inner-approximations of the Pontryagin difference for constrained zonotopes [13], we propose *least-squares-based algorithms to compute CCGs that inner-approximate the Pontryagin difference between a CCG minuend and a symmetric, convex, and compact subtrahend*. To facilitate the use of the result in [13], we design a systematic approach to obtain a constrained zonotopic inner-approximation to a CCG. These algorithms enable fast and scalable computation of RC sets for additive uncertainty sets that are symmetric, convex, and compact.

The main contributions of this paper are: 1) a tractable inner-approximation of the CCG with a constrained zonotope, 2) a tractable inner-approximation of the Pontryagin difference between a CCG minuend and a symmetric, convex, and compact subtrahend, and 3) an inner-approximation of RC sets using the proposed Pontryagin difference approximations.

II. Preliminaries & Problem Statement

Let $\mathcal{B}_{p,m}$ be a m-dimensioned unit ball in ℓ_p-norm for $p \in \{1, 2, \infty\}$, and $\|\cdot\|_p$ is the ℓ_p-norm of a vector. We denote by $0_{n\times m}$ and $1_{n\times m}$ the matrices of zeros and ones in $\mathbb{R}^{n\times m}$, respectively, I_n to be the n-dimensional identity matrix, $\mathbb{N}_{[a:b]}$ is the subset of natural numbers between (and including) $a, b \in \mathbb{N}$, $a \leq b$, the Cartesian product $\bigtimes_{i\in\mathbb{N}_{[1:p]}} \mathcal{X}_i = \mathcal{X}_1 \times \cdots \times \mathcal{X}_p$, and e_i is the standard axis vectors of $\mathbb{R}^n$. Let $M \in \mathbb{R}^{m\times n}$ be a matrix with full row rank, then, $M^\dagger = M^\top (MM^\top)^{-1}$ denotes its (right) pseudoinverse, and $x = M^\dagger v$ solves the system of linear equation $Mx = v$ for any vector $v \in \mathbb{R}^m$.

*Corresponding author

D. Silvestre is with School of Science and Technology from the NOVA University of Lisbon (NOVA Lisbon), 2829-516 Caparica, Portugal. He is also affiliated with the Institute for Systems and Robotics (ISR), under Laboratory for Robotics and Engineering Systems (LARSyS), and with the CTS - Centro de Tecnologia e Sistemas part of LASI. dsilvestre@fct.unl.pt

A. P. Vinod is with Mitsubishi Electric Research Laboratories, Cambridge, MA 02139, USA. Email: abraham.p.vinod@ieee.org.

Additionally, there exists a matrix $F \in \mathbb{R}^{n\times(n-m)}$ such that $x = M^\dagger v + Fz$ for any $z \in \mathbb{R}^{n-m}$ characterizes the entire solution set of the system $Mx = v$, and F may be obtained via QR decomposition [24, App. C.5]. Given a sequence of matrices $M_1, \cdots, M_n$, $\mathrm{blkdiag}(M_1, \cdots, M_n)$ denotes the block diagonal matrix obtained with the matrices $M_1, \cdots, M_n$. A set $\mathcal{S} \subset \mathbb{R}^n$ is said to be *symmetric* about any $c \in \mathbb{R}^n$, if $c + x \in \mathcal{S}$ implies $c - x \in \mathcal{S}$ for any $x \in \mathbb{R}^n$.

A. Set representations

Let $\mathcal{C}$ be a convex and compact polytope in $\mathbb{R}^n$. We consider two representations of $\mathcal{C}$ — *H-Rep polytope* (1a) and *constrained zonotope* (CZ) (1b),

$$\mathcal{C} = \{x \mid H_C x \leq k_C\}, \tag{1a}$$

$$\mathcal{C} = \{G_C\xi + c_C \mid A_C\xi = b_C,\ \xi \in \mathcal{B}_\infty\}, \tag{1b}$$

with $H_C \in \mathbb{R}^{L_C\times n}$, $k_C \in \mathbb{R}^{L_C}$, $G_C \in \mathbb{R}^{n\times N_C}$, $c_C \in \mathbb{R}^n$, $A_C \in \mathbb{R}^{M_C\times N_C}$, $b_C \in \mathbb{R}^{M_C}$, and $\mathcal{B}_\infty$ is a unit hypercube in $\mathbb{R}^{N_C}$. We use the shorthand notation $\mathcal{C} = (G_C, c_C, A_C, b_C)$ to denote a polytope $\mathcal{C}$ in CZ representation (1b) Other representations of $\mathcal{C}$, apart from (1), include vertex representation (V-Rep polytope) [3] and AH-polytopes [25]. See [5], [13] for more details, including the equivalence of the representations in (1).

A broad class of convex and compact sets that are symmetric about a point $c_S \in \mathbb{R}^n$ may be cast as affine transformations of unit norm balls $\mathcal{B}_p$ with $p \geq 1$, i.e.,

$$\mathcal{S} = G_S\mathcal{B}_p + c_S, \tag{2}$$

for appropriately defined $G_S \in \mathbb{R}^{n\times N_S}$ and $p \in \{1, 2, \infty\}$. Notably, (2) characterizes zonotopes $\mathcal{Z}$ and ellipsoids $\mathcal{E}$ for $p = \infty$ and $p = 2$ respectively,

$$\mathcal{Z} \triangleq G_Z\mathcal{B}_\infty + c_Z, \qquad \mathcal{E} \triangleq G_E\mathcal{B}_2 + c_E. \tag{3}$$

By definition, $\mathcal{Z}, \mathcal{E}$ are symmetric about c_Z, c_E. We denote sets in the form of (2) using (G, c) where p is made clear from context. We denote their corresponding *dual norm* with p^*, which satisfies $1/p + 1/p^* = 1$ [26, Sec. A.1.6].

Recently, *constrained convex generator* (CCG) was proposed in [21] that generalizes the above set representations. Specifically, we define a CCG $\mathcal{G}$ as follows,

$$\mathcal{G} = \{G_G\xi + c_G \mid A_G\xi = b_G, \xi \in \mathfrak{C}_1 \times \cdots \times \mathfrak{C}_p\} \subset \mathbb{R}^n, \tag{4}$$

$$= \left\{G_G\xi + c_G \;\middle|\; A_G\xi = b_G, \Pi_j\xi \in \mathfrak{C}_j, \forall j \in \mathbb{N}_{[1:p]}\right\}, \tag{5}$$

with the *latent space* $\mathfrak{C}_1 \times \cdots \times \mathfrak{C}_p \subseteq \mathbb{R}^{N_G}$ is the Cartesian product of p sets $\mathfrak{C} \in \{\mathcal{B}_p\}_{p\geq 1}$ for any p index of a norm ball and Π_j are appropriately defined matrices for the necessary projections. Clearly, (4) subsumes representations (1b) (with $\mathfrak{C} = \mathcal{B}_\infty$) and (2) and (3) (with $\mathfrak{C} = \mathcal{B}_p$ with an appropriate p and A_G, b_G omitted).

B. Set operations

For any sets $\mathcal{C}, \mathcal{S} \subseteq \mathbb{R}^n$ and $\mathcal{W} \subseteq \mathbb{R}^m$, and a matrix $R \in \mathbb{R}^{m\times n}$, we define the set operations (affine map, Minkowski sum $\oplus$, intersection with inverse affine map $\cap_R$, and Pontryagin difference $\ominus$):

$$R\mathcal{C} \triangleq \{Ru \mid u \in \mathcal{C}\}, \tag{6a}$$

$$\mathcal{C} \oplus \mathcal{S} \triangleq \{u + v \mid u \in \mathcal{C},\ v \in \mathcal{S}\}, \tag{6b}$$

$$\mathcal{C} \cap_R \mathcal{W} \triangleq \{u \in \mathcal{C} \mid Ru \in \mathcal{W}\}, \tag{6c}$$

$$\mathcal{C} \ominus \mathcal{S} \triangleq \{u \mid \forall v \in \mathcal{S}, u + v \in \mathcal{C}\}. \tag{6d}$$

Since $\mathcal{C} \cap \mathcal{S} = \mathcal{C} \cap_{I_n} \mathcal{S}$, (6c) also includes the standard intersection. For any $x \in \mathbb{R}^n$, we use $\mathcal{C} + x$ and $\mathcal{C} - x$ to denote $\mathcal{C} \oplus \{x\}$ and $\mathcal{C} \oplus \{-x\}$ respectively for brevity. All of the set operations (6) preserve convexity and compactness of the sets involved [3], and several computational tools exist for performing these set operations for polytopes, ellipsoids, and constrained zonotopes [27], [28]. However, the computational complexity and accuracy of these operations depend on the set representation used, and existing approaches may not accommodate nonlinear convex constraints without introducing additional complexity and approximations.

From [21], CCGs alleviate these limitations, and admit closed-form expressions for most of the operations in (6).

$$\begin{aligned}
R\mathcal{C} &= (RG_C, Rc_C, A_C, b_C, \xi_C),\\
\mathcal{C} \oplus \mathcal{S} &= \left(\begin{bmatrix} G_C & G_S\end{bmatrix}, c_C + c_S, \begin{bmatrix} A_C & 0\\ 0 & A_S\end{bmatrix}, \begin{bmatrix} b_C\\ b_S\end{bmatrix}, \xi_C \times \xi_S\right),\\
\mathcal{C} \cap_R \mathcal{W} &= \left(\begin{bmatrix} G_C & 0\end{bmatrix}, c_C, \begin{bmatrix} A_C & 0\\ 0 & A_W\\ RG_C & -G_W\end{bmatrix}, \begin{bmatrix} b_C\\ b_W\\ c_W - Rc_C\end{bmatrix}, \xi_C \times \xi_W\right),\\
\mathcal{C} \cap \mathcal{H} &= \left(\begin{bmatrix} G_C & 0\end{bmatrix}, c_C, \begin{bmatrix} A_C & 0\\ p^\top G_C & d_m/2\end{bmatrix}, \begin{bmatrix} b_C\\ \frac{q + p^\top c_C - \|p^\top G_C\|_1}{2}\end{bmatrix}, \xi_C \times \mathcal{B}_\infty\right),
\end{aligned} \tag{7}$$

where $\mathcal{H} = \{x \mid p^\top x \leq q\} \subset \mathbb{R}^n$ is an halfspace.

To the best of our knowledge, the Pontryagin difference (6d) involving a CCG minuend does not have a closed-form expression, similar to (7) [6], [9], [13]. In fact, given a constrained zonotope $\mathcal{C}$ (a special type of CCG) and a zonotope $\mathcal{Z}$, it is impossible to find a polynomial-size constrained zonotope $\mathcal{C} \ominus \mathcal{Z}$ in polynomial-time, unless P=NP [9, Prop. 1]. On the other hand, a tractable inner-approximation of the Pontryagin difference between a constrained zonotopic minuend and a convex, compact, symmetric subtrahend (2) was recently proposed in [13].

Lemma 1: (PONTRYAGIN DIFFERENCE WITH A CONSTRAINED ZONOTOPE MINUEND [13, CORR. 1]) Given a full-dimensional, constrained zonotopic minuend $\mathcal{C} \subset \mathbb{R}^n$ and a subtrahend $\mathcal{S} = G_S\mathcal{B}_p + c_S \subset \mathbb{R}^n$ for $p \geq 1$, define a constrained zonotope $\mathcal{M}^- = (G_{M^-}, c_{M^-}, A_{M^-}, b_{M^-})$ with

$$G_{M^-} = G_C D, \qquad c_{M^-} = c_C - c_S, \tag{8a}$$

$$A_{M^-} = A_C D, \qquad b_{M^-} = b_C, \tag{8b}$$

and $D \in \mathbb{R}^{N_C\times N_C}$ is a diagonal matrix with

$$D_{ii} = 1 - \left\| e_i^\top [G_C; A_C]^\dagger [G_S; 0_{M_C\times N_S}] \right\|_{p^*}. \tag{9}$$

defined for every $i \in \mathbb{N}_{[1:N_C]}$. Then, we have $\mathcal{M}^- \subseteq \mathcal{C} \ominus \mathcal{S}$, provided $D_{ii} \geq 0$ for every $i \in \mathbb{N}_{[1:N_C]}$.

A key focus of this work is to leverage Lemma 1 to compute Pontryagin difference involving CCG. Specifically, we propose a tractable algorithm to inner-approximate $\mathcal{G} \ominus \mathcal{S}$ by addressing the following problem.

Problem 1: For any CCG $\mathcal{G}$, design an algorithm to compute a constrained zonotope $\mathcal{C}^- \subseteq \mathcal{G}$.

With a tractable solution to Problem 1, we can use Lemma 1 to compute the Pontryagin difference of a CCG with a symmetric, convex, and compact set. Additionally, we show that the proposed solution to Problem 1 casts the CCG representation with homogenous linear constraints in the latent space (renders b_G in (4) to zero), that may be of independent interest.

C. Robust controllable set

Consider a linear time-varying system,

$$x_{t+1} = A_t x_t + B_t u_t + F_t w_t, \tag{10}$$

with state $x_t \in \mathbb{R}^n$, input $u_t \in \mathcal{U}_t \subset \mathbb{R}^m$, disturbance $w_t \in \mathcal{W}_t \subset \mathbb{R}^p$, and appropriately defined time-varying matrices A_t, B_t, and F_t.

Definition 1: (T-STEP RC SET) [3, Defn. 10.18] Given (10), a set of (possibly time-varying) state constraints $\{\mathcal{X}_t\}_{t=0}^{T-1}$ with $\mathcal{X}_t \subseteq \mathbb{R}^n$ for each t, and a goal set $\mathcal{G} \subset \mathbb{R}^n$, the T-step robust controllable (RC) set is

$$\mathcal{K} = \left\{ x_0 \in \mathcal{X}_0 \;\middle|\; \begin{array}{c} \forall t \in \mathbb{N}_{[0:T-1]},\ \exists u_t \in \mathcal{U}_t,\ \forall w_t \in \mathcal{W}_t, \\ x_{t+1} = A_t x_t + B_t u_t + F_t w_t, \\ x_t \in \mathcal{X}_t,\ x_T \in \mathcal{G} \end{array} \right\}.$$

Informally, the T-step RC set $\mathcal{K} \subset \mathbb{R}^n$ is the set of initial states that, despite the additive disturbance $w_t \in \mathcal{W}_t$, can be steered using $u_t \in \mathcal{U}_t$ to reach the goal set $\mathcal{G}$ at time step T, while staying within the state constraints $\mathcal{X}_t$ at all intermediate time steps $t \in \mathbb{N}_{[0:T-1]}$. The T-step RC set is also known as robust reachability of target tube [4], backward reachable set [9], or backward reach-avoid set [10]. The following set recursion yields $\mathcal{K} = \mathcal{K}_0$,

$$\mathcal{K}_t = \mathrm{Pre}(\mathcal{K}_{t+1}) \cap \mathcal{X}_t, \qquad \forall t \in \mathbb{N}_{[0:T-1]}, \tag{11a}$$

$$\mathrm{Pre}(\mathcal{K}_{t+1}) \triangleq \{x \mid A_t x \in (\mathcal{K}_{t+1} \ominus F_t \mathcal{W}_t) \oplus (-B_t \mathcal{U}_t)\}, \tag{11b}$$

with $\mathcal{K}_T \triangleq \mathcal{G}$. Note that (11) uses all set operations in (6).

For H-Rep/V-Rep polytopes $\mathcal{X}, \mathcal{G}, \mathcal{U}$, and $\mathcal{W}$, we can compute RC sets using existing results for polytopic set operations [27], [28]. However, when using H-Rep/V-Rep polytopes for high-dimensional systems (10) or over long horizons T, we face numerical challenges when implementing (11), since it requires a combination of Minkowski sum and intersection operations. See [3], [16]–[18] for a detailed discussion. On the other hand, constrained zonotopes can compute RC sets tractably, and the computed sets are exact when $\mathcal{W} = \emptyset$ and inner-approximation otherwise [13], [27]. However, both approaches require that the sets involved $\mathcal{U}, \mathcal{X}, \mathcal{G}$ are polytopic. In several practical applications, one may need to accommodate convex, non-polytopic constraint sets $\mathcal{U}, \mathcal{X}, \mathcal{G}$. For example, when $\mathcal{U}$ represents control input with bounded energy, $\mathcal{U}$ is an ellipsoid. While such sets may be approximated as polytopes via ray-shooting, the inaccuracies may compound over the set recursion (11) (see Section IV). For such applications, CCG are a natural choice of representation due to their expressivity.

Problem 2: Compute a CCG $\mathcal{K}^-$ that inner-approximates $\mathcal{K}$ asin Defn. 1, where for every $t \in \mathbb{N}_{[0:T-1]}$, $\mathcal{X}_t, \mathcal{U}_t$, and $\mathcal{G}$ are CCG, and $\mathcal{W}_t$ are symmetric, convex, and compact sets.

III. PONTRYAGIN DIFFERENCE FOR CONSTRAINED CONVEX GENERATORS (CCGS)

In this section, we first show that a naive application of the equivalence of norms may be severely conservative. We then address Problem 1 via tractable inner-approximations of a CCG using a CZ. Finally, we conclude with a discussion on how the proposed approaches may be adapted to address Problem 2.

A. An example demonstrating a naive application of the equivalence of norms may be severely conservative

Consider the case of a CCG $\mathcal{X} \in \mathbb{R}^2$,

$$\mathcal{X} = (G_X, 0_2, A_X, 1_{20}, \mathcal{B}_{2,m_1} \times \mathcal{B}_{2,m_2} \times \mathcal{B}_{\infty,m_3}). \tag{12}$$

for some appropriate G_X, A_X and $\mathcal{B}_{p,m}$ denotes a m-dimensional unit ball in the ℓ_p norm. A naive inner-approximation of $\mathcal{X}$ may be obtained by inner-approximating $\mathcal{B}_{2,m_1} \times \mathcal{B}_{2,m_2} \times \mathcal{B}_{\infty,m_3}$ via the equivalence of norms. Recall that for any $x \in \mathbb{R}^n$,

$$\|x\|_\infty \leq \|x\|_2 \leq \|x\|_1 \leq \sqrt{n}\|x\|_2 \leq n\|x\|_\infty. \tag{13}$$

Consequently, we can define matrix $T = \mathrm{blkdiag}\left(\frac{1}{\sqrt{m_1}} I_{m_1}, \frac{1}{\sqrt{m_2}} I_{m_2}, I_{m_3}\right)$ to obtain a constrained zonotopic, inner-approximation of $\mathcal{X}$,

$$\mathcal{X}_{\mathrm{cz}} = (G_X T, 0_2, A_X T, 1_{20}) \subseteq \mathcal{X}. \tag{14}$$

As a concrete example of why (14) may be too conservative in practice, we consider an example illustrated in Figure 1 with $m_1 = 11, m_2 = 7, m_3 = 32$, and G_X, A_X generated randomly. Additionally, we consider a CCG $\mathcal{W}$ similar to $\mathcal{X}$ with an additional constraint parameterized by $\alpha \in \mathbb{R}, \alpha \geq 0$ in the latent space, i.e.,

$$\mathcal{W}(\alpha) = \left(G_X, 0_2, \begin{bmatrix} & A_X & \\ 1 & 1 & 0_{1\times(m_2+m_3)} \end{bmatrix}, \begin{bmatrix} 1_{20} \\ \frac{2}{\sqrt{m_1}} + \alpha \end{bmatrix}, \mathcal{B}_{2,m_1} \times \mathcal{B}_{2,m_2} \times \mathcal{B}_{\infty,m_3}\right). \tag{15}$$

As seen from Figure 1, $\mathcal{X}, \mathcal{W}(0), \mathcal{W}(0.1)$ are quite similar, and $\mathcal{X}_{\mathrm{CZ}}, \mathcal{W}_{\mathrm{CZ}}(0)$ provide valid inner-approximations. However, $\mathcal{W}_{\mathrm{CZ}}(\alpha)$ is empty for $\alpha > 0$, since the α-parameterized hyperplane in the latent dimensions of $\mathcal{W}$ is tangent to the latent set constraints on ξ_1, ξ_2 at $\alpha = 0$,

$$\begin{bmatrix} 1 & 1 & 0_{1\times m-2} \end{bmatrix} T\xi = \frac{2}{\sqrt{m_1}} + \alpha \tag{16}$$

$$\iff \frac{\xi_1}{\sqrt{m_1}} + \frac{\xi_2}{\sqrt{m_1}} = \frac{2}{\sqrt{m_1}} + \alpha \iff \xi_1 + \xi_2 = 2 + \alpha. \tag{17}$$

Since ξ (after affine transformation with T) must lie in $\mathcal{B}_\infty$ for $\mathcal{W}_{\mathrm{CZ}}(\alpha)$, we conclude $\xi_1 = \xi_2 = 1$ when $\alpha = 0$ and $\mathcal{W}_{\mathrm{CZ}}(\alpha) = \emptyset$ for $\alpha > 0$. Such constraints arise naturally in practice (e.g., for the double integrator dynamics

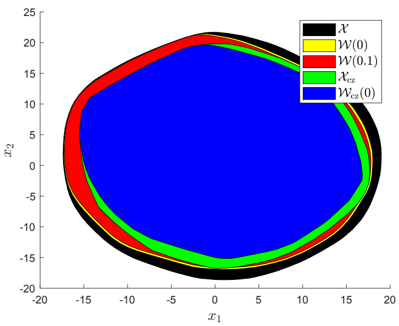

Fig. 1: Depiction of a random CCG $\mathcal{X}$ and the counterexample using a modified $\mathcal{W}(0)$ and $\mathcal{W}(0.1)$ with the corresponding CZ approximations.

considered in Section IV, this would happen in the second iteration of the set recursion (11)), we need a more robust inner-approximation method that is not sensitive to such constraints.

B. Inner-approximation of CCGs using CZs

We now present our main result that addresses Problem 1. The proposed approach is based on the idea of first finding a representation of the CCG such that the affine constraints in the latent space are homogeneous (i.e., b_G is zero), which means that any subsequent approximation of the generator sets using the equivalence of norms as discussed in Section III-A will not result in the empty set. The pseudo-code for such an approach is provided in Algorithm 1. We use $\mathrm{qr\_decomposition}(\cdot)$ to denote a function that returns the pseudo-inverse of the argument and a matrix spanning the orthogonal space (see [26, Appendix C.5] for this relationship) and use $\mathrm{box}(A, b)$ to denote a function that returns the box (i.e., the hyperrectangle) that overbounds the polytope $\{x : Ax \leq b\}$.

Theorem 1 (Inner-approximation for CCGs): Consider a CCG $\mathcal{X} = (G, c, A, b, \{\mathfrak{C}_1, \cdots, \mathfrak{C}_p\})$ with p generator sets with sizes $m_1, \cdots, m_p$ and n_c constraints. Without loss of generality, let $\mathfrak{C}_1 = \mathcal{B}_\infty$. Assume that $m_1 > n_c$. Then, CCG $\mathcal{X}^-$ produced by Algorithm 1 satisfies $\mathcal{X}^- \subseteq \mathcal{X}$. Additionally, a sufficient condition for non-empty $\mathcal{X}^-$ is $c_i = \|\Pi_i A^\dagger b\|_{\ell_i} \leq 1, \quad i \in \mathbb{N}_{[1:p]}$.

Proof: The CCG $\mathcal{X}$ corresponds to the definition

$$\mathcal{X} = \left\{G\xi + c \mid A\xi = b, \xi \in \mathfrak{C}_1 \times \cdots \times \mathfrak{C}_p\right\}, \tag{18}$$
$$= \left\{G\xi + c \mid \xi = A^\dagger b + Fz, \xi \in \mathfrak{C}_1 \times \cdots \times \mathfrak{C}_p\right\}, \tag{19}$$

where (19) was obtained from the well-known parameterized solution of under-constrained system of linear equations using QR decomposition (see [26, Appendix C.5]). By substituting the expression of ξ,

$$\mathcal{X} = \left\{G(A^\dagger b + Fz) + c \;\middle|\; \|\Pi_i(A^\dagger b + Fz)\|_{\ell_i} \leq 1,\ i \in \mathbb{N}_{[1:p]}\right\}.$$

Using triangle inequality and definitions of c', $c_1, \cdots, c_p$ in lines 3 and 13 of Algorithm 1,

$$\mathcal{X} \supseteq \mathcal{X}^- \triangleq \left\{GFz + c' \mid \|\Pi_i Fz\|_{\ell_i} \leq 1 - c_i,\ i \in \mathbb{N}_{[1:p]}\right\}. \tag{20}$$

Algorithm 1 CCG inner-approximation of $\mathcal{X}$

Input: CCG $\mathcal{X} = (G, c, A, b, \mathfrak{C}_1 \times \cdots \times \mathfrak{C}_p)$ with p generator sets with sizes $m_1, \cdots, m_p$ and n_c constraints.
Output: CCG $\mathcal{X}^- = (G^-, c', A^-, 0, \mathfrak{C}_2 \times \cdots \times \mathfrak{C}_p \times \mathfrak{C}_1)$ such that $\mathcal{X}^- \subseteq \mathcal{X}$

1: $\begin{bmatrix} A^\dagger & F \end{bmatrix} = \mathrm{qr\_decomposition(A)}$
2: **for** $i = 1$ to p **do**
3: $\quad c_i = \|\Pi_i A^\dagger b\|_{\ell_i}$
4: **end for**
5: $P = \mathrm{blkdiag}(\Pi_2, \cdots, \Pi_p)F$
6: $\begin{bmatrix} P^\dagger & Q \end{bmatrix} = \mathrm{qr\_decomposition(P)}$
7: $\Lambda = \mathrm{blkdiag}\left((1 - c_2)I_{m_2}, \cdots, (1 - c_p)I_{m_p}\right)$
8: $H = \begin{bmatrix} \Pi_1 F P^\dagger \Lambda & \Pi_1 F Q \\ -\Pi_1 F P^\dagger \Lambda & -\Pi_1 F Q \end{bmatrix}$
9: $\tilde{Z} = \mathrm{box}(H, (1 - c_1)1)$
10: $\tilde{Z}_{1:(m-m_1),1:(m-m_1)} = I$
11: $G^- = \begin{bmatrix} GF \begin{bmatrix} P^\dagger \Lambda & Q \end{bmatrix} \tilde{Z} & 0 \end{bmatrix}$
12: $c' = GA^\dagger b + c$
13: $A^- = \begin{bmatrix} H\tilde{Z} & -(1 - c_1)I \end{bmatrix}$
14: **return** $\mathcal{X}^- = (G^-, c', A^-, 0, \mathfrak{C}_2 \times \cdots \times \mathfrak{C}_p \times \mathfrak{C}_1)$

Given that ℓ_1 is the only ∞-norm ball (assumed without loss of any generality), we can create the block diagonal matrix,

$$P = \mathrm{blkdiag}(\Pi_2, \cdots, \Pi_p) \in \mathbb{R}^{(\sum_{i=2}^p m_i) \times (m - n_c)} F. \tag{21}$$

Since we have assumed that $m_1 > n_c \iff (m - n_c) > (\sum_{i=2}^p m_i)$, the system of linear equations $Pz = w = \begin{bmatrix} w_2^\intercal & \cdots & w_p^\intercal \end{bmatrix}^\intercal$ is also under-constrained. Consequently, we set up a change of variables from z to w using the QR decomposition once again. Specifically, we use $z = P^\dagger w + Q\phi$ with an auxiliary variable ϕ, and we obtain,

$$\mathcal{X}^- = \left\{GF(P^\dagger w + Q\phi) + c' \;\middle|\; \begin{array}{c} \|\Pi_i F(P^\dagger w + Q\phi)\|_{\ell_i} \leq 1 - c_i, \\ i \in \mathbb{N}_{[1:p]} \end{array}\right\},$$

By construction, the change of variables also satisfies,

$$\Pi_i F P^\dagger w = w_i, \quad \Pi_i F Q = 0,$$

for every $i \in \mathbb{N}_{[1:p]}$. Consequently, we have

$$\mathcal{X}^- = \left\{GF(P^\dagger w + Q\phi) + c' \;\middle|\; \begin{array}{c} \|\Pi_1 F(P^\dagger w + Q\phi)\|_\infty \leq 1 - c_1, \\ \|w_i\|_{\ell_i} \leq 1 - c_i, i \in \mathbb{N}_{[2:p]} \end{array}\right\}.$$

Next, we use matrix Λ as in line 7 of Algorithm 1 to redefine $w \to \Lambda w$, and obtain

$$\mathcal{X}^- = \left\{GF(P^\dagger \Lambda w + Q\phi) + c' \;\middle|\; \begin{array}{c} \|\Pi_1 F(P^\dagger \Lambda w + Q\phi)\|_\infty \leq 1 - c_1, \\ \|w_i\|_{\ell_i} \leq 1, i \in \mathbb{N}_{[2:p]} \end{array}\right\}. \tag{22}$$

Since $\mathcal{S}_\infty \triangleq \{[w, \phi] \mid \|\Pi_1 F(P^\dagger \Lambda w + Q\phi)\|_\infty \leq 1 - c_1\}$ may be represented in H-rep as $H\begin{bmatrix} w \\ \phi \end{bmatrix} \leq (1 - c_1)$ with an appropriately defined H (see Line 8 of Algorithm 1), we can use [5, Thm. 1] to express $\mathcal{S}_\infty$ as a constrained zonotope,

$$\mathcal{S}_\infty = \left(\begin{bmatrix} G_0 & 0 \end{bmatrix}, 0, \begin{bmatrix} HG_0 & -(1 - c_1)I \end{bmatrix}, 0\right). \tag{23}$$

Here, G_0 is a generator matrix of a zonotope containing the set $\mathcal{S}_\infty$ as required by [5, Thm. 1] (see Line 9 of

Algorithm 2 CZ inner-approximation to CCG $\mathcal{G}$

Input: CCG $\mathcal{G} = (G, c, A, b, \mathfrak{C}_1 \times \cdots \times \mathfrak{C}_p)$ with p generator sets with sizes $m_1, \cdots, m_p$ and n_c constraints.

Output: CZ $\mathcal{Z} = (G_z, c_z, A_z, 0)$ such that $\mathcal{Z} \subseteq \mathcal{G}$

$\mathcal{G}_h = \left(G_h, c_h, A_h, 0, \bigtimes_{i\in\mathbb{N}_{[2:p]}} \mathcal{B}_{\ell_i,s_i} \times \mathcal{B}_\infty, s_1\right)$ with generator sizes $s_i, i \in \mathbb{N}_{[1:p]}$ as in Algorithm 1

$T = \begin{bmatrix} \text{blkdiag}\left(\frac{1}{\ell_2\sqrt{s_2}} I_{s_2}, \ldots, \frac{1}{\ell_p\sqrt{s_p}} I_{s_p}\right) & 0 \\ 0 & I_{s_1} \end{bmatrix}$,

return $\mathcal{Z} = (G_h T, c_h, A_h T, 0)$

Algorithm 1). Denoting the latent space variables of $\mathcal{S}_\infty$ in (23) by φ and substituting (23) in (22), we obtain

$$\mathcal{X}^- = \left\{ GF \begin{bmatrix} P^\dagger\Lambda & Q \end{bmatrix} \begin{bmatrix} w \\ \phi \end{bmatrix} + c' \;\middle|\; \begin{array}{c} \begin{bmatrix} w \\ \phi \end{bmatrix} = \begin{bmatrix} G_0 & 0 \end{bmatrix} \varphi, \\ \begin{bmatrix} HG_0 & -(1-c_1)I \end{bmatrix} \varphi = 0, \\ \|\varphi\|_\infty \leq 1, \|w_i\|_{\ell_i} \leq 1, i \in \mathbb{N}_{[2:p]} \end{array} \right\}.$$

Finally, substituting for $[w, \;\; \phi]^\top$ and collecting terms to define $\tilde{Z}$ as in line 10 of Algorithm 1, we obtain

$$\mathcal{X}^- = \left\{ \begin{bmatrix} GF \begin{bmatrix} P^\dagger\Lambda & Q \end{bmatrix} \tilde{Z} & 0 \end{bmatrix} \varphi \;\middle|\; \begin{array}{c} \begin{bmatrix} H\tilde{Z} & -(1-c_1)I \end{bmatrix} \varphi = 0, \\ \|\varphi\|_\infty \leq 1, \|w_i\|_{\ell_i} \leq 1, i \in \mathbb{N}_{[2:p]} \end{array} \right\},$$

the CCG $\mathcal{X}^-$ returned by Algorithm 1.

The proof for sufficient condition for non-emptiness of $\mathcal{X}^-$ follows from the structure of (20), i.e., $c' \in \mathcal{X}^-$ when $c_i \leq 1$ for every $i \in \mathbb{N}_{[1:p]}$ (which corresponds to $z = 0$). ■

Remark 2: In the statement of Theorem 1, we forced $m_1 > n_c$ to ensure that $z = Pw$ with P in (21) is under-constrained. One alternative could have been to introduce additional approximation by converting other norms into ∞-norm in order to satisfy this assumption, resulting in further conservativeness. Empirically, we observed the assumption of $m_1 > n_c$ holds almost always, especially when most of the sets involved in the set operations are polytopes resulting in more ℓ_∞-norm (see Section IV).

Algorithm 2 combines the discussion in Section III-A and Theorem 1 to obtain a constrained zonotopic inner-approximation of a CCG, and thereby address Problem 1.

C. Inner-approximation of the RC sets using CCGs

Problem 2 can be addressed using the steps outlined in the pseudo-code in Algorithm 3. Since CZs are a special case of CCGs, all the operations in the iteration (11) can be carried out in CCG representation, while approximating the Pontryagin difference using Algorithm 2 and Lemma 1. For the case of a non-singular matrix A_t, the linear map with the inverse reduces the number of generators and constraints from the resulting $\mathcal{K}_t^-$. See [13] for more details.

IV. Numerical simulations

In this section, we illustrate the proposed method for calculating the Pontryagin difference of a CCG with a unit ℓ_∞-norm ball. Additionally, we illustrate the application of the proposed method to calculate the T-step robust control set for a double integrator system in 2D. The simulations

Algorithm 3 RC-sets computation in CCG form

Input: Horizon T, Goal set $\mathcal{G}$, Intermediate safe set $\{\mathcal{X}_t\}_{t=0}^{T-1}$, system dynamics (A_t, B_t, F_t), and sets describing the bounds for actuation $\{\mathcal{U}_t\}_{t=0}^{T-1}$ and disturbance $\{\mathcal{W}_t\}_{t=0}^{T-1}$.

Output: RC-sets $\mathcal{K}_t^-, t \in \mathbb{N}_{[0:T]}$

$\mathcal{K}_T^- = \mathcal{G}$
for $t = N-1$ to 0 **do**
  Get $\mathcal{Z}_{t+1}$ with Algorithm 2 from $\mathcal{K}_{t+1}$
  $\tilde{\mathcal{Z}}_{t+1} = \mathcal{Z}_{t+1} \ominus F_t\mathcal{W}_t$ using Lemma 1
  if A_t is singular **then**
    $\mathcal{K}_t^- = \mathcal{X}_t \cap_{A_t} \left(\tilde{\mathcal{Z}}_{t+1} \oplus (-B_t\mathcal{U}_t)\right)$
  else
    $\mathcal{K}_t^- = \mathcal{X}_t \cap \left[A_t^{-1}\left(\tilde{\mathcal{Z}}_{t+1} \oplus (-B_t\mathcal{U}_t)\right)\right]$
  end if
end for
return $\mathcal{K}_t^-, t \in \mathbb{N}_{[0:T]}$

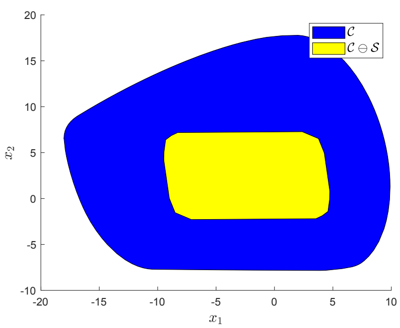

Fig. 2: Example on a randomly selected CCG $\mathcal{C}$ by removing the unit ℓ_∞-norm ball.

were performed in MATLAB R2021b on a laptop with an Intel Core i7-8550U CPU 1.80GHz and 12GB RAM. The set implementations resort to the ReachTool toolbox available at `https://github.com/danielmsilvestre/ReachTool` [29] that uses YALMIP for set plotting.

A. Pontryagin Difference of Randomly Generated Sets

In this subsection, we illustrate the type of sets produced by the proposed algorithms for randomly generated instances of CCGs by uniformly sampling the entries of G and A in the interval $[0, 1]$. The subtrahend was selected to be $\mathcal{B}_\infty$. In Fig. 2, it is depicted the set $\mathcal{C}$ and the calculated inner-approximation for the Pontryagin difference. In order to assess the conservatism of the proposed solution, we illustrate in Fig. 3 the original set $\mathcal{C}$ in comparison with $\mathcal{M} \oplus \mathcal{S}$. The instance for $\mathcal{C}$ was generated with 10 generator variables as we observed very similar outcomes even if the number of generators was doubled while the number of constraints was maintained.

B. T-Step Robust Control Set for a 4D linear system

In this section, we consider a system modeled as a double integrator in 2 dimensions in discrete-time given by:

$$x_{k+1} = Ax_k + Bu_k + Fw_k \tag{24}$$

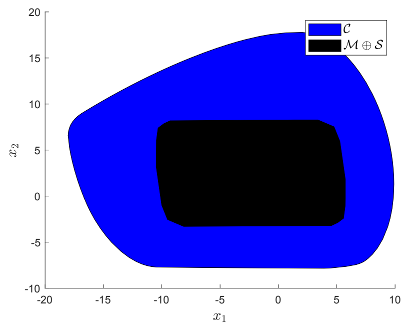

Fig. 3: Comparison between the randomly selected $\mathcal{C}$ and the result of $\mathcal{M} \oplus \mathcal{B}_\infty$.

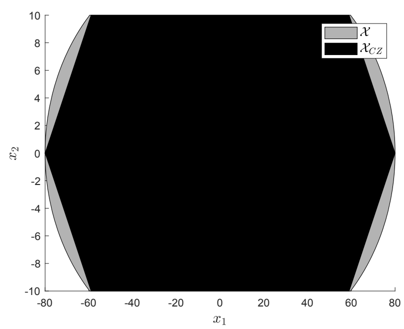

Fig. 4: Mission environment that is encoded in the first two coordinates of the set $\mathcal{X}$. To compare with the results in [13], we approximate CCG $\mathcal{X}$ as a polytope written as a CZ.

where the matrices are given by

$$A = \begin{bmatrix} I_2 & \Delta T I_2 \\ 0 & I_2 \end{bmatrix}, \quad B = \begin{bmatrix} (\Delta T)^2/2 I_2 \\ \Delta T I_2 \end{bmatrix}, \text{ and } F = I_4. \tag{25}$$

For a double integrator, the first two entries of the state are going to be the position of the vehicle and will be constrained by the mission environment and is therefore considered the generic CCG depicted in Fig. 4. As a comparison, we considered a polytopic approximation of the CCG to facilitate an implementation with CZs [13]. See Fig. 4 illustrates an inner approximation resorting to a ray along the x coordinate. Acceleration is assumed 4 m/s whereas the disturbance forces can affect both position and velocity with a maximum magnitude of 0.1. Therefore, both sets $\mathcal{U}$ and $\mathcal{W}$ are ellipsoids as they encode ℓ_2-norm constraints. Given that the Pontryagin difference for CZs is possible for any $\mathcal{S}$ set that is symmetric with respect to a point, the set $\mathcal{W}$ being an ellipsoid does not pose any complication. On the other hand, both the constraint set $\mathcal{X}$ and the input set $\mathcal{U}$ need to be inner-approximated to render a RC set. In this simulation, we resorted to inscribing a hypercube within the ℓ_2 unit ball within the latent space which amounts to replacing $\|\xi\|_2 \leq 1$ by $\|\xi\|_\infty \leq 1$ and multiplying the G matrix by $\frac{1}{\sqrt{n}}$.

The objective of the controller would be to drive the state of the system to the origin of the reference frame with a position error vector with maximum magnitude of 1 meter and a velocity vector with norm at most 20 m/s. This constraints were encoded in the goal set CCG $\mathcal{G}$ and inner

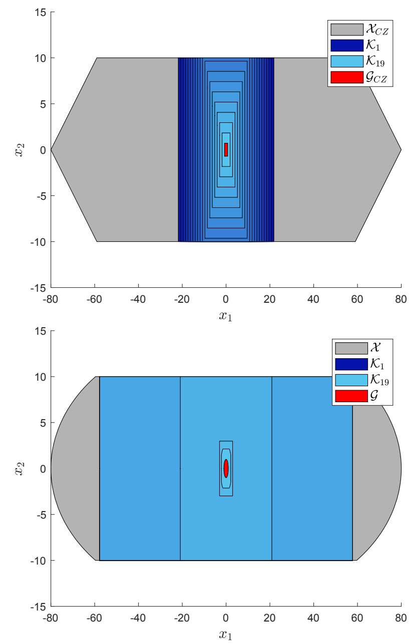

Fig. 5: Evolution of the RC set for 20 discrete time steps for the double integrator example. (Top) using CZs [13] and (Bottom) proposed using CCGs.

	# Generators	# Constraints	# Doubles	Avg. Time [s]
CZ [13]	202	160	33292	0.0013
CCG (this work)	126	84	11186	11.0509

TABLE I: Number of generators, constraints and double variables required to store the representation of $\mathcal{K}_1^-$ and the average computing time for all 20 iterations.

approximated with the ∞-norm for the CZ implementation. In Fig. 5 (Top), it is depicted the evolution of the RC sets projected onto the position coordinates for 20 time steps. It is shown in the figure, the RC sets computed with [13] are quite conservative as the result of the inner approximation required for all bounds involving energy-like constraints defined using the ℓ_2 norm. The same simulation was conducted with Algorithm 1 and produces the RC sets whose projection onto the position coordinates is depicted in Fig. 5 (Bottom). In this case, after the backwards iteration in (11) of 4 iterations, the RC set has reached a steady-state and all remaining $\mathcal{K}_i, 1 \leq i \leq 15$ are overlapped by $\mathcal{K}_{16}$.

From the previous results, the proposed method produces better inner-approximations of the RC sets in comparison with the propagation in CZ format. However, another important consideration is the growth of the data structures associated with the CZ and CCG representation that can prevent the adoption of the technique. A summary of the main metrics is provided in Table I for set $\mathcal{C}_1^-$ as is the one with the highest complexity and it is reported the average computing time to obtain the description. For the case of using CZs, the set $\mathcal{K}_1$ requires 202 generators and 160 constraints whereas using the proposed method in CCG format requires 126 generators and 84 constraints. This means that the CZ approach requires storing 33292 double variables whereas the CCG format only uses 11186 doubles to store all numerical values for the matrices and vectors of each representation. Therefore, the proposed method not only produces more accurate representations of the RC sets but also reduces the memory requirements to store the data representation. Even though the proposed solution with CCG produces more accurate sets with a smaller data structure, its computing time increases essentially due to a lack of closed-form expression for the Pontryagin difference with a CCG minuend (for a CZ minued, Lemma 1 provides a closed-form expression). Additionally, there are two steps in the implementation of Algorithm 1 that require solving optimization problems. The first is the elimination of redundant constraints in the H matrix in Line 8 from Algorithm 1, which requires solving 2 optimization programs per row of H to determine if it can be removed. Additionally, the step of computing a zonotope to contain the polytope defined by H was also implemented by solving 2 optimization programs per each coordinate of the polytope. Therefore, if further research into conversion of zonotopes to CZ format is performed, this computational time drops significantly. Moreover, since the computation of RC sets is often done offline to validate the feasibility or robustness of a system, the increase in computing time may be acceptable given the advantages in accuracy and memory footprint.

V. Conclusion

In this paper, we have tackled the problem of computing Robust Controllable (RC) sets for linear systems in discrete time in the presence of both ellipsoidal and polytopic bounds. In such a setup, adopting strategies in the literature using CZs induces a larger growth in the data structures size due to the approximation of the ℓ_2 bounds. We have proposed an algorithm that first converts the constraints of a CCG to a homogeneous form to avoid being overly conservative when approximating any norm by the ∞-norm. Then, by leveraging QR decomposition and recent results for Pontryagin difference involving a CZ minuend, we proposed an algorithm to compute an inner approximation for the Pontryagin difference for a CCG minuend. Using numerical simulations, we showed that the proposed method is accurate for both randomly generated sets but also for practical applications. In particular, for a double integrator dynamics it was show to outperform the current state-of-the-art using CZs, while retaining a smaller memory footprint.

References

[1] W. Langson, I. Chryssochoos, S. Raković, and D. Q. Mayne, "Robust model predictive control using tubes," *Automatica*, vol. 40, no. 1, pp. 125–133, 2004.

[2] D. Mayne, M. Seron, and S. Raković, "Robust model predictive control of constrained linear systems with bounded disturbances," *Automatica*, vol. 41, no. 2, pp. 219–224, 2005.

[3] F. Borrelli, A. Bemporad, and M. Morari, *Predictive control for linear and hybrid systems*. Cambridge Univ. Press, 2017.

[4] D. Bertsekas and I. Rhodes, "On the minimax reachability of target sets and target tubes," *Automatica*, vol. 7, pp. 233–247, 1971.

[5] J. Scott, D. Raimondo, G. Marseglia, and R. Braatz, "Constrained zonotopes: A new tool for set-based estimation and fault detection," *Automatica*, vol. 69, pp. 126–136, 2016.

[6] V. Raghuraman and J. Koeln, "Set operations and order reductions for constrained zonotopes," *Automatica*, 2022.

[7] F. Xu, S. Olaru, and M. Seron, "Observer gain optimization for minimization of minimal robust positively invariant set and invariant set-based fault detection," *Automatica*, 2024.

[8] F. Gruber and M. Althoff, "Scalable robust safety filter with unknown disturbance set," *IEEE Trans. Auto. Ctrl.*, vol. 68, no. 12, pp. 7756–7770, 2023.

[9] L. Yang, H. Zhang, J. Jeannin, and N. Ozay, "Efficient backward reachability using the Minkowski difference of constrained zonotopes," *IEEE Trans. Comp.-Aided Design Integ. Circ. Syst.*, vol. 41, no. 11, pp. 3969–3980, 2022.

[10] J. Gleason, A. Vinod, and M. Oishi, "Lagrangian approximations for stochastic reachability of a target tube," *Automatica*, vol. 125, 2021.

[11] A. Vinod, A. Weiss, and S. Di Cairano, "Abort-safe spacecraft rendezvous under stochastic actuation and navigation uncertainty," in *Proc. Conf. Dec. & Ctrl.*, 2021.

[12] A. Vinod, J. Gleason, and M. Oishi, "SReachTools: a MATLAB stochastic reachability toolbox," in *Proc. Hybrid Syst.: Comp. & Ctrl.*, 2019, pp. 33–38. [Online]. Available: https://sreachtools.github.io/

[13] A. Vinod, A. Weiss, and S. Di Cairano, "Projection-free computation of robust controllable sets with constrained zonotopes," *Automatica*, vol. 175, p. 112211, 2025.

[14] H. Ahn, K. Berntorp, P. Inani, A. Ram, and S. Di Cairano, "Reachability-based decision-making for autonomous driving: Theory and experiments," *IEEE. Trans. Ctrl. Syst. Tech.*, vol. 29, no. 5, pp. 1907–1921, 2020.

[15] N. Malone, H. Chiang, K. Lesser, M. Oishi, and L. Tapia, "Hybrid dynamic moving obstacle avoidance using a stochastic reachable set-based potential field," *IEEE Trans. Rob.*, vol. 33, no. 5, pp. 1124–1138, 2017.

[16] F. Blanchini and S. Miani, *Set-theoretic analysis of dynamic systems*. Springer, 2015.

[17] M. Althoff, G. Frehse, and A. Girard, "Set propagation techniques for reachability analysis," *Annual Rev. Ctrl., Rob., & Auto. Syst.*, vol. 4, no. 1, pp. 369–395, 2021.

[18] E. Kerrigan, *Robust constraint satisfaction: Invariant sets and predictive control*. University of London, 2000.

[19] F. Rego and D. Silvestre, "Cooperative navigation using constrained convex generators," *Euro. J. Ctrl.*, vol. 81, p. 101163, 2025.

[20] D. Silvestre, "Closed-form approximations for the minimal robust positively invariant set using constrained convex generators," *IFAC-PapersOnLine*, vol. 58, no. 25, pp. 126–131, 2024.

[21] ——, "Constrained convex generators: A tool suitable for set-based estimation with range and bearing measurements," *IEEE Control Systems Letters*, vol. 6, pp. 1610–1615, 2022.

[22] M. Althoff, "An introduction to CORA," in *Proc. Sec. Verif. Cont. Hybrid Syst.*, December 2015, pp. 120–151.

[23] D. Silvestre, "Exact set-valued estimation using constrained convex generators for uncertain linear systems," *IFAC-PapersOnLine*, vol. 56, no. 2, pp. 9461–9466, 2023.

[24] S. Boyd and L. Vandenberghe, *Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares*. Cambridge Univ. Press, 2018.

[25] S. Sadraddini and R. Tedrake, "Linear encodings for polytope containment problems," in *Proc. Conf. Dec. & Ctrl.* IEEE, 2019, pp. 4367–4372.

[26] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge Univ. Press, 2004.

[27] A. Vinod, "pycvxset: A Python package for convex set manipulation," in *Proc. Amer. Ctrl. Conf.*, 2025. [Online]. Available: https://merlresearch.github.io/pycvxset/

[28] M. Herceg, M. Kvasnica, C. Jones, and M. Morari, "Multi-Parametric Toolbox 3.0," in *Proc. Euro. Ctrl. Conf.*, 2013. [Online]. Available: https://www.mpt3.org/

[29] D. Silvestre, "Comparison of recent advances in set-membership techniques: Application to state estimation, fault detection and collision avoidance," *Euro. J. Ctrl.*, vol. 81, p. 101161, 2025.