Velocity Estimation with Single-Photon Lidar

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Abstract

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ABSTRACT

Accurate velocity estimation is increasingly important for many lidar applications. However, estimating velocity with pulsed lidar conventionally requires fitting an object's position over time, which requires multiple distance measurements and is complicated by the possibility of lateral motion. In this work, we present the technique of Doppler Single-Photon Lidar, which measures the velocity of a moving target from the Doppler shift in the pulse repetition frequency. Importantly, this velocity estimation can achieve high accuracy despite erroneous distance measurements, such as when a target is beyond the unambiguous range of the lidar system. We demonstrate the effect of the pulse repetition rate on parameter estimation performance through a pair of experiments, highlighting a potential tradeoff between distance and velocity accuracy.

Keywords: single-photon lidar, velocity, Doppler shift, time-correlated single-photon counting, maximum likelihood estimation

1. INTRODUCTION

Single-photon lidar (SPL) uses time-correlated single-photon counting (TCSPC) to measure distances from the time delays between pulsed laser emissions and single-photon detections. The picosecond-resolution measurements allow sub-centimeter distance resolution, and the single-photon sensitivity enables sensing under photon-starved conditions such as long-range measurements. This capability under extreme conditions makes SPL a promising technology for terrain mapping, autonomous navigation, etc. We recently introduced a technique called Doppler Single-Photon Lidar, which can estimate the velocity of a moving target using SPL.[1] The key observation is that the periodic pulsing of the laser acts as amplitude modulation,[2] and target motion induces a Doppler shift in the pulse repetition frequency. One potential limitation of Doppler SPL is the inverse relationship between the unambiguous range and the pulse frequency. A higher pulse rate is usually desirable for detecting more photons and improving distance estimates, but objects that are farther than the unambiguous range appear aliased.[3] In this work, we examine the effect of the pulse frequency on both distance and velocity estimation.

2. METHOD

We consider an object that is initially located at a distance z and moving with velocity v away from the lidar system. The laser transmits n_r pulses with shape $h(t)$ and repetition period t_r between pulses. The detector is triggered by N individual photon arrivals over the acquisition time $t_\mathrm{a} = t_\mathrm{r} n_\mathrm{r}$, and their absolute detection times $\mathcal{T} = (T_i)_{i=1}^N$ (measured with respect to the start of acquisition at time $t = 0$) are recorded by a time tagger. The background intensity b describes the photon detection rate per second from ambient light and dark counts. The background flux $B = bt_\mathrm{r}$ integrates the intensity to get the expected number of background detections in one pulse repetition period t_r. Similarly, the signal flux S is the mean number of detected laser photons per transmitted pulse. We assume the total flux $S + B$ is low enough that detector dead times are negligible. The signal-to-background ratio (SBR) is defined as S/B.

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According to Ref. 1, the photon detection times can be described as an inhomogeneous Poisson process with intensity function

$$\lambda(t) = S\left[\sum_{n=0}^{n_{\mathrm{r}}-1} h\left(t - \frac{2z}{c-v} - n\frac{c+v}{c-v}t_{\mathrm{r}}\right)\right] + b, \quad t \in [0, t_{\mathrm{a}}] \tag{1}$$

Given the intensity function, the approximate log-likelihood of the detection times is straightforward to derive[4] as

$$\mathcal{L}(S, b, z, v; \mathcal{T}) = -n_{\mathrm{r}}(S + bt_{\mathrm{r}}) + \sum_{T\in\mathcal{T}} \log\left[S\, h\left(T \bmod t_{\mathrm{r}} - \frac{2vT}{c} - \frac{2z}{c}\right) + b\right]. \tag{2}$$

Then joint maximum likelihood estimators (MLEs) of the flux levels, distance, and velocity can be determined by solving

$$\hat{S}, \hat{b}, \hat{z}, \hat{v} = \underset{S,b,z,v}{\arg\max}\, \mathcal{L}(S, b, z, v; \mathcal{T}). \tag{3}$$

We note that the modulo operation $T \bmod t_{\mathrm{r}}$, which results from the periodic pulsing, causes distance aliasing (i.e., ambiguity over integer multiples of the repetition period) for objects beyond the unambiguous range.

Solving for the maximum likelihood estimators is a challenging problem due to the non-concavity of the log-likelihood (2). One useful approach is to seek an initial estimate of each parameter before performing numerical optimization (3). An initial velocity estimate can be made using the observation that motion induces a Doppler shift in the pulse repetition frequency. Consider that the transmitted frequency is $f_{\mathrm{r}} = 1/t_{\mathrm{r}}$, whereas the repetition frequency in the received pulse train intensity (1) is $f'_{\mathrm{r}} = (c-v)/[t_{\mathrm{r}}(c+v)]$. The received frequency f'_{r} can be estimated using flux probing,[5] i.e., Fourier analysis of the photon timestamps. Assuming the transmitted frequency f_{r} is known, the initial velocity estimate is $\hat{v} = c(f_{\mathrm{r}} - f'_{\mathrm{r}})/(f_{\mathrm{r}} + f'_{\mathrm{r}})$. Notably, this initial estimate is made independently of the flux and distance parameters.

3. RESULTS

We performed a pair of experiments to compare the estimation ability of our Doppler SPL technique for different repetition frequencies f_{r}. The SPL system consists of a pulsed laser at 443 nm, a silicon single-photon avalanche diode (SPAD) detector, and a time tagger. The test scene is a 3-meter linear translation stage controlled by a stepper motor. A draw-wire linear encoder connected to the translation stage provides reference measurements of distance and velocity. Attached to the translation stage is a diffuse white card, and the optical axis of the SPL system is aligned with the translational motion of the card. To emulate a distance longer than the limit set by the length of the translation stage, we add an electronic delay of 1855 ns between the time tagger channel synchronized to the laser pulse times and the channel recording SPAD detections. The stream of photon time stamps was divided into "frames" of 50 ms (20 frames/s), and the parameter estimates were recovered independently for each frame.

The first experiment was performed with a repetition period $t_{\mathrm{r}} = 3780\,\mathrm{ns}$ ($f_{\mathrm{r}} \approx 265\,\mathrm{kHz}$), and parameter estimation results are shown in the top row of Fig. 1. No ambient light was intentionally introduced for the experiment, keeping the background flux extremely low. Both the signal and background flux varied with the motion of the target, with the SBR in the range [1.20, 30.35]. The estimated equivalent distance (assuming the added electronic delay was instead due to a longer time of flight) achieves sub-centimeter root mean square error (RMSE) compared to the reference measurement, and the velocity RMSE is approximately $16\,\mathrm{cm\,s^{-1}}$.

The second experiment, shown in the bottom row of Fig. 1, was performed with a repetition period $t_{\mathrm{r}} = 94.5\,\mathrm{ns}$ ($f_{\mathrm{r}} \approx 10.6\,\mathrm{MHz}$), or a factor of 40× more pulses. Since the laser peak power was held constant, the signal flux estimate is roughly 40× higher over the same observation window of 3870 ns. The background also has a proportional increase, suggesting that most of the light contribution estimated as "background" was in fact laser light that had undergone multiple reflections in the test enclosure. A major difference between the two experiments with different repetition rates is the unambiguous range. For $t_{\mathrm{r}} = 94.5\,\mathrm{ns}$, the unambiguous range is roughly 14.2 m, causing the longer emulated distance to appear aliased and resulting in a large estimation error. However, the MLE, and in particular the Fourier-based initialization, are able to achieve accurate velocity estimates independent of distance errors such as those due to aliasing. Specifically, the increased pulse rate leads to a velocity RMSE of roughly $4\,\mathrm{cm\,s^{-1}}$.

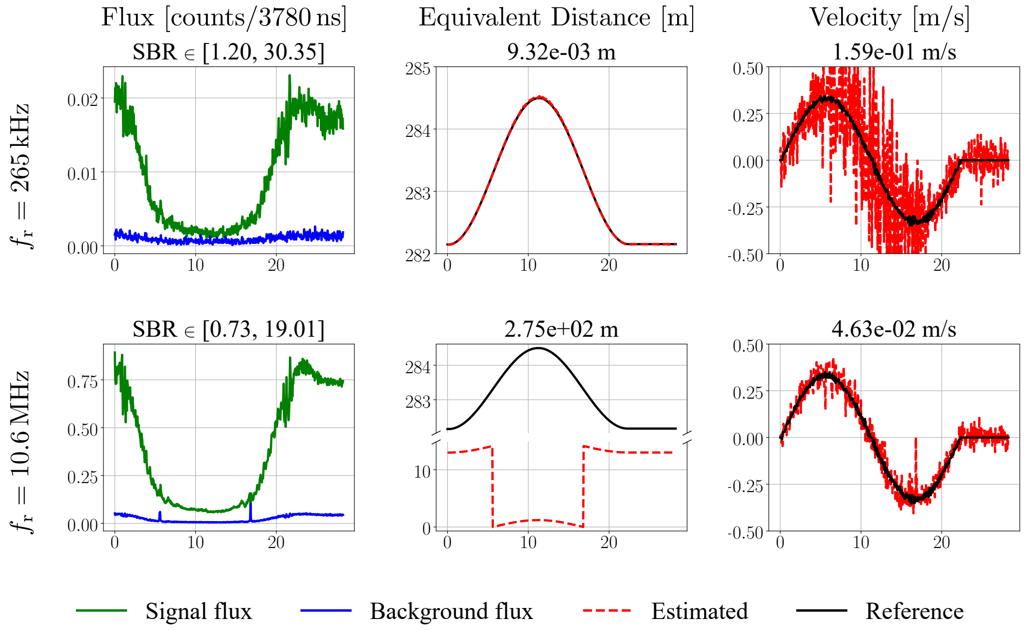

Figure 1: Experimental measurements of a dynamic target are made by fixing the laser's peak power and changing the pulse repetition frequency. For easy comparison between the two experiments, we show the signal and background flux as the mean number of detections over a fixed time (3780 ns). The lower repetition frequency achieves good distance recovery but a noisy velocity estimate, whereas the higher frequency results in inaccurate (aliased) distances but lower-variance velocity estimates.

4. CONCLUSION

In this paper, we demonstrated the effect of the pulse repetition frequency on the Doppler SPL estimates of photon flux, distance, and velocity. With more emitted pulses—and more detected photons—Doppler SPL can achieve greater accuracy velocity estimates. However, increasing the repetition frequency also comes with the risk of causing distance measurements to become aliased. This tradeoff could be broken by leveraging a target's continuous motion to unwrap the aliased distance measurements[3] or using techniques to recover velocity from non-periodic pulse sequences with longer unambiguous range.[6]

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