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TR2025-121 August 20, 2025

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IEEE Conference on Automation and Science Engineering 2025

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I. INTRODUCTION

Off-the-shelf industrial robotic manipulators usually come with specifications to fulfill the requirements of a broad range of customers. Their designs are often optimized for general-purpose applications [1], which implies sub-optimality for a specific application. Application-oriented robot design, which optimizes robot for a specific application, offers a great potential to deliver more cost-effective solutions, e.g., higher productivity, less energy consumption, and lower initial cost. Robot design is multidisciplinary in nature, involving battery [2], structural mechanics [3], kinematics [4], dynamics [5], thermodynamics [6], and control [7]. The design objectives are also multidisciplinary, e.g., weight [2], [3], [5], [6], [8], energy consumption [6], [9], [10], task completion time [9], [10] or workspace maximization [8]. Much of the established robot design methods have been discipline-specific. Some typical examples include, kinematic design optimization with a posture determination [4], kinematic design to optimize the position and orientation for a specific operation [11], multi-objective design for workspace, path

planning or lightweight optimization [12], [13], and PID gain tuning for system performance [7]. Co-design, which reconciles the coupling and conflict of subsystems at an early stage [6], is of interest for its potential to overcome sub-optimality that results from a discipline-specific design process. Its use can be found in applications such as manipulators [1], [3], [5], [14], [15], legged robots [16], [17], aerial/ground manipulators [2], service robotic arms [8] and industrial robots [6], just to name a few. Particularly, the work [15] co-designs the drivetrain and joint trajectories of an industrial robot, where the drivetrain is parameterized in terms of the motor shaft length and gearbox ratio. The works [14], [18] simultaneously optimize trajectories and controllers. The works [2], [5] take the gearbox or motor selection into account. One of the main challenges in co-design is the heavy computational burden [18]. Consequentially, most co-design work restricts the number of axis or motor design freedom [15], [19].

Contributions: This paper proposes a systematic modeling-simulation-optimization pipeline to perform application-driven co-design of the motions (joint trajectories) and actuators of a six degrees of freedom (DOF) robotic manipulator to improve performance (e.g., productivity or energy efficiency). Each actuator is assumed to be a surface permanent magnet synchronous motor (SPMSM) and the application is characterized by a set of tasks. The co-design problem differs from the state-of-the-art in that the motor design involves parameters representing the geometric shape other than selecting motors from a product catalogue. The proposed co-design process tackles the computational challenge through employing: 1) magnetic equivalent circuit (MEC) modeling to establish the analytic mapping from motor design to dynamic model parameters; 2) a differentiable simulator of motor and arm dynamics based on CasADi [20] where the analytical formula of the gradient and Hessian can be derived via auto-differentiation; 3) direct collocation-based integration of motor and arm dynamics to balance accuracy and efficiency; and 4) a two-step process to trade-off computational efficiency and system optimality, where the motion and motor design is solved for each task, and then the candidate motor designs corresponding to all tasks are projected to reach the final motor design.

Notation throughout this paper follows [21]. Section II describes the system modeling and problem formulation. Section III presents the main results. Section IV shows the simulation, and Section V concludes the work.

A. Stein is with the Department of Mechanical Engineering at the Louisiana State University, Pleasant Hall, 1146, Baton Rouge, LA 70802. This work was done while he was a research intern at Mitsubishi Electric Research Laboratories (MERL), Cambridge, MA 02139. (email: astein@lsu.edu).

Y. Wang, Y. Sakamoto, and B. Wang are with (MERL). (email: {yebinwang, sakamoto, bwang}@merl.com).

H. Fang is with the Department of Mechanical Engineering, University of Kansas, Lawrence, KS 66045. (email: fang@ku.edu).

II. SYSTEM MODELING AND PROBLEM FORMULATION

This section describes the dynamic modeling of the SPMSM and 6DOF manipulator, and formulates the co-design problem. Particularly, the dynamic models preserve the analytical dependence on motor's physical design parameters and thus allows for co-design.

A. SPMSM Design Parameterization and Modeling

Motor design involves tens of parameters. For illustration purpose, we only consider the design freedom as illustrated in Fig. 1, while fixing the following design parameters:

- Number of pole pairs $P = 4$
- Number of slots $Q = 12$
- Height of tooth tip $h_{tip} = 2$ mm
- Width of air gap $\delta = 0.5$ mm
- Number of winding turns per tooth $n_s = 50$
- Number of coils connected in parallel $C_p = 1$
- Remanent flux density of the magnet $B_r = 1.38$ T
- Relative recoil permeability of the magnet $\mu_r = 1.05$
- Filling factor $f_f = 0.55$
- Maximum limitation for flux density $B_{max} = 1.5$ T
- Magnet width in electric angle $\alpha_m = \pi$

Physical meanings of design variables, denoted by $\Xi = [L, R_{ro}, R_{so}, h_m, h_{sy}, w_{tooth}, b_0]^T$, can be found in Table I. With zero skewness assumption, a slot/pole ratio of 12/8 yields a winding factor of $k_{w1} = 0.866$.

The dynamic model of the SPMSM is given below [22]:

$$\frac{di_d}{dt} = -\frac{R}{L_d}i_d + P\omega i_q + \frac{u_d}{L_d} \quad (1a)$$

$$\frac{di_q}{dt} = -\frac{R}{L_q}i_q - P\omega \left(i_d + \frac{\Phi_m}{L_q} \right) + \frac{u_q}{L_q}, \quad (1b)$$

where i_d and i_q are the currents in d- and q-axis, respectively, and ω is the rotor velocity. For an SPMSM, its d-axis and q-axis inductance are equal, i.e., $L_d = L_q$. The motor produces a torque $\tau = \frac{3}{2}P(L_d i_d + \Phi_m)i_q$.

Given motor design Ξ , we follow the MEC modeling, detailed in Appendix, to establish analytical mapping from Ξ to the dynamic model parameters R, L_d, L_q, Φ_m . Table I shows that for a particular motor design, the dynamic model parameters from the MEC modeling are comparable to those obtained by applying a finite element method (FEM).

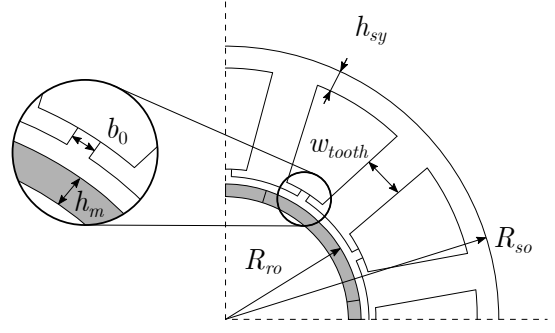


Fig. 1: The cross-section of the magnetic design of the SPMSM. (Note: The axial stack length L is not illustrated.)

TABLE I: Motor design and comparison results.

Motor design variables Ξ			
Parameter	Initial Value	Unit	
Axial length of core L	20	mm	
Outer radius of rotor R_{ro}	18	mm	
Outer radius of stator R_{so}	30	mm	
Height of magnet h_m	3	mm	
Stator yoke h_{sy}	5	mm	
Width of tooth w_{tooth}	7	mm	
Slot opening b_0	2	mm	
Number of winding turns per tooth n_s	30	-	
Dynamic model parameter values from MEC and FEM			
Parameters	FEM	MEC	Unit
Magnetic flux Φ_m	0.0303	0.0288	Wb
Inductance L_d	0.714	0.698	mH
Inductance L_q	0.708	0.698	mH
Resistance R	-	0.621	Ohm

The motor design variables Ξ are subject to the following constraints:

$$[20, 10, 10, 1, 5, 5, 1] \leq \Xi \leq [100, 100, 100, 5, 10, 20, 10] \quad (2a)$$

$$\arcsin\left(\frac{w_{tooth}}{2(R_{ro} + \delta)}\right) + \arcsin\left(\frac{b_0}{2(R_{ro} + \delta)}\right) \leq \frac{\pi}{Q} \quad (2b)$$

$$h_{ss} > 0 \quad (2c)$$

$$m_{stator} + m_{rotor} \leq 3 \quad (2d)$$

$$D_{wire} \geq 0.6, \quad (2e)$$

where the constraints in (2b)-(2e) enforce the maximum tooth width, minimal slot height, maximum weight and minimal wire diameter of each motor. The motor is additionally subject to the following operational constraints:

$$-3A \leq i_d \leq 3A, \quad -3A \leq i_q \leq 3A \quad (3a)$$

$$-100V \leq u_d \leq 100V, \quad -100V \leq u_q \leq 100V \quad (3b)$$

$$\frac{k_p \Phi_m}{w_{tooth} L} \leq 1.5 \text{ T}, \quad \frac{1}{\sqrt{3}} \frac{k_p \Phi_m}{h_{sy} L} \leq 1.5 \text{ T}, \quad (3c)$$

where (3a)-(3b) restrict the currents and voltages in d- and q-axis, and (3c) constrains the magnetic fluxes in the tooth and the stator yoke. The torque ripples are not modelled.

B. 6DOF Open-Chain Robot Manipulator

According to [21], the dynamic model for the 6DOF manipulator, without tip force, can be written as follows

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau, \quad (4)$$

where $\theta = [\theta_{L_1} \ \dots \ \theta_{L_6}]^\top \in \mathbb{R}^6$, $\dot{\theta}$, and $\ddot{\theta}$ are the angles, velocities, accelerations of all links, respectively, and τ are torques applied on all links; M , C and G are the link inertia matrix, Coriolis forces, and gravitational force. The units of θ , $\dot{\theta}$, $\ddot{\theta}$ are rad, rad/s, rad/s², respectively. The constraints on the angles of velocities are given by:

$$-2\pi \leq \theta_{L_1}, \theta_{L_4}, \theta_{L_6} \leq 2\pi \quad (5a)$$

$$-0.6\pi \leq \theta_{L_2}, \theta_{L_3}, \theta_{L_5} \leq 0.6\pi \quad (5b)$$

$$-100\pi \leq \dot{\theta}_{R_k} \leq 100\pi, \quad (5c)$$

where subscripts L_k and R_k stand for the k th link and rotor, respectively. Given the gearbox ratio Z being the ratio of the rotor speed to the associated link speed, we have $\omega = \dot{\theta}Z$.

C. Problem Formulation

To perform application-oriented co-design, we first characterize an application as a set of n tasks: $\mathcal{T} \triangleq \{\mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_n\}$. Take pick and place in warehouses as an example. A task \mathcal{T}_i is uniquely represented by a tuple (x_0, x_f, M_p) , where $x_0 = (\theta_0, \dot{\theta}_0)$, $x_f = (\theta_f, \dot{\theta}_f)$, and M_p are the initial state, the final state, and the inertia matrix of the payload, respectively. We thus have the co-design problem as follows.

Problem 1: Given a task set \mathcal{T} , the motor design parameterization Ξ , the motor model and constraints (1) - (3), and the manipulator model and constraints (4)-(5), determine the optimal control u_i^* of task \mathcal{T}_i for $1 \leq i \leq n$ and optimal motor design Ξ^* for all tasks such that a certain cost function $J(u_1, \dots, u_n, \Xi)$ is minimized.

The control inputs u of the motor and manipulator dynamics aggregates the voltages in d- and q-axis of all six axes; and Ξ aggregates the motor design parameters of all six axes. For a time-optimal co-design problem, the cost function is $J = \sum_{i=1}^n t_{f,i}$, where $t_{f,i}$ is the final time of task \mathcal{T}_i . For energy-optimal co-design, the cost function is $J = \sum_{i=1}^n \int_0^{T_{f,i}} u_i^T(t)u_i(t)dt$ where $T_{f,i}$ is the final time for task \mathcal{T}_i . Problem 1, designed to determine u as functions over a certain time interval, is infinite-dimensional. Thanks to the high dimension of manipulator dynamics, optimal control theory, e.g., minimum principle or dynamic programming, is unlikely to be applicable. We adopt the well-established direct transcript [23]–[25] to reduce Problem 1 into a finite-dimensional numerical optimization problem.

Remark 1: If \mathcal{T} only contains one task, i.e., $n = 1$, Problem 1 is similar to that considered in [15] except that Problem 1 allows more motor design freedom.

III. MAIN RESULTS

This section presents the proposed approach to Problem 1: two-step co-design process, a differentiable simulator, and ABA with gearbox and motor inertia. The analysis of computational complexity and convergence is briefly discussed.

A. Two-Step Co-Design Process

When $n > 1$, solving Problem 1 at one time incurs much higher computation burden than solving for $(\hat{u}_i^*, \hat{\Xi}_i^*)$ individually n times. When n becomes large, solving Problem 1 in one shot gets computationally intractable. Therefore we propose a two-step process: 1) solve for $(\hat{u}_i^*, \hat{\Xi}_i^*)$ corresponding to task \mathcal{T}_i , $1 \leq i \leq n$; 2) reconcile $(\hat{\Xi}_1^*, \dots, \hat{\Xi}_n^*)$ to obtain Ξ^* and solve for u_i^* based on Ξ^* .

1) *Co-Design for Task \mathcal{T}_i :* The procedure is illustrated by Fig. 2, where the red and green boxes mark the manipulator and motor, respectively. It consists of two parts: 1) given u and motor design Ξ , a differentiable simulator simulates the motor and manipulator dynamics to get state trajectory $x(t)$, $i_d(t)$, $i_q(t)$ and calculates the gradient of the cost function w.r.t. (u, Ξ) ; 2) the optimizer assesses the state trajectory against boundary conditions, dynamic and operational constraints, and if no violation occurs, updates control and motor design based on the gradient. The initial guess of Ξ and u is the mean of the upper and lower bound of the motor design variables as shown in (2a) and (3a)-(3b) respectively.

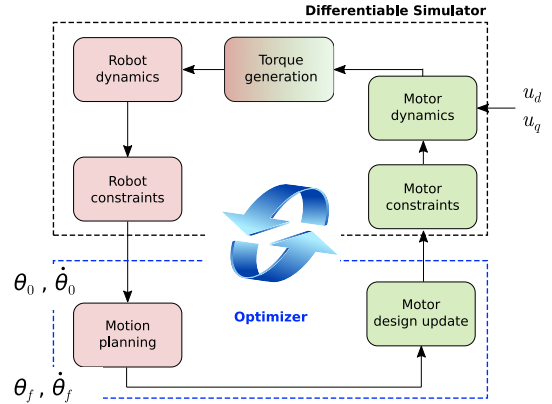


Fig. 2: Co-design procedure for single task: the robot manipulator (in red) and the motor (in green).

2) *Reconciliation of Motor Designs:* Applying the procedure as shown in Fig. 2 to all tasks yields n pairs of motion and motor: $(\hat{u}_i^*, \hat{\Xi}_i^*)$, $1 \leq i \leq n$. In a real world, only one motor design is adopted. Hence we need to come up with the motor design Ξ^* based on $(\hat{\Xi}_1^*, \dots, \hat{\Xi}_n^*)$ systematically.

The reconciliation procedure is illustrated by Fig. 3. Assume that we have $(\hat{u}_i^*, \hat{\Xi}_i^*)$ for \mathcal{T}_i , $1 \leq i \leq n$. Then, the motor designs for all cases are weighted equally to get the average value $\bar{\Xi}^*$. Note that even if the motor design $\hat{\Xi}_i^*$ satisfies the constraints (2) and (3c), $\bar{\Xi}^*$ might not, and

the following constrained optimization problem needs to be solved to obtain a feasible motor design:

Problem 2: Find the feasible motor design Ξ^* which is closest to the mean value $\bar{\Xi}^*$ by solving

$$\Xi^* = \arg \min_{\Xi \in \Omega} \sum_{k=1}^{n_{\Xi}} \frac{(\bar{\Xi}_k^* - \Xi_k)^2}{(\bar{\Xi}_k^*)^2},$$

where n_{Ξ} is the dimension of Ξ ; $\bar{\Xi}_k^*$, Ξ_k are the k th element of the vectors $\bar{\Xi}^*$ and Ξ , respectively, and Ω is the feasible domain of Ξ induced by constraints (2) and (3c).

If all elements in Ξ^* are independent from each other and Ω is in the form of upper and lower bounds for each design variable, then Problem 2 can be reduced to a simple projection $\bar{\Xi}_k$ to its closest bound. This is however not true owing to the coupling constraints (2) and (3c).

Finally, the motor design Ξ^* is used to update the motor and manipulator models, based on which the trajectories u_i^* for $1 \leq i \leq n$ can be determined.

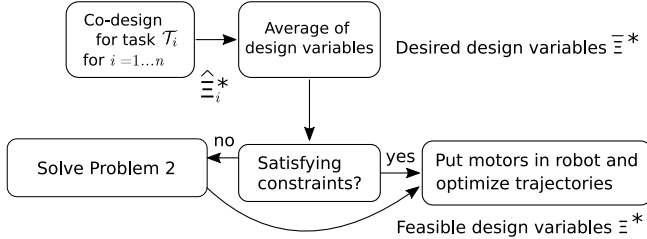


Fig. 3: Reconciliation to obtain a unique motor design.

B. Differentiable Simulator

Overcoming the computational challenge entails high efficiency of the simulator and optimization, which is governed by the number of time steps of integration and decision variables. We resort to CasADi [20] to implement a differentiable simulator of both the motor and robotic arm dynamics. The optimization benefits from this treatment because CasADi natively supports the derivation of analytical gradient formula as long as all functions are implemented in such a way that all outputs are differentiable w.r.t. inputs.

We wish to reduce the number of time steps, which compromises simulation accuracy and numerical stability. This conflict is worsened by the fact that the time scale of the motor dynamics is orders of magnitude faster than that of the arm dynamics. In virtue of the unsatisfactory performance for explicit numerical integration methods such as Runge-Kutta to deal with stiff dynamics, we adopt the direct collocation method to integrate the forward dynamics of both the motor and robotic arm in CasADi. This allows us to reduce the number of time steps. Below is a short sketch. Discretizing control over time interval $[0, t_f]$ yields

$$\begin{aligned} [u_d(t), u_q(t)] &= [u_{d,w}, u_{q,w}]_{12 \times 1}, \\ t &\in [t_w, t_{w+1}], \quad w = 0, \dots, n_w - 1, \end{aligned} \quad (6)$$

where the index n_w represents the total number of discretization points. The vector $[u_{d,w}, u_{q,w}]$ describes the voltage inputs for all six motors. The list of all collocation points is:

$$t_{w,j} = t_w + \Delta t_w \gamma_j, \quad \text{for } w = 0, \dots, n_w - 1; \quad j = 0, \dots, d.$$

where $\Delta t_w = t_{w+1} - t_w$, γ remarks the nodes, and the degree d for the Lagrangian polynomials is set to 1.

C. ABA with Gearbox and Motor Inertia

We employ Alg. 1 for forward dynamics simulation, where p , A , M , G , F_{tip} , \mathcal{V} and Z are the biased force, screw axis, homogeneous transformation matrix, spatial inertia matrix, wrench at the tip, twist, and gear ratio, respectively. The ABA algorithm in [26] is extended to account for motor inertia and gear ratio. As in [21], we assume that the $(k+1)$ th motor is mounted on the k th link, and thus the k th link inertia G_{L_k} in Alg. 1 contains the stator of the $(k+1)$ th motor. The motor rotor and gearbox in the k th axis is treated as an individual rigid body, with the inertia being denoted as G_{R_k} , and all related kinematic and dynamic quantities such as the twist are updated separately. The flexibility and backlashes of the gears are ignored. The spatial vector for gravitational acceleration is $g = -[0, 0, 0, 0, 0, 9.81]^\top$.

D. Discussions

By directly transcribing Problem 1 and solving it for all tasks at one time, we are able to obtain a solution converging to a local minimum. The proposed two-step process, albeit being much more computationally efficient, does not necessarily converge to a stationary point. As a result, the proposed algorithm yields sub-optimal design.

Let us take $n_w = 50$, which means that solving Problem 1 for task \mathcal{T}_i involves $m = 1842$ decision variables. Solving Problem 1 for n tasks at one time involves n times more decision variables. So does the number of constraints. Since CasADi uses IPOPT to solve optimization problem based on gradient and Hessian, which possess a complexity of $O(m)$ and $O(m^2)$ with auto-differentiation, one can appreciate the computational advantage of the two-step process.

This work can be readily extended to co-design of the geometric parameters of robot links. All robot link parameters appear in the model (4) analytically, which implies that the cost function is an analytical function of the link parameters. Therefore, auto-differentiation remains viable.

IV. SIMULATION

Simulation compares the results of applying three co-design methods to pick-and-place application with time-optimal and energy-optimal cost functions: 1) an empirical co-design for a task \mathcal{T}_e with a medium stroke $\lambda = 0.6$ and payload $M_p = 2$ kg; 2) a worst-case co-design for a task \mathcal{T}_w with the longest stroke $\lambda = 1$ and maximum payload $M_p = 4$ kg; and 3) the multi-task two-step based co-design using

Algorithm 1: ABA for a 6DOF open-chain manipulator with gear boxes and motor inertia.

Inputs: $\theta_{L_k}, \dot{\theta}_{L_k}, M_{L_k, L_{k-1}}, M_{R_k, L_{k-1}}, M_{R_k, L_k}, A_{L_k}, A_{R_k}, \tau_{L_k}, F_{tip}, G_{L_k}, G_{R_k}$;
Output: $\dot{\theta}_{L_k}, k \in \{1, 2, \dots, 6\}$;
Ensure: $\mathcal{V}_{pre} = 0, \dot{\mathcal{V}}_0 = -g$;
 // update matrices and initialize inertia/biased forces of articulated bodies;
for $k = 1$ **to** 6 **do**
 Calculate homogeneous transformation matrix:
 $T_{L_k, L_{k-1}} = \text{vTSE3}(-A_{L_k}, \theta_{L_k}) M_{L_k, L_{k-1}}$;
 $T_{R_k, L_{k-1}} = \text{vTSE3}(-A_{R_k}, \theta_{L_k}) M_{R_k, L_{k-1}}$;
 Calculate adjoint matrices:
 $Ad_{T_{L_k, L_{k-1}}}, Ad_{T_{R_k, L_{k-1}}}$;
 Calculate the twists:
 $\mathcal{V}_{L_k} = Ad_{T_{L_k, L_{k-1}}} \mathcal{V}_{pre} + A_{L_k} \dot{\theta}_{L_k}$;
 $\mathcal{V}_{R_k} = Ad_{T_{R_k, L_{k-1}}} \mathcal{V}_{pre} + A_{R_k} \dot{\theta}_{L_k}$;
 Calculate Lie bracket of twist: $ad_{\mathcal{V}_{L_k}}, ad_{\mathcal{V}_{R_k}}$;
 Calculate:
 $\xi_{L_k} = ad_{\mathcal{V}_{L_k}} A_{L_k} \dot{\theta}_{L_k}, \xi_{R_k} = ad_{\mathcal{V}_{R_k}} A_{R_k} \dot{\theta}_{L_k}$;
 Initialize the spatial inertia matrix:
 $\mathcal{I}_{L_k}^A = G_{L_k}, \mathcal{I}_{R_k}^A = G_{R_k}$;
 Initialize the biased force:
 $p_{L_k}^A = ad_{\mathcal{V}_{L_k}} \mathcal{I}_{L_k}^A \mathcal{V}_{L_k}, p_{R_k}^A = ad_{\mathcal{V}_{R_k}} \mathcal{I}_{R_k}^A \mathcal{V}_{R_k}$;
 Prepare for next iteration: $\mathcal{V}_{pre} = \mathcal{V}_{L_k}$;
 // update spatial inertia matrix and biased force;
for $k = 6$ **to** 1 **do**
 Calculate:
 $\mu_{L_k} = \tau_{L_k} - A_{L_k} p_{L_k}^A, \mu_{R_k} = \tau_{R_k} - A_{R_k} p_{R_k}^A$;
 $D_k = 1 / (A_{L_k}^\top \mathcal{I}_{L_k}^A A_{L_k} + A_{R_k}^\top \mathcal{I}_{R_k}^A A_{R_k})$;
 if $k > 1$ **then**
 $\mathcal{I}_{L_{k-1}}^A = \mathcal{I}_{L_{k-1}}^A + Ad_{T_{L_k, L_{k-1}}}^\top (\mathcal{I}_{L_k}^A - \mathcal{I}_{L_k}^A A_{L_k} D_k A_{L_k}^\top) Ad_{T_{L_k, L_{k-1}}} + Ad_{T_{R_k, L_{k-1}}}^\top (\mathcal{I}_{R_k}^A - \mathcal{I}_{R_k}^A A_{R_k} D_k A_{R_k}^\top) Ad_{T_{R_k, L_{k-1}}}$;
 $p_{L_{k-1}}^A = p_{L_{k-1}}^A + Ad_{T_{L_k, L_{k-1}}}^\top (p_{L_k}^A + \mathcal{I}_{L_k}^A \xi_{L_k} + \mathcal{I}_{L_k}^A A_{L_k} D_k (\mu_{L_k} - A_{L_k}^\top \mathcal{I}_{L_k}^A \xi_{L_k})) + Ad_{T_{R_k, L_{k-1}}}^\top (p_{R_k}^A + \mathcal{I}_{R_k}^A \xi_{R_k} + \mathcal{I}_{R_k}^A A_{R_k} D_k (\mu_{R_k} - A_{R_k}^\top \mathcal{I}_{R_k}^A \xi_{R_k}))$;
 // update joint accelerations;
 $\dot{\mathcal{V}}_{k, pre} = \dot{\mathcal{V}}_0$ **for** $k = 1$ **to** 6 **do**
 $\dot{\mathcal{V}}_i = Ad_{T_{L_k, L_{k-1}}} \dot{\mathcal{V}}_{k, pre}$;
 $\ddot{\theta}_{L_k} = D_k (\tau_{L_k} - A_{L_k}^\top (\mathcal{I}_{L_k}^A (\dot{\mathcal{V}}_{L_k} + \xi_{L_k}) + p_{L_k}^A) - A_{R_k}^\top (\mathcal{I}_{R_k}^A (\dot{\mathcal{V}}_{L_k} + \xi_{R_k}) + p_{R_k}^A))$;
 $\dot{\mathcal{V}}_{L_k} = \mathcal{V}_{L_k} + A_{L_k} \ddot{\theta}_{L_k} + \xi_{L_k}$;
 Prepare for next iteration: $\dot{\mathcal{V}}_{k, pre} = \dot{\mathcal{V}}_{L_k}$;
Return: $\ddot{\theta}_{L_k}, k \in [1, 2, \dots, 6]$;

25 tasks with varying strokes and payloads. Note that both the empirical and worst-case co-designs solve Problem 1 for tasks \mathcal{T}_e and \mathcal{T}_w , respectively, by conducting motor design

instead of merely motor selection as in literature, e.g., [15].

A. Application-specific Task Set \mathcal{T}

Without loss of generality, we specify the task set \mathcal{T} by numerating initial states \mathbf{x}_0 over five choices

$$\mathbf{x}_0 = \lambda [\mathbf{1}_6 \quad \mathbf{0}_6]^\top, \text{ for } \lambda \in \{0.2, 0.4, 0.6, 0.8, 1\},$$

and payload M_p over five options $M_p \in \{0, 1, 2, 3, 4\}$ kg. Here $\mathbf{1}_6$ is a vector of ones in \mathbb{R}^6 , and $\mathbf{0}_6$ is a vector of zeros in \mathbb{R}^6 . For each choice of \mathbf{x}_0 , we change M_p (a solid ball) to be 0, 1, 2, 3, 4 kg to define five tasks. Eventually, we obtain 25 tasks to characterize the application, all of which have the same final state $\mathbf{x}_f = [\mathbf{0}_6 \quad \mathbf{0}_6]^\top$.

B. Simulation Results

Fig. 4 illustrates that the empirical outperforms the multi-task for neighboring tasks of \mathcal{T}_e in terms of the final time and energy consumption, whereas the multi-task performs better once the discrepancy between test task and \mathcal{T}_e gets relatively large (area in green). Note that task \mathcal{T}_e is ideally picked for the range of λ and M_p when comparing it to the multi-task design where every task is equally weighted to derive the motor design. Fig. 5 shows the comparison

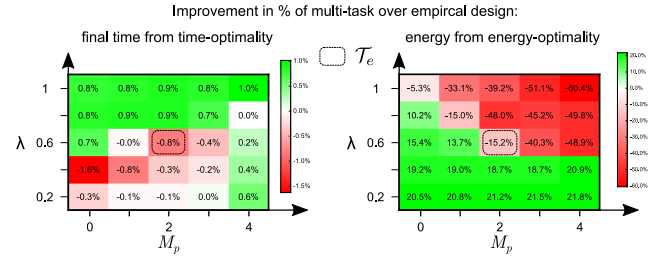


Fig. 4: Percentages of improvement (green) of multi-task method over empirical design.

between the multi-task and the worst-case design in terms of the final time (left plot) and energy consumption (right plot). As expected the worst-case design performs better for \mathcal{T}_w . However, considering the whole space of λ and M_p , the multi-task method proves to provide good results, especially in terms of energy consumption for small initial conditions and light payloads. It should be noted that the final time-or energy consumption is seemingly more on the initial conditions λ than on the payload, because the payload puts less stress on motors than the manipulator does, which explains why the worst-case design performs better for long-stroke tasks. The co-design results for 25 tasks are shown as box whisker charts in Fig. 6, where red and blue mark the time and energy-optimal solutions. For illustration purposes, we provide the axial stack length L and the outer stator radius R_{so} . The worst-case design results are shown as red and green circles for the time and energy-optimal designs, respectively. It can be seen that for the stack length of motor in axis 1, 2, and 3, the multi-task yields time-optimal solutions close to the worst-case solution, with motor in axis

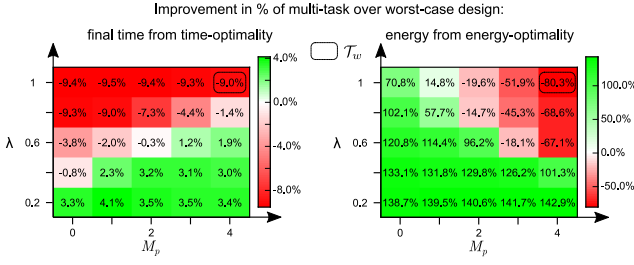


Fig. 5: Percentages of improvement (green) of multi-task method over worst-case design.

2 exactly matching the worst-case design. A similar trend can be observed for the outer stator radius. Motor in axis 2 revolves around the y -axis, counteracting the gravity. The time-optimal solution requires large accelerations. Therefore, the motor needs a long stack length to create high torques but a small outer stator radius to have a small inertia. Overall, the multi-task method offers more flexibility for future research to stochastically analyse different strokes and payloads to derive an optimal motor design. The empirical and worst-case design only considers one specific task where it could be seen that specifically in terms of energy-optimality the multi-task method outperforms these designs.

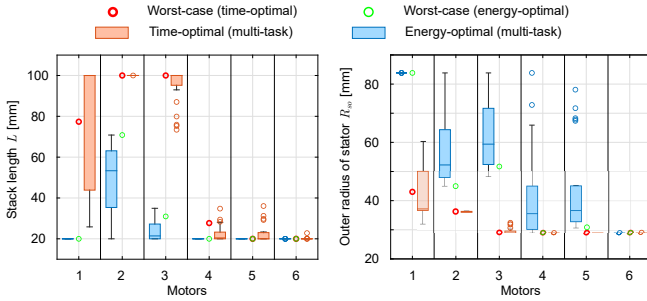


Fig. 6: Stack length and outer stator radius of the motors from 25 tasks (box whisker charts) versus worst-case design (circle): time-optimal (red) and energy-optimal (blue).

V. CONCLUSIONS AND FUTURE WORK

This paper dealt with joint optimization of motions and motors of a manipulator to achieve high performance for a particular application characterized by a set of tasks. We overcame the computational challenge of this multidisciplinary design problem by 1) proposing a two-step design process to resolve the scalability issue arising from multiple tasks of an application; 2) conducting analytical modeling of motor and manipulator to establish a differentiable mapping from motor design and motion to an objective function; 3) developing a differentiable simulator based on direct collocation to balance computation efficiency and simulation accuracy. Simulation was performed to validate the effectiveness of the application-oriented co-design paradigm. Future work includes 1) incorporating weight and friction of the gear box and perform co-design of the gear box, motor, arm,

and control policy; and 2) exploring stochastic framework to obtain optimal motor design for multiple tasks.

ACKNOWLEDGMENT

The authors thank Drs. Scott Bortoff, Diego Romeres, Marcel Menner, and Jinyun Zhang for helpful discussions.

APPENDIX

A. MEC Modeling for the SPMSM

The MEC modeling is provided in this section. More details can be found in [27].

1) *Dimensional specifications:* The cylindrical rotor weight can be calculated as $m_{rotor} = \rho_{iron}\pi R_{ro}^2 L$, and the inertia in its principal axes are $I_{xx} = \frac{1}{2}\rho_{iron}\pi R_{ro}^4 L$ and $I_{yy} = I_{zz} = \frac{1}{12}\rho_{iron}\pi R_{ro}^2 L (3R_{ro}^2 + L^2)$. The slot height is $h_{ss} = R_{so} - h_{sy} - R_{ro} - \delta - h_{tip}$. Assuming a rectangular tooth cross-sectional area (see Fig. 1), the slot width is $b_{ss} = \frac{A_{slot}}{h_{ss}}$ where

$$A_{slot} = \frac{\pi \left((R_{so} - h_{sy})^2 - (R_{ro} + \delta + h_{tip})^2 \right)}{Q} - w_{tooth} h_{ss}.$$

2) *Stator and rotor weight:* For a given filling factor of $f_f = 0.55$ the copper area can be calculated as $A_{cu} = A_{slot} f_f$. For a concentrated winding type and the assumption that one winding is a complete turn around a tooth, a single coil area is given by $A_{coil} = \frac{A_{cu}}{2n_s}$. The stator and rotor weight can be calculated as follows:

$$\begin{aligned} m_{stator} &= \rho_{iron}\pi R_{so}^2 L - \rho_{iron}\pi R_{ro}^2 L \dots \\ &\quad - \rho_{iron} A_{cu} L Q + \rho_{cu} A_{coil} L_{coil} n_s Q \\ m_{rotor} &= \rho_{iron}\pi R_{ro}^2 L. \end{aligned}$$

3) *Resistance:* The arc span τ_s per slot is given by $\tau_s = \frac{2\pi(R_{ro} + \delta)}{Q}$. The average length of the end-winding of the coil is $L_{end,av} = \frac{1}{2} (w_{tooth} (2 - \frac{\pi}{2}) + \frac{\pi\tau_s}{2})$ and the coil length $L_{coil} = 2L + 2L_{end,av}$. The resistance per tooth is given by $R_1 = \frac{n_s^2 \rho_c L_{coil}}{A_{slot} f_f}$, where n_s is the number of windings per tooth. Finally, the phase resistance can be calculated as $R = \frac{q_1}{C^2} R_1$, where C is the number of coils connected in parallel and $q_1 = \frac{Q}{m}$ is the slots per phase (here $C = 1$ and $m = 3$).

4) *Flux:* Carter's coefficient k_C is given as $k_C = \frac{t_{pitch}}{t_{pitch} - \gamma\delta}$ and the magnetic flux density across the gap is defined as B_g :

$$\gamma = \frac{\left(\frac{b_0}{\delta}\right)^2}{5 + \frac{b_0}{\delta}}, \quad t_{pitch} = \frac{2\pi R_{ro}}{Q}, \quad B_g = B_r \frac{\frac{h_m}{\mu_r}}{\frac{h_m}{\mu_r} + \delta k_C}.$$

The flux density of the first harmonics is $B_{g,1} = \frac{4}{\pi} B_g$, and the flux per tooth per single turn is $\Phi_1 = B_{g,1} \frac{2\pi R_{ro}}{Q} L$. Without skewness, we have the flux linkage $\Phi_m = k_w \frac{q_1}{C} n_s \Phi_1$, where $k_w = k_p k_d$ is the winding factor where $k_p =$

$\sin\left(\frac{\pi P}{Q}\right)$ and $k_d = \frac{\sin\left(\frac{\pi}{6}\right)}{q_{pm} \sin(\pi/(6q_{pm}))} \cdot q_{pm} = \frac{q_1}{gcd(q_1, 2P)}$ is the slots per pole per phase and $gcd(\cdot)$ means the great common divisor.

5) *Permeance*: The permeance of the magnetic path across the air gap and the slot opening, denoted by p_g and p_{so} , respectively, can be calculated as follows:

$$p_g = \frac{\left(\frac{2\pi R_{ro}\mu_0}{Q}\right)L}{\delta + \frac{h_m}{\mu_r}}, \quad p_{so} = \frac{\mu_0 h_{tip} L}{b_0}.$$

The permeance of the curved magnetic path from tip to tip is:

$$p_{tt} = \frac{\mu_0 (\delta + h_m) L}{\frac{\pi}{2} (\delta + h_m)}.$$

6) *Inductance*: The d- and q-axis inductance is given by $L_{dq} = \frac{q_1}{C^2} n_s^2 L_1$, where $L_1 = p_g + 3p_{so} + 3p_{tt}$ is the inductance per turn and per tooth.

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