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Physics Informed Gaussian Process Regression Methods for Robot Inverse Dynamics Identification

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***Abstract*— In this extended abstract we present two recent contributions in the context of Physics Informed black-box inverse dynamics identification using Gaussian Processes (GPs). The first contribution consists in a novel kernel, named *Geometrically inspired Polynomial Kernel* (GIP) for single joint GP-based inverse dynamics identification. Driven by the fact that the inverse dynamics can be described as a polynomial function on a suitable input space, the GIP kernel restricts the regression problem to a finite-dimensional space which contains the inverse dynamics function, thus leading to improved data efficiency and generalization properties. The second contribution consists in the derivation of a multidimensional GP framework, named Lagrangian GPR, which overcomes the single joint approach and learns the inverse dynamics in a multidimensional setting. Exploiting the properties of GPs in connection with linear operators, Lagrangian GPR allows to impose by design the known symmetric structure of the Euler-Lagrange equation on the learned models. Moreover, since information is shared between different degrees of freedom (DOFs), this approach strongly improves data efficiency and generalization properties.**

I. INTRODUCTION

Inverse dynamics identification is a fundamental but challenging task in robotics. In the recent years black-box learning techniques have drawn the attention of the robotics community. Among them, a promising framework is represented by Gaussian Process Regression (GPR) [1]. Despite their ability to approximate very complex dynamics, however, GPR based black-box methods have two main limitations:

(i) they typically require large amount of samples and in general do not show satisfying generalization properties;
(ii) each torque component is typically modeled by an independent GP, which ignores the correlations and symmetries between different degrees of freedom (DOFs).

In order to address the aforementioned issues, a promising research line is represented by the so called *Physics-Informed* methods, which are based on the idea of embedding insights from physics as model prior. Instead of learning the inverse dynamics in a completely unstructured manner, which makes the problem unnecessarily hard, geometrical and physical properties are exploited both to improve learning performance and to impose physical consistency.

In this work we present our recent developments in the context of Physics-Informed GPR-based inverse dynamics identification of robot manipulators. In particular:

(i) inspired by the property of the inverse dynamics map components of being a polynomial function in an augmented input space, we proposed in [2] a novel black-box polynomial kernel named Geometrically Inspired Polynomial (GIP) kernel. This kernel aims at improving generalization and data efficiency by constraining the regression problem, for each DOF, into a finite dimensional space that contains the inverse dynamics equations.
(ii) Inspired by the fact that the inverse dynamics equations are obtained applying a linear operator to the Lagrangian function, we derived a multidimensional GPR framework, named Lagrangian GPR, where the inverse dynamics is learned considering correlation between different DOFs. Whitin this framework, symmetries of the system are imposed by design. Moreover, information is shared between different DOFs, which leads to improved data efficiency and generalization properties.

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II. ROBOT INVERSE DYNAMICS

Consider a robotic arm with n joints, connected by $n+1$ links and let $\boldsymbol{q} \in \mathbb{R}^n$ be the vector of joint positions. The inverse dynamics is defined as the function mapping the robot state $\boldsymbol{x} = (\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) \in \mathbb{R}^{3n}$ into the vector of generalized torques $\boldsymbol{\tau} \in \mathbb{R}^n$. Under the rigid body and energy conservation assumptions, the robot dynamics can be derived from the Lagrangian mechanics. Let the Lagrangian be the difference between kinetic energy T and potential energy V, namely $L = T - V$. Then, the system dynamics satisfies the Euler-Lagrange (EL) equation, expressed as

$$\boldsymbol{\tau} = \frac{d}{dt}(\nabla_{\dot{\boldsymbol{q}}} L(\boldsymbol{q}, \dot{\boldsymbol{q}})) + \nabla_{\boldsymbol{q}} L(\boldsymbol{q}, \dot{\boldsymbol{q}}) \tag{1}$$

where $\nabla_{\mathbf{x}} f(\cdot)$ denotes the gradient of f with respect to $\mathbf{x}$.

III. GAUSSIAN PROCESS REGRESSION

GPR can be employed to approximate an unknown function $f : \mathbb{R}^d \to \mathbb{R}$, given a training dataset $\mathcal{D} = \{X, \boldsymbol{y}\}$. The input samples are collected in $X \in \mathbb{R}^{Q \times N}$ while $\boldsymbol{y} \in \mathbb{R}^N$ contains the corresponding output measurements, with N being the number of observations and Q the input dimensions.

The output measurements $\boldsymbol{y}$ are assumed to be generated by the following probabilistic model

$$\boldsymbol{y} = \begin{bmatrix} f(\boldsymbol{x}_1) \\ \vdots \\ f(\boldsymbol{x}_N) \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix} = \boldsymbol{f}(X) + \boldsymbol{w}, \tag{2}$$

where $\boldsymbol{w}$ is i.i.d Gaussian noise with standard deviation σ. The unknown function f is modeled a priori as a zero-mean GP,

namely $\boldsymbol{f}(X) \sim \mathcal{N}(0, \mathbb{K})$. Each element of the covariance matrix $\mathbb{K}$, known also as *kernel matrix*, is defined through a kernel function $k(\cdot, \cdot)$. In particular, the element of $\mathbb{K}$ in position (h, j) is equal to $k(\boldsymbol{x}_h, \boldsymbol{x}_j)$. Given the observations $\mathcal{D}$ and a new input location $\boldsymbol{x}_*$, it can be proved that the posterior distribution of $f(x_*)$ is Gaussian, with mean

$$\hat{f}_* = \boldsymbol{k}_*^T \boldsymbol{\alpha} \tag{3}$$

and covariance

$$\mathbb{V}[f_*] = k(\boldsymbol{x}_*, \boldsymbol{x}_*) - \boldsymbol{k}_*^T (\mathbb{K}^{(i)} + \sigma^2 I)^{-1} \boldsymbol{k}_*, \tag{4}$$

where

$$\boldsymbol{k}_* = [k(\boldsymbol{x}_*, \boldsymbol{x}_1), \ldots, k(\boldsymbol{x}_*, \boldsymbol{x}_N)]^T$$

and

$$\boldsymbol{\alpha} = (\mathbb{K} + \sigma_i^2 I)^{-1} \boldsymbol{y}.$$

The maximum a posteriori estimator coincides with the posterior mean $\hat{f}_*$. Within the GPR framework, a fundamental role is played by the kernel function: the whole complexity of GPR reduces to the choice of the correct kernel function for the problem at hand.

A. Single joint GPR Inverse dynamics identification

The GPR framework described above can be applied to the inverse dynamics identification problem. In particular, within the so called single-joint approach, each joint is considered individually and its torque $\boldsymbol{\tau}^{(i)}$ is modeled as an unknown function of the robot state $\boldsymbol{x} \in \mathbb{R}^{3n}$. Thus, the inverse dynamics identification problem reduces to solve n independents GPR problems.

IV. Geometrically Inspired Polynomial kernel

As already pointed out, the choice of the kernel is a crucial aspect in GPR. State of the art methods for single-joint Inverse dynamics identification rely on the so called *Squared Exponential* (SE) kernel [1]. This kernel function, however, capture only similarities between data and it ignores existing relations imposed by the physics and geometry of the problem. As a consequence, it shows the same data inefficiency and poor generalization issues mentioned above.

The Geometrically Inspired Polynomial (GIP) kernel, instead, is based on the property that each component $\boldsymbol{\tau}^{(i)}$ in eq. (1) is a polynomial function in a proper transformation of the GP input, fully characterized only by the type of each joint. Specifically, $\boldsymbol{q}$ is mapped in $\tilde{\boldsymbol{q}}$, the vector composed by the concatenation of the positions of prismatic joints and the sines and cosines of the positions of revolute joints. As proved in [2], the inverse dynamics eq. (1) is composed by polynomial functions in $\ddot{\boldsymbol{q}}$, $\dot{\boldsymbol{q}}$ and $\tilde{\boldsymbol{q}}$, where the elements of $\ddot{\boldsymbol{q}}$ have maximum relative degree of one, whereas the ones of $\dot{\boldsymbol{q}}$ and $\tilde{\boldsymbol{q}}$ have maximum relative degree two. To exploit this property, the GIP kernel is defined through the sum and the product of different polynomial kernels [3], hereafter denoted as $k_P^{(p)}(\cdot, \cdot)$ where p is the degree of the polynomial kernel. In particular,

$$K_{GIP}(\boldsymbol{x}, \boldsymbol{x}') = (k_P^{(1)}(\ddot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}') + k_P^{(2)}(\dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}')) k_Q(\tilde{\boldsymbol{q}}, \tilde{q}'). \tag{5}$$

where k_Q is given by the product of polynomial kernels with degree two. In this way, the GIP kernel allows defining a regression problem in a finite-dimensional function space where (1) is contained. From experimental results, the GIP kernel showed better data efficiency and generalization properties if compared to other black-box estimators. If compared to parametric models, instead, it shows similar generalization properties while requiring less prior information.

V. Lagrangian GPR Framework

The single joint approach is simpler and more computationally efficient, but it ignores known correlation between joints. As a result, the learned model is not guaranteed to respect the structure and symmetries imposed by the EL equation in (1). Our idea is to model the Lagrangian function as a zero-mean GP and then to exploit the invariance property of GPs under linear operators [4] to obtain a multidimensional GP framework for Inverse dynamics identification.

First, let $L(\boldsymbol{q}, \dot{\boldsymbol{q}}) \sim \mathcal{GP}(0, k_L(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{q}', \dot{\boldsymbol{q}}'))$. Then consider the following proposition.

Proposition 1: Let $\boldsymbol{\mathcal{T}}_{\boldsymbol{x}}$ be a linear operator and f be a GP such that

$$f(\boldsymbol{x}) \sim \mathcal{GP}(m(\boldsymbol{x}), k(\boldsymbol{x}, \boldsymbol{x}')). \tag{6}$$

Then, $g = \mathcal{T}_{\boldsymbol{x}} f$ is a (possibly multidimensional) GP such that

$$g(\boldsymbol{x}) \sim \mathcal{GP}(m_g(\boldsymbol{x}), k_g(\boldsymbol{x}, \boldsymbol{x}')) \tag{7}$$

with mean $m_g(\boldsymbol{x}) = \boldsymbol{\mathcal{T}}_{\boldsymbol{x}} m(\boldsymbol{x})$ and kernel $k_g(\boldsymbol{x}, \boldsymbol{x}') = \boldsymbol{\mathcal{T}}_{\boldsymbol{x}} k(\boldsymbol{x}, \boldsymbol{x}') \boldsymbol{\mathcal{T}}_{\boldsymbol{x}'}$.

Defining the Lagrangian operator as $\mathcal{L}_{(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})} = \frac{d}{dt} \nabla_{\dot{\boldsymbol{q}}} + \nabla_{\boldsymbol{q}}$, from eq. (1) we have that $\boldsymbol{\tau} = \mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) L(\boldsymbol{q}, \dot{\boldsymbol{q}})$. Moreover, applying proposition 1 we obtain

$$\boldsymbol{\tau} \sim \mathcal{GP}(0, k_{\boldsymbol{\tau}}(\boldsymbol{x}, \boldsymbol{x}')) \tag{8}$$

with $k_{\boldsymbol{\tau}}(\boldsymbol{x}, \boldsymbol{x}') = \mathcal{L}_{(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})} k_L(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{q}', \dot{\boldsymbol{q}}')) \mathcal{L}_{(\boldsymbol{q}', \dot{\boldsymbol{q}}', \ddot{\boldsymbol{q}}')}$, where $\boldsymbol{x} = (\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$ is the robot state. Note that this way we obtained a multidimensional GP. The extension of the GPR presented in section III to the multidimensional setting is straightforward [5].

Preliminary results showed that this approach provides improved data effeciency and generalization properties with respect to single joint methods. Moreover, the remaining degree of freedom represented by the choice of the lagrangian kernel k_L can be exploited to further improve the learning performances or to impose other constraints on the model, e.g. the positivity of the kinetic energy.

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