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# Stable Adaptive Estimation for Speed-sensorless Induction Motor Drives: A Geometric Approach

Yebin Wang, Senior Member, IEEE, Akira Satake, Sota Sano, and Shinichi Furutani

Abstract—Rotor speed estimation is one of the key problems in speed-sensorless motor drives. Adaptation-based approaches, assuming the rotor speed as a parameter and based on the original coordinates, admit simple estimator designs, albeit suffer from the lack of guaranteed convergence of estimation error dynamics. Focusing on stable speed estimation, this paper proposes a new algorithm based on transforming the motor model into an adaptive observer form via a change of state coordinates. The resultant adaptive estimator renders globally exponentially convergent estimation error dynamics, under persistent excitation condition. The proposed algorithm is advantageous for its guaranteed stability, ease of tuning, and robustness. Experiments demonstrate its effectiveness.

*Index Terms*—Speed-sensorless, motor drives, induction motors, adaptive estimation

# I. INTRODUCTION

Speed-sensorless motor drives, with rotor position/speed unmeasured, are practically favored due to reduced cost and improved reliability [1], [2]. Speed-sensorless control design is however challenging due to the absence of the speed sensor as well as nonlinearity in motor dynamics, and thus has attracted tremendous theoretical interests [3]–[5]. Commercialized motor drives suffer performance degradation from the elimination of the speed sensor, as well as loss of stability in certain operation region. State estimation is identified as the main bottleneck to speed regulation performance.

Adaptation-based approaches are widely used [6]–[8]. They circumvent the difficulty caused by nonlinear dynamics via treating the speed as an unknown parameter. Treating a state as an unknown parameter necessarily compromises the speed control performance. This disadvantage serves as thrust for work without the parameter assumption, to name a few, [5], [9]–[16]. Relying on singular perturbation analysis, work [5] establishes local stability results. Work [9]–[12] relies on a triangular observable form (TOF) where the system dynamics have triangular state dependence [17]. Nonlinear terms with general state dependence however have to be treated as disturbances to ensure that the transformed system dynamics admit the TOF. Such a treatment necessarily results in conservative design. Resorting to high gain observer design

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based on a non-triangular observable form, work [16] relaxes the restriction on the system structure, however is difficult to implement in practice.

Balancing theoretical guarantee and practical effectiveness, this paper revisits the adaptation idea, and develops a normal form-based estimation algorithm. It targets on two issues of conventional adaptation-based approaches: lack of global convergence guarantee [18]; and non-trivial tuning of speed adaptation gains. This work first puts the motor model into an adaptive observer form (AOF) by a parameterdependent state transformation; and then performs adaptive state estimation in the new coordinates. The resultant estimation error dynamics are Globally Exponentially Stable (GES) under the well-known Persistent Excitation Condition (PEC). In addition to guaranteed stability, the proposed algorithm also enjoys ease of tuning and robustness. Effectiveness of the proposed algorithm is validated by experiments. Compared with [19], this work casts light on general benefits and disadvantages of conducting estimation in transformed coordinates

The rest of this paper is organized as follows. Problem formulation is provided in Section II. Section III presents the proposed speed-sensorless estimation algorithm. Experimental results in Section IV verify that the proposed algorithm is meaningful and effective in practice. This paper is concluded by Section V.

For the rest of this paper, letting  $\zeta$  be a dummy variable, we denote  $\hat{\zeta}$  as its estimate,  $\zeta^*$  as its reference,  $\tilde{\zeta} = \zeta - \hat{\zeta}$  as the estimation error, and  $e_{\zeta} = \zeta^* - \hat{\zeta}$  as the tracking error.

## II. PRELIMINARIES

#### A. Problem Statement

The induction motor model in a frame rotating at an angular speed  $\omega_1$  is given by

$$\dot{i}_{ds} = -\gamma i_{ds} + \omega_1 i_{qs} + \beta (\alpha \Phi_{dr} + \omega \Phi_{qr}) + \frac{u_{ds}}{\sigma}$$

$$\dot{i}_{qs} = -\omega_1 i_{ds} - \gamma i_{qs} + \beta (\alpha \Phi_{qr} - \omega \Phi_{dr}) + \frac{u_{qs}}{\sigma}$$

$$\dot{\Phi}_{dr} = -\alpha \Phi_{dr} + (\omega_1 - \omega) \Phi_{qr} + \alpha L_m i_{ds}$$

$$\dot{\Phi}_{qr} = -\alpha \Phi_{qr} - (\omega_1 - \omega) \Phi_{dr} + \alpha L_m i_{qs}$$

$$\dot{\omega} = \mu (\Phi_{dr} i_{qs} - \Phi_{qr} i_{ds}) - \frac{T_l}{J}$$

$$y = [i_{ds} \quad i_{qs}],$$
(1)

where  $i_{ds}, i_{qs}, \Phi_{dr}, \Phi_{qr}, \omega$  are system state,  $u_{ds}$  and  $u_{qs}$  are control input, y is system output,  $T_l$  external load torque, and

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all the rest notations are constant parameters. Table I lists definitions of notations. The frame with  $\omega_1 = 0$  is called the stator or stationary frame. Readers are referred to [20] for details on motor modeling. Speed-sensorless state estimation problem can be formulated as follows.

Problem 1: Given the induction motor model (1), reconstruct the full state based on stator voltages  $u_{ds}$ ,  $u_{qs}$  and stator currents  $i_{ds}$ ,  $i_{qs}$ .

Description
stator currents in $d$ - and $q$ -axis
rotor fluxes in d- and q-axis
rotor angular speed
stator voltages in d- and q-axis
angular speed of a rotating frame
rotor flux amplitude reference
rotor angular speed reference
references of stator currents in d- and q-axis
load torque
inertia
stator, mutual, and rotor inductances
stator and rotor resistances
$\frac{\frac{L_s L_r - L_m^2}{L_r}}{R_r / L_r}$
$R_r/L_r$
$L_m/(\sigma L_r)$
$R_s/\sigma + \alpha\beta L_m$
$3pL_m/(2JL_r)$
number of pole pairs

The model (1) is nonlinear, and uniform observability [17] is necessary to facilitate convergent state estimation for arbitrary control input. Analysis in [2], [21] corroborates the existence of operation regimes where the model is neither observable nor detectable. In other words, the model (1) is non-uniformly observable. Lack of local (uniform) observability poses fundamental limitations to Problem 1. Involving mechanical dynamics implies the knowledge of customer information for instance, load inertia and toque profile, which are hardly available in practice. This motivates us to adopt the adaptation idea which circumvents mechanical parameters.

Assumption 2.1: The rotor speed is a constant parameter, i.e.,  $\dot{\omega} = 0$ .

With Assumption 2.1, (1) is reduced to a linear timeinvariant (LTI) system with unknown parameter

$$\begin{aligned} \dot{x} &= A(\omega)x + Bu\\ \dot{\omega} &= 0 \end{aligned} (2) \\ y &= Cx, \end{aligned}$$

where  $x = [i_{ds}, i_{qs}, \phi_{dr}, \phi_{qr}]^{\top}$ , and

$$A(\omega) = \begin{bmatrix} -\gamma & 0 & \alpha\beta & \beta\omega \\ 0 & -\gamma & -\beta\omega & \alpha\beta \\ \alpha L_m & 0 & -\alpha & -\omega \\ 0 & \alpha L_m & \omega & -\alpha \end{bmatrix}, \ B = \frac{1}{\sigma} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Work [2] shows that (2) is state observable if and only if

$$\sqrt{\Phi_{dr}^2(t) + \Phi_{qr}^2(t)} \neq 0.$$
 (3)

*Remark 2.2:* Observerability condition (3) has the nature of 'instantaneous', and thus allows observer design with arbitrarily fast convergent rate. This is in fact weakened. In fact, PEC, in the form of

$$\int_{t}^{t+T} \sqrt{\Phi_{dr}^{2}(t) + \Phi_{qr}^{2}(t)} \mathrm{d}t \ge \epsilon > 0, \quad \forall t \in [0, +\infty),$$

is required to admit adaptive observer design with convergent estimation error dynamics. Given the linearly parameterized model (2), solving Problem 1 via adaptive observer design entails that (i) the x-system in (2) is state observable; (ii)  $\omega$  is identifiable. One can readily verify that (i) holds. Identifiability of  $\omega$  is reduced to the PEC.

# B. Baseline Adaptive Estimator & Stability Analysis

Let a short notation A be in place of  $A(\omega)$  for a concise presentation. The baseline adaptive estimator is given by

$$\dot{\hat{x}} = \hat{A}\hat{x} + Bu + L(y - \hat{y})$$

$$\dot{\hat{\omega}} = \lambda\beta(\tilde{i}_{ds}\hat{\Phi}_{qr} - \tilde{i}_{qs}\hat{\Phi}_{dr})$$

$$\hat{y} = C\hat{x},$$
(4)

where  $\hat{A} = A(\hat{\omega})$ , L is the observer gain matrix, and  $\lambda > 0$  is adaptive gain. The resultant estimation error dynamics are

$$\tilde{x} = (A - LC)\tilde{x} + \Delta A\hat{x} 
\dot{\tilde{\omega}} = -\lambda\beta(\tilde{i}_{ds}\hat{\Phi}_{qr} - \tilde{i}_{qs}\hat{\Phi}_{dr}),$$
(5)

where  $\tilde{x} = x - \hat{x}$  is the state estimation error, and  $\Delta A = A - \hat{A}$ .

The speed adaptation law in (4) is designed to ensure negative definiteness of the time derivative for the Lyapunov function candidate:  $V = \tilde{x}^{\top} \tilde{x} + (\omega - \hat{\omega})^2 / \lambda$ . Next we illustrate that the adaptive law in (4) does not suffice to guarantee the stability, even though the following assumption holds *Assumption 2.3:* 

(A-1) There exists a matrix L such that  $(A - LC)^{\top} + (A - LC) = -Q$ , with Q negative definite for all  $\omega$  in a bounded set.

(A-2) System (2) is persistently excited. Given (5), the time derivative of V is

$$\begin{split} \dot{V} &= \tilde{x}^{\top} ((A - LC)^{\top} + (A - LC)) \tilde{x} + \frac{2}{\lambda} \tilde{\omega} (\dot{\omega} - \dot{\omega}) \\ &+ \tilde{\omega} \left( 2\beta (\tilde{i}_{ds} \hat{\Phi}_{qr} - \tilde{i}_{qs} \hat{\Phi}_{dr}) + (\tilde{\Phi}_{qr} \hat{\Phi}_{dr} - \tilde{\Phi}_{dr} \hat{\Phi}_{qr}) \right) \\ &= \tilde{x}^{\top} ((A - LC)^{\top} + (A - LC)) \tilde{x} \\ &+ \tilde{\omega} \left( 2\beta (\tilde{i}_{ds} \hat{\Phi}_{qr} - \tilde{i}_{qs} \hat{\Phi}_{dr}) - \frac{2\dot{\omega}}{\lambda} + \tilde{\Phi}_{qr} \hat{\Phi}_{dr} - \tilde{\Phi}_{dr} \hat{\Phi}_{qr} \right) \\ &= \tilde{x}^{\top} ((A - LC)^{\top} + (A - LC)) \tilde{x} + \tilde{\omega} d(\tilde{x}, \hat{x}), \end{split}$$

where  $d(\tilde{x}, \hat{x}) = \tilde{\Phi}_{qr} \hat{\Phi}_{dr} - \tilde{\Phi}_{dr} \hat{\Phi}_{qr}$  is non-definite. Even under Assumption 2.3,  $\dot{V}$  is not necessarily negative for all non-zero  $\tilde{x}$  and  $\tilde{\omega}$ . In other words, stability of the estimation error dynamics result from the baseline adaptive estimator (4) cannot be established. Additionally, it is non-trivial to determine L such that Assumption 2.3 holds for all  $\omega \in \Omega \in \mathbb{R}$ with  $\Omega$  being a compact set.

## **III. STABLE ADAPTIVE ESTIMATION**

This section conducts stable adaptive estimation, in order to address the stability issue and challenge of tuning L. This is achieved by first transforming (2) into an AOF with a parameter-dependent state transformation, and then design adaptive observer in the transformed coordinates.

# A. Adaptive Observer Form

The proposed design pivots on the following AOF

$$\dot{z} = A_z z + \psi(y, u)\Theta + B_z u$$
  

$$y = C_z z,$$
(6)

where  $z \in \mathbb{R}^n$  is the state,  $\Theta \in \mathbb{R}^s$  the vector of unknown parameters,  $y \in \mathbb{R}^p$ ,  $u \in \mathbb{R}^m$ ,  $B_z$  is a constant matrix, and for  $i, j \in \{1, ..., p\}$ ,

$$A_{z} = \begin{bmatrix} A_{11} & \dots & A_{1p} \\ \vdots & A_{ij} & \vdots \\ A_{p1} & \dots & A_{pp} \end{bmatrix}, \quad C_{z} = \begin{bmatrix} C_{z1} & \dots & \mathbf{0} \\ \vdots & C_{zi} & \vdots \\ \mathbf{0} & \dots & C_{zp} \end{bmatrix}$$
$$A_{ij} = \begin{cases} \begin{bmatrix} * & \mathbf{I} \\ * & \mathbf{0} \\ * & \mathbf{0} \\ * & \mathbf{0} \end{bmatrix}, \quad i = j$$
$$C_{zi} = \begin{bmatrix} 1 & \mathbf{0} \end{bmatrix}.$$

Given a system in the AOF, one can design an adaptive state estimator which yields stable estimation error dynamics [22], [23]. Work [24, Lem. II.1] establishes that a linear time-invariant (LTI) system such as the linear motor model (2) is transformable to the AOF via a linear state transformation if and only if the model is state observable.

#### B. Transform to Adaptive Observer Form

Next we construct a linear state transformation z = Txwhich puts the model (2) into the AOF. As an intermediate step, we transforms the model into the Luenberger observable canonical form

$$\dot{z} = \begin{bmatrix} A_{11}(\omega) & A_{12}(\omega) \\ A_{21}(\omega) & A_{22}(\omega) \end{bmatrix} z + \begin{bmatrix} B_{z1}(\omega) \\ B_{z2}(\omega) \end{bmatrix} u$$
$$\dot{\omega} = 0 \qquad (7)$$
$$y = \begin{bmatrix} C_{z1} & 0 \\ 0 & C_{z2} \end{bmatrix} z,$$

where for  $i, j \in \{1, 2\}$ ,

$$A_{ij}(\omega) = \begin{cases} \begin{bmatrix} * & 1 \\ * & 0 \\ \\ * & 0 \\ \\ * & 0 \end{bmatrix}, & i \neq j \\ C_{zi} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

To this end, one verifies that the model (2) is state observable with a set of observability indices  $\{2,2\}$ . The corresponding

state transformation  $\mathbf{T}x$  is determined by the following procedure.

Step-(i) Compute an observability matrix  $Q_o$ 

$$Q_o = \begin{bmatrix} C_1 \\ C_1 A \\ C_2 \\ C_2 A \end{bmatrix},$$

where  $C_1 = [1, 0, 0, 0]$  and  $C_2 = [0, 1, 0, 0]$ . Step-(ii) Solveing starting vectors  $g_1, g_2$  from

$$(Q_o g_1)^{\top} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} (Q_o g_2)^{\top} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix},$$

we have

$$g_1 = [0, 0, \frac{\alpha}{\alpha^2 + \omega^2}, \frac{\omega}{\alpha^2 + \omega^2}]^\top$$
$$g_2 = [0, 0, \frac{-\omega}{\alpha^2 + \omega^2}, \frac{\alpha}{\alpha^2 + \omega^2}]^\top$$

Step-(iii) Calculate the inverse of the state transformation

$$\mathbf{T}^{-1}z = \begin{bmatrix} Ag_1 & g_1 & Ag_2 & g_2 \end{bmatrix} x$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ -\frac{1}{\beta} & \frac{\alpha}{\beta(\alpha^2 + \omega^2)} & 0 & \frac{-\omega}{\beta(\alpha^2 + \omega^2)}\\ 0 & \frac{\omega}{\beta(\alpha^2 + \omega^2)} & -\frac{1}{\beta} & \frac{\alpha}{\beta(\alpha^2 + \omega^2)} \end{bmatrix} z,$$

which also implies the state transformation as follows

$$\mathbf{T}x = \begin{bmatrix} 1 & 0 & 0 & 0\\ \alpha & \omega & \alpha\beta & \beta\omega\\ 0 & 1 & 0 & 0\\ -\omega & \alpha & -\beta\omega & \alpha\beta \end{bmatrix} x$$

With the state transformation  $z = \mathbf{T}x$ , the model (2) is transformed to

$$\dot{z} = \bar{A}(\omega)z + \bar{B}(\omega)u$$
  
$$\dot{\omega} = 0$$
  
$$y = \bar{C}z,$$
  
(8)

where the system matrices  $\bar{A} = \mathbf{T}A\mathbf{T}^{-1}, \bar{B} = \mathbf{T}B, \bar{C} = C\mathbf{T}^{-1}$  are given as follow

$$\bar{A}(\omega) = \begin{bmatrix} -\gamma - \alpha & 1 & -\omega & 0\\ -\alpha(\gamma - \alpha\beta L_m) & 0 & -\omega(\gamma - \alpha\beta L_m) & 0\\ \omega & 0 & -\gamma - \alpha & 1\\ \omega(\gamma - \alpha\beta L_m) & 0 & -\alpha(\gamma - \alpha\beta L_m) & 0 \end{bmatrix}$$

$$\bar{B}(\omega) = \frac{1}{\sigma} \begin{bmatrix} 1 & 0\\ \alpha & \omega\\ 0 & 1\\ -\omega & \alpha \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 1 & 0 & 0\\ 0 & 0 & 1 & 0 \end{bmatrix}.$$
(9)

The transformed system (8) is in the Luenberger observable canonical form. Taking into consideration  $y_1 = z_1, y_2 = z_3$ ,

one can further rearrange the transformed system (8) in the form of the AOF (6) with matrices given by  $C_z = \overline{C}$ , and

$$A_{z} = \begin{bmatrix} -\gamma - \alpha & 1 & 0 & 0 \\ -\alpha \kappa & 0 & 0 & 0 \\ 0 & 0 & -\gamma - \alpha & 1 \\ 0 & 0 & -\alpha \kappa & 0 \end{bmatrix}$$
(10)  
$$\psi(y, u) = \begin{bmatrix} -y_{2} \\ -\kappa y_{2} + \frac{u_{q}}{\sigma} \\ y_{1} \\ \kappa y_{1} - \frac{u_{d}}{\sigma} \end{bmatrix}, \quad B_{z} = \frac{1}{\sigma} \begin{bmatrix} 1 & 0 \\ \alpha & 0 \\ 0 & 1 \\ 0 & \alpha \end{bmatrix},$$

where  $\kappa = \gamma - \alpha \beta L_m$ . The derivation of the model in the form of (6) is completed.

# C. Adaptive Observer Design

Given the model in the form of (6), a multitude of adaptive observers can be utilized to fulfill adaptive state estimation. For the illustration purpose, we employ the result in [23] and take the adaptive observer below

$$\hat{z} = A_z \hat{z} + \psi(y, u) \hat{\omega} + B_z u + L_z(y - \hat{y}) + M \hat{\omega}$$

$$\dot{M} = (A_z - L_z C_z) M + \psi(y, u)$$

$$\dot{\omega} = \lambda M^\top C_z^\top (y - \hat{y})$$

$$\hat{y} = C_z \hat{z},$$
(11)

where  $L_z \in \mathbb{R}^{4 \times 2}$  is a constant gain matrix. Considering  $M = [M_{11}, M_{21}, M_{31}, M_{41}]^{\top}$ , the adaptive law in (11) is simplified as

$$\hat{\omega} = \lambda (M_{11}\tilde{y}_1 + M_{31}\tilde{y}_2).$$

For the sake of self-completeness, the stability analysis of the resultant estimation error dynamics is offered below. Let

$$\eta = z - M\omega,$$

and define  $\hat{\eta} = \hat{z} - M\hat{\omega}, \tilde{\eta} = \eta - \hat{\eta}, \tilde{\omega} = \omega - \hat{\omega}$ . We have  $\dot{\tilde{\eta}} = (A_z - L_z C_z)(\tilde{\eta} + M\tilde{\omega}) + \psi(y, u)\tilde{\omega} - M\dot{\tilde{\omega}} - \dot{M}\tilde{\omega} - M\dot{\tilde{\omega}}$  $= (A_z - L_z C_z)\tilde{\eta}.$ 

Finally we have the estimation error dynamics

$$\dot{\tilde{\eta}} = (A_z - L_z C_z) \tilde{\eta}$$
  

$$\dot{M} = (A_z - L_z C_z) M + \psi(y, u) \qquad (12)$$
  

$$\dot{\tilde{\omega}} = \rho M^\top C_z^\top C_z (\tilde{\eta} + M \tilde{\omega}).$$

Globally exponential stability of the zero solution of (12) is guaranteed if the following PEC holds: there exist positive finite constants  $\rho_1, \rho_2, T$  such that for any t > 0

$$0 < \rho_1 \le \int_t^{t+T} M^\top(t) C_z^\top C_z M(t) \mathrm{d}t \le \rho_2 < +\infty.$$

Remark 3.1: We have the integrated function in the PEC as  $M_{11}^2(t) + M_{31}^2(t)$ . From the *M*-dynamics, one learns that  $M_{11}$  is excited by  $y_2$  and  $u_q$ , whereas  $M_{31}$  by  $y_1$  and  $u_d$ . If  $u_d$  and  $u_q$  are zero for a long period, then the PEC is not satisfied; otherwise, the PEC generally holds. That is to say, the PEC is barely a restriction.

Note that the gain matrix  $L_z$  is to stabilize  $A_z$ , which is always possible in the virtue of state observability. Particularly,  $L_z$  could be constant, instead of being  $\omega$ -dependent for the baseline. This affirms that the proposed design offers ease of tune.

Remark 3.2: A speed-sensorless control algorithm should be robust w.r.t. offsets and noises in actuators and sensors. Thanks to the GES, the estimation error dynamics (12) is robust to bounded additive disturbances, under the persistent excitation condition. This fact can be easily observed by combining the fact that the undisturbed part of (12) is GES and input-to-state stable w.r.t. additive disturbances.

#### IV. EXPERIMENTAL VALIDATION

We perform closed-loop experiments, where either the proposed (11) or the baseline estimation algorithm 4 runs in closed-loop, and the other runs in open-loop. Experiments demonstrate that during the operation where the motor is persistently excited, the proposed algorithm leads to improved speed tracking performance, by effectively rejecting uncertainties arising from model mismatches, sensors and the voltage source inverter.

The testbed comprises Matlab/Simulink®, dSPACE® ACE Kit DS1104, a Myway®DC-AC inverter, and a Marathon®three-phase AC induction motor with an inertial load. During experiments, the dSPACE executes the data acquisition and real-time estimation and control tasks. Both the control loop and the PWM operates at 4kHz. The motor has parameter values: rated power 0.18KW,  $R_s = 11.05\Omega, R_r = 6.11\Omega, L_s = L_r = 0.3165H, L_m =$  $0.2939H, J = 1.2e - 3kgm^2$ . Note that although a higher PWM frequency might improve system performance, it however overruns the dSPACE and leads to compilation failure.

# A. Tracking Controller and State Estimator

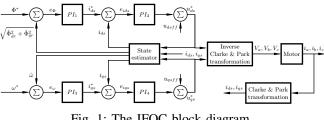


Fig. 1: The IFOC block diagram.

The tracking controller implements an indirect field oriented control (IFOC) shown in Fig. 1. Four Proportional and Integral (PI) controllers  $PI_i, 1 \leq i \leq 4$  are used to regulate the speed, the rotor flux amplitude, the q-axis stator current, and the *d*-axis stator current, respectively. Feedforward  $u_{dsff} = -\sigma \omega_1 i_{qs}$  and  $u_{qsff} = \sigma (\omega_1 i_{ds} + \beta \omega \hat{\Phi}_{dr})$  are employed to improve the tracking control performance.

For the baseline estimator (4), its speed adaptation gain  $\lambda\beta$  and flux observer gain L are tuned by trial and error to balance the harmonics reduction during steady state and fast estimation during transient. For the proposed estimator

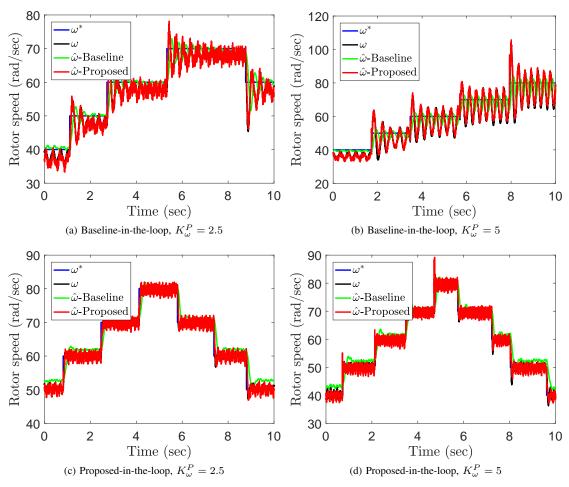


Fig. 2: Speeds: solid blue-reference; solid black-measured; solid green- estimated from baseline; solid red-estimated from the proposed algorithm

(11), its flux observer gain  $L_z$  is determined by placing the poles  $A_z - L_z C_z$  at p = -400, in order to achieve robust flux estimation against model mismatches, whereas the speed adaptation gain is tuned by trial and error, with its order of magnitude being approximately  $p^2$ . The flux field angle is estimated according to the following equation

$$\dot{\rho} = \omega + \frac{\alpha i_{qs}^*}{i_{ds}^*}, \quad \rho(0) = 0.$$

The estimated flux field angle is used in the Clarke/Park transformation and its inverse blocks.

# B. Experimental Results

With a focus on the bandwidth of speed estimation (related to the speed tracking), we conduct tests where the reference speed includes step changes, and examine how fast two estimation algorithms converge. Baseline and proposed (11) algorithms work well when the gain of the speed controller is low. As the proportional gain, denoted by  $K_{\omega}^{P}$ , of the speed controller gradually increases, the speed tracking performance result from the baseline degrades more significantly. This is shown in Fig. 2.

Fig. 3 compares speed tracking responses of two closedloop systems against steps of reference speed. Oscillation in Fig. 3(a) is largely due to harmonics in estimated speed induced by model mismatches and uncertainties in current sensors and the voltage source inverter. Fig. 3(c) infers that system with the proposed algorithm in the loop performs reliably. Fig. 3(b) indicates that the proposed estimator (11) in open-loop quickly tracks the speed. Fig. 3(d) says that baseline tracks the speed, albeit slowly.

During experiment, we observed significant harmonics in the rotor speed curves when the speed tracking control block adopts a large proportional gain. This indeed means that the speed estimation is not fast enough. We attribute the difficulty in cranking up speed controller gain to the presence of large model mismatches in motor dynamics. Typically, the estimator gain should be large enough to attenuate model mismatches, which however amplifies measurement noises. Such an apparent trade-off prevents us from using large gains in estimator, and renders slow and biased speed estimation. We stress that with the same controller, the proposed algorithm renders better speed tracking performance, over the baseline algorithm, where similar levels of tuning efforts are invested

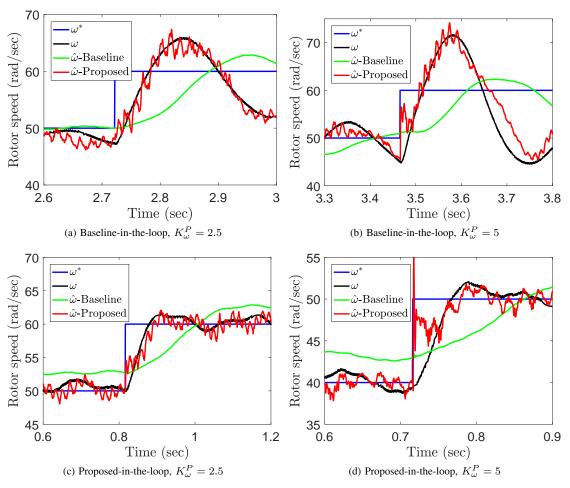


Fig. 3: Speeds: solid blue-reference; solid black-measured; solid green- estimated from baseline; solid red-estimated from the proposed algorithm

in the proposed and baseline algorithms. This corroborate that the proposed algorithm, under the assumption of persistent excitation condition, is relatively more robustness.

*Remark 4.1:* The proposed algorithm ensures stability under the condition of persistent excitation, which essentially requires that the rotor flux is time-varying for a certain period of time interval. In other words, the proposed algorithm is not suitable in the operation regions when the rotor flux barely changes. As far as computation complexity, it presents a heavier load compared to baseline. In fact, it is evident that the baseline estimator involves integrating 6 first order ordinary differential equations (ODEs) at every sampling time, whereas the proposed estimator involves 21 first order ODEs, because the auxiliary signal M is a  $4 \times 4$  matrix, and it is generated by integrating 16 first order ODEs. Consequently, the proposed algorithm possesses a comparable computation load against Kalman filter-based estimators.

*Remark 4.2:* Experiment in this work remains preliminary, in the sense that the induction motor is tested in a small operation region. Specifically, the induction motor is merely coupled with an inertia load, and not subject to external torque. We leave extensive experimental validation for future

work.

# V. CONCLUSION AND FUTURE WORK

This paper proposed and verified a new estimation algorithm for speed-sensorless motor drives. The proposed algorithm first transforms the motor model into an adaptive observer form by a change of state coordinates, and then performs adaptive observer design in the new coordinates. Globally exponential stability can be obtained for the estimation error dynamics in the original coordinates, as long as the PEC holds. Closed-loop experiments verified the effectiveness and advantages: guaranteed stability and ease of tuning.

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