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TR2019-130 November 13, 2019

### Abstract

Naive estimation of horizontal wind velocity over complex terrain using measurements from a single wind-LiDAR introduces a bias due to the assumption of uniform velocity in any horizontal plane. While Computational Fluid Dynamics (CFD)-based methods have been proposed for bias removal, there exist several issues in the implementation. For instance, the upstream atmospheric boundary layer thickness or direction are unknown. Conventional CFD-based corrections use trial and error to estimate the bias. Such approaches not only become numerically intractable for complicated flows, e.g. when the number of unknowns is large, but they also suffer from the fact that there is no guarantee for optimality of the obtained results. We propose a direct-adjoint-loop (DAL) optimization based framework to estimate such unknown parameters in a systematic way. For the validation of the method, we performed an experimental study using DIABREZZA LiDAR on a complex terrain for two wind directions of northwesterly (NW) and southeasterly (SE). The slope error improved from -0.09 to -0.02 for SE and from -0.09 to +0.04 for NW.

*NAWEA/WindTech Conference*

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# Improving LiDAR performance on a complex terrain using CFD-based correction and direct-adjoint-loop optimization

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**Abstract.** Naïve estimation of horizontal wind velocity over complex terrain using measurements from a single wind-LiDAR introduces a bias due to the assumption of uniform velocity in any horizontal plane. While Computational Fluid Dynamics (CFD)-based methods have been proposed for bias removal, there exist several issues in the implementation. For instance, the upstream atmospheric boundary layer thickness or direction are unknown. Conventional CFD-based corrections use trial and error to estimate the bias. Such approaches not only become numerically intractable for complicated flows, e.g. when the number of unknowns is large, but they also suffer from the fact that there is no guarantee for optimality of the obtained results. We propose a direct-adjoint-loop (DAL) optimization based framework to estimate such unknown parameters in a systematic way. For the validation of the method, we performed an experimental study using DIABREZZA LiDAR on a complex terrain for two wind directions of northwesterly (NW) and southeasterly (SE). The slope error improved from -0.09 to -0.02 for SE and from -0.09 to +0.04 for NW.

## 1. Introduction

The vertical wind profiling LiDARs retrieve the horizontal velocity,  $V_h$ , by processing the line-of-sight (LOS) velocities at each height assuming homogeneous  $V_h$  over the sampled volume. In order to retrieve the wind velocity vector at a given height, it is assumed that the flow remains homogeneous over the sampled volume at a given height. Using such assumption, the three components of the wind vector velocity are reconstructed using trigonometric relations from the radial LOS velocities given by LiDAR as written below

$$\begin{aligned}
v_x^L &= \frac{LOS_2 - LOS_4}{2\sin\varphi}, \\
v_y^L &= \frac{LOS_1 - LOS_3}{2\sin\varphi}, \\
w &= LOS_0, \\
V_h^L &= \sqrt{(v_x^L)^2 + (v_y^L)^2}
\end{aligned} \tag{1}$$

where the definition of homogeneous velocity components  $v_x^L, v_y^L, w$ , azimuth angle  $\theta$ , elevation angle  $\varphi$  are given in Fig. 1. Also shown in Fig. 1 is the direction of transmitted beams from the LiDAR along which LOS velocities are measured.

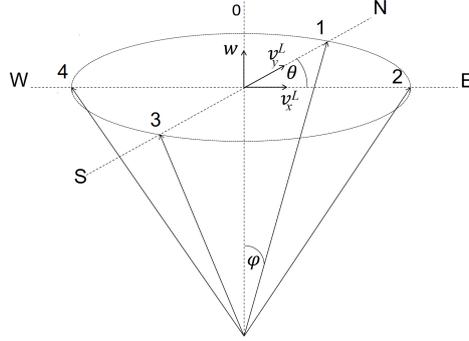
The excellent performance of a single LiDAR such as DIABREZZA is demonstrated on a flat terrain in terms of regression error and LiDAR availability when Eq. 1 is used [1]. However, on a complex terrain, the bias due to the homogeneous assumption may increase [2]. The error is due to the variation of vertical velocity,  $w$ . In this case, the un-biased velocity components  $v_x, v_y$  to the first order, at each height are given by

$$\begin{aligned}
v_x &= v_x^L - z \frac{dw}{dx}, \\
v_y &= v_y^L - z \frac{dw}{dy}, \\
V_h &= \sqrt{(v_x)^2 + (v_y)^2}
\end{aligned} \tag{2}$$

As indicated by Eq. 2, the corrected components of wind vector velocity can be retrieved from the ones that are evaluated based on homogeneous assumption minus the gradients of vertical velocity at each direction multiplied by the elevation. The main challenge for the correction of the LiDAR measurements in case of distortions in the wind vector, e.g. flow over a complex terrain, is that it is not feasible to compute the vertical velocity gradients solely based on the LOS data. Instead, other fluid models such as computational fluid dynamics (CFD) should be used to estimate  $\frac{dw}{dx}$  and  $\frac{dw}{dy}$  at each height above the LiDAR,  $z$ .

There are examples in the literature [3, 4, 5, 6] in which linear models or CFD simulations were used to correct the LiDAR measurements to retrieve  $V_h$  using an approach similar to employing Eq. 2 on a complex terrain; however, as we see in the next section, there are non-trivial challenges for performing the numerical simulation in such cases. For instance, Hofsäß et al. [6], which proposed an extension of Eq. 2, assumed a linear model to relate the gradient of the wind vector velocity to the known variables. They demonstrated that the use of the gradient of velocity for LiDAR retrieval improves the reconstruction accuracy. However, the linear model uses many assumptions and is not as accurate as CFD models. In this paper, we propose the use of direct-adjoint-looping (DAL) optimization method [7] to overcome the difficulties of CFD simulations when the operating conditions, e.g. the inlet velocity profile, are unknown. Adjoint method gives the gradient of the cost function, which is used for estimation and data-assimilation, with a computational cost comparable to that of a single CFD simulation regardless of the number of unknowns.

The rest of the paper is organized as follows. In Section 2, we first discuss the governing equations and the numerical method for the CFD simulation. We then formulate the continuous adjoint equations used in the DAL method and discuss the implementation for atmospheric flows for LiDAR correction. In Section 3, the experimental campaign is described. In Section 4, we discuss the results of CFD-based optimization and correction on LiDAR retrieval in terms of regression error and uncertainty percentage. In Section 5, we provide the concluding remarks.



**Figure 1.** Five beam directions (0-4) of the LiDAR and definition of azimuth angle  $\theta$  and elevation angle  $\varphi$ . The radial beams are along the LOS velocities. Also shown are the wind velocity components  $v_x, v_y$ , and  $w$  with respect to their orientation to North (N), East (E), South (S) and West (W) directions.

## 2. Correction method

### 2.1. Governing equations

The flow is governed by steady-state incompressible Navier-Stokes equations described below (using Einstein notation)

$$\begin{aligned} \frac{\partial v_j}{\partial x_j} &= 0, \\ \frac{\partial v_i v_j}{\partial x_j} + \frac{\partial p_i}{\partial x_i} - \frac{\partial}{\partial x_j} (\nu_{eff} \frac{\partial v_i}{\partial x_j}) &= 0 \end{aligned} \quad (3)$$

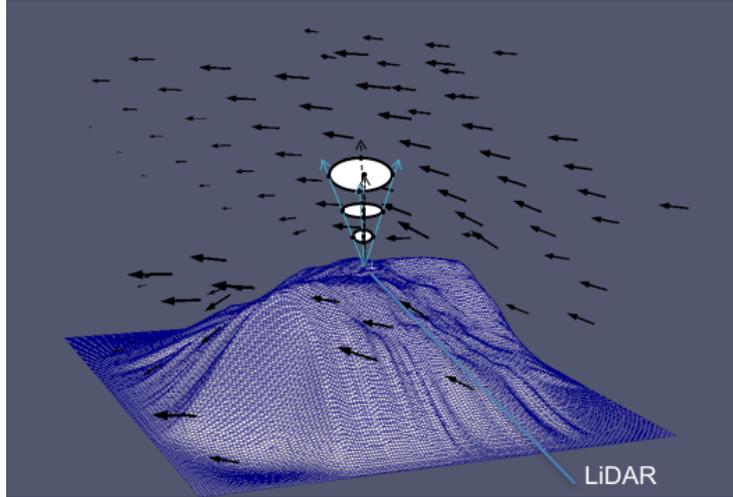
with  $\mathbf{v}, p$  as time-averaged velocity and pressure and  $\nu_{eff}$  is the effective viscosity. In this study, we use two-equation turbulence model with Reynolds Averaged-Navier-Stokes (RANS) formulation to calculate  $\nu_{eff}$  and flow field variables.

Boundary conditions are as follows: at the surface of the terrain the no-slip boundary condition is used i.e.  $\mathbf{v} = 0$ . For the outlet, we use the zero Neumann boundary condition i.e. we set the normal gradient of velocity to be zero  $(n_i \partial / \partial x_i) v_i = 0$  with  $\mathbf{n}$  as the normal unit vector. Consistent with various studies dealing with CFD simulation of airflow in the complex terrain [8], we use the standard log-profile model for boundary condition at inlet, which is given below

$$\begin{aligned} \mathbf{v}_{in} &= \frac{\mathbf{v}^*}{\kappa} \ln \left( \frac{z - z_g + z_0}{z_0} \right), \\ \mathbf{v}^* &= \kappa \frac{\mathbf{v}_{ref}}{\ln \frac{z_{ref} + z_0}{z_0}} \end{aligned} \quad (4)$$

with  $\kappa = 0.41$  as von Karman's constant,  $\mathbf{v}^*$  as friction velocity,  $\mathbf{v}_{ref}, z_{ref}$  as some reference velocity and height,  $z_0$  as surface roughness height, and  $z_g$  as ground height.

In Eq. 4,  $\mathbf{v}_{ref}$  needs to be determined at a certain elevation such that the inlet velocity profile becomes determined. Thus, a major challenge in solution of equations 3 and 4 over complex terrain is that the value for  $\mathbf{v}_{ref}$  is unknown and needs to be estimated. Our approach is to take advantage of the fact that LOS velocities are available. Hence, one can use such data and



**Figure 2.** A typical simulation of airflow over a complex terrain. Velocity vectors are denoted by black vectors. The location of LiDAR is highlighted at the tip of the upsides down cone.

assimilate it into Navier-Stokes equations for the best estimation of the unknown parameters. In this paper, we propose an optimization-based solution for LiDAR data-assimilation to estimate such unknowns.

### 2.2. Details of Numerical Solver

We use OpenFOAM [9] for CFD simulations, which is based on a finite-volume method with a collocated grid arrangement and offers object-oriented implementations that suit the employed continuous adjoint formulation used for optimization as well as simulation of atmospheric flows on a complex terrain. Pressure and velocity are decoupled using the SIMPLE algorithm technique in the state/adjoint equations. For the convection terms, second order Gaussian integration is used with the Sweby limiter [10] for numerical stability of external flows while maintaining the accuracy. For diffusion, Gaussian integration with central-differencing-interpolation is used. A steady-state and incompressible solver with  $k - \epsilon$  Reynolds-averaged Navier Stokes (RANS) turbulence closure model was employed on discretized domain comprising of 4.2 million nodes mapping a terrain swathe of 1.6km 2.5 km 2.5 km. Fig. 2 shows a typical snapshot of the solution on the numerical domain used in this study.

It should be noted that rough wall-functions for the atmospheric boundary layer (ABL) is used in OpenFOAM to be consistent with formulation of [8]. OpenFOAM and RANS turbulence closure models have been tested for complex terrain simulations in numerous examples (see for instance [11, 12]).

### 2.3. Optimization using continuous adjoint method

As discussed in Section 2.1., in order to carry out a correct CFD simulation on a complex terrain, the inlet boundary conditions need to be estimated, since the values of  $\mathbf{v}_{ref}$  can greatly impact the solution of equations 3. Conventional methods use trial and error for such a purpose. But they lack any optimally guarantee for the results and also the computational cost is greatly larger. In this study, we propose an optimization algorithm to estimate the unknowns for CFD simulation in a systematic way. The optimization problem is formulated as

$$\begin{aligned} \min_{\mathbf{v}_{ref}} \mathcal{J} &= \sum_i^N \gamma_i (LOS_i - LOS_{CFD,i})^2, \\ \text{s.t. } \mathcal{R}(\mathbf{v}, p, \mathbf{v}_{ref}) &= 0, \end{aligned} \quad (5)$$

where coefficients  $\gamma_i$  are the weighting factors in the cost function and  $\mathcal{R}$  denotes the constraints arising from the state governing equations, i.e. Navier-Stokes Eq. 3 and the inlet boundary condition 4. Inspired by Eq. 2, we set  $\gamma_i$  to be proportional to the height above the LiDAR. Moreover,  $N$  is the total number of the measurements. As shown in Fig. 1, for each elevation there are 5 LOS measurements. Hence, the total number of available data is 5 times the number of ranges over which LOS is measured. In this study, at 20 elevations data is measured so in overall  $N = 100$ . LOS, as depicted in Fig. 1, is the radial line-of-sight-velocity. We hence define the cost function  $\mathcal{J}$  as the square of difference between LOS values measured by LiDAR and CFD simulations at each measurement point. The unknown parameters, i.e.  $\mathbf{v}_{ref}$  at inlet of the domain, are determined as minimizers of such cost function defined in the constrained optimization problem of Eq. 5. We use a direct-adjoint-looping (DAL) method [7], briefly explained below, to solve the optimization problem and estimate the CFD unknowns.

In order to solve the optimization problem, we define Lagrangian  $\mathcal{L}$  to enforce the Navier-Stokes equations and log-profile boundary conditions, as

$$\min_{\mathbf{v}_{ref}} \mathcal{L} = \mathcal{J} + \langle \mathcal{P}, \mathcal{R} \rangle, \quad (6)$$

where  $\mathcal{P} = (\mathbf{u}, p_a)$  is the vector of adjoint variables,  $\mathbf{u}$  as adjoint velocity, and  $p_a$  as adjoint pressure. We use the notation  $\langle f, g \rangle = \int_{\mathbb{D}} fg dV$  with  $\mathbb{D}$  as the whole domain. The adjoint variables are interpreted as Lagrange multipliers to enforce the state equations Eq. 3 and 4. To ensure (at least locally) the optimality of the solution, we enforce  $\delta \mathcal{L} = \delta_{\mathbf{v}_{ref}} \mathcal{L} + \delta_{\mathbf{v}, p} \mathcal{L} = 0$ , where  $\delta$  denotes variation of a dependent variable. We choose the adjoint variables such that  $\delta_{\mathbf{v}, p} \mathcal{L} = 0$ . The sensitivity equations with respect to unknown parameter is then obtained as  $\delta \mathcal{L} = \delta_{\mathbf{v}_{ref}} \mathcal{L}$ . This idea is the core of the adjoint method (please refer to appendix A of [7] for details of our derivation).

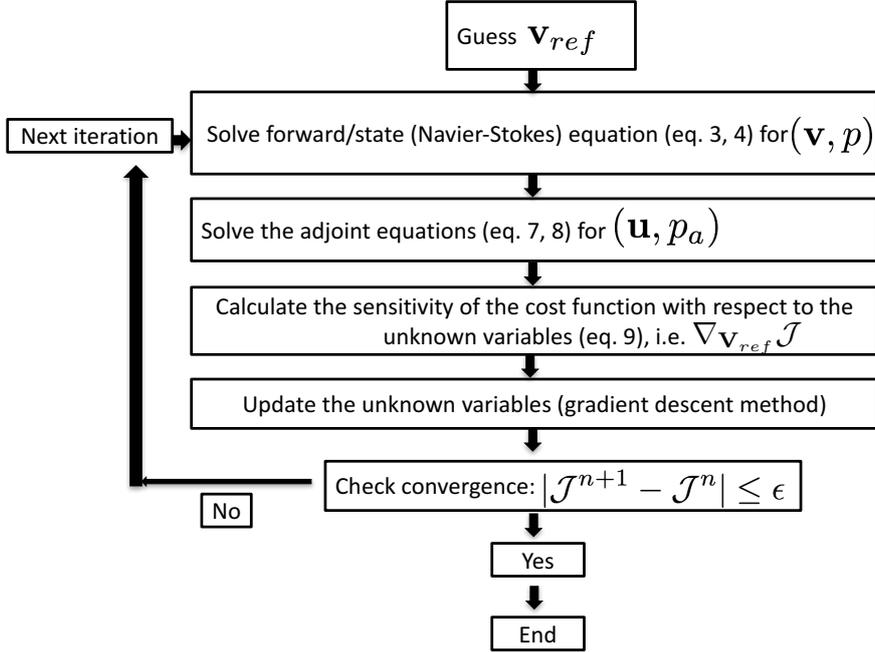
By enforcing that first order variations with respect to the state variables vanish at optimal solutions, i.e.,  $\delta_{\mathbf{v}, p} \mathcal{L} = 0$ , we obtain the adjoint equations

$$\begin{aligned} \frac{\partial u_j}{\partial x_j} &= 0, \\ u_j \frac{\partial v_j}{\partial x_i} - v_j \frac{\partial u_i}{\partial x_j} - \frac{\partial}{\partial x_j} (\nu_{eff} \frac{\partial u_i}{\partial x_j}) + \frac{\partial p_a}{\partial x_i} + \frac{\partial \mathcal{J}}{\partial \mathbf{v}} &= 0 \end{aligned} \quad (7)$$

The adjoint boundary conditions are

$$\begin{aligned} \text{inlet} : \mathbf{u} &= 0, (n_i \partial / \partial x_i) p_a = 0, \\ \text{outlet} : v^n u^t + \nu_{eff} (n_i \partial / \partial x_i) u^t &= 0, \\ p_a &= v^n u^n + \nu_{eff} (n_i \partial / \partial x_i) (u^n), \\ \text{terrain} : \mathbf{u} &= 0, (n_i \partial / \partial x_i) p_a = 0, \end{aligned} \quad (8)$$

where  $u^n$  and  $u^t$  are the normal and tangential component of adjoint velocity, respectively.



**Figure 3.** Flow chart for the Direct-Adjoint-Looping method.

In deriving Eqs. 7 and 8, we use the ‘frozen turbulence’ hypothesis; that is we ignore the variation of turbulent eddies while solving the adjoint equations [13]. An assessment of the validity of this assumption can be done for a given problem by comparing adjoint sensitivities to those computed using a finite-difference method. For the range of Reynolds number considered in this study, we realized such an assumption is indeed an appropriate one.

After computation of the adjoint field we obtain the sensitivity of the cost function with respect to inlet velocity and temperature, i.e. the design variables, as follows

$$\nabla_{\mathbf{v}_{ref}} \mathcal{J} = p_a|_{in} \mathbf{n} - \nu_{eff}(\mathbf{n} \cdot \nabla) v_a|_{in}, \quad (9)$$

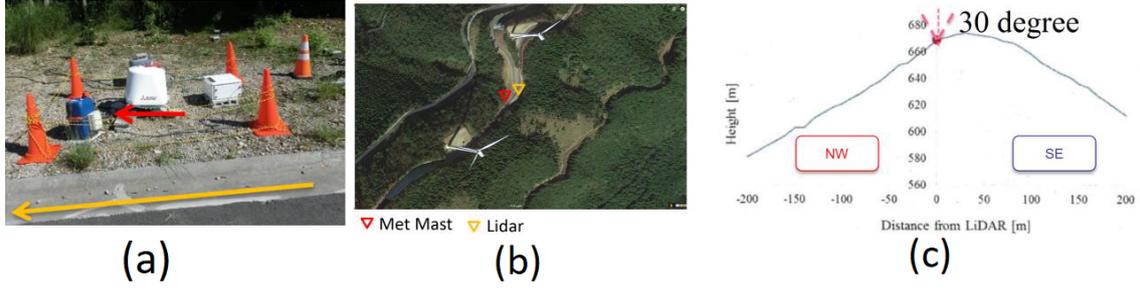
Please note that Eq. 9 is a vector equation that deals with all components of  $\mathbf{v}_{ref}$ . Once the value of the gradient is known, we can use the gradient descent approach to find the minimizer of Eq. 5, which gives the value of unknown parameters required for Eq. 4.

#### 2.4. DAL algorithm and implementation

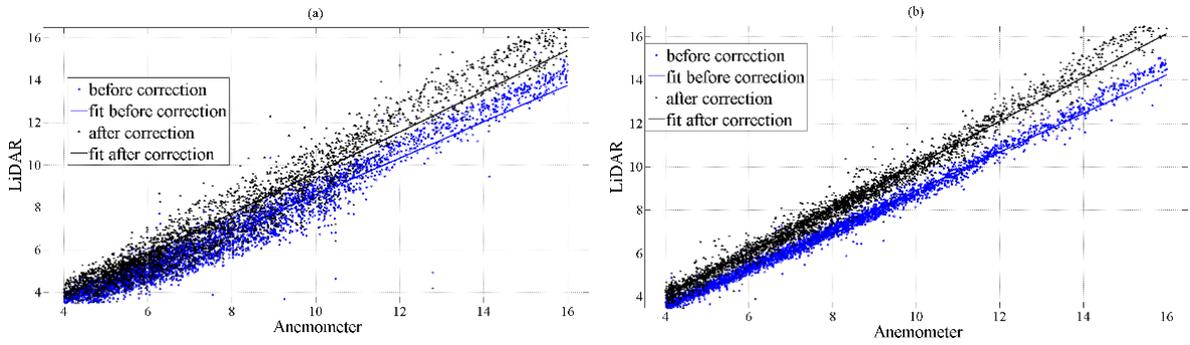
We illustrate the iterative solution procedure schematically in Fig. 3. Determination of a solution begins with an initial guess for the the direction and magnitude of  $\mathbf{v}_{ref}$ . The set of ‘direct’ state and adjoint equations are solved in a loop and the subsequent sensitivity calculation is used to obtain the next guess for the optimal values of the unknowns, i.e. the inlet boundary condition. This process is repeated until the convergence criterion for the cost functional is satisfied, i.e.  $|\mathcal{J}^{n+1} - \mathcal{J}^n| \leq \epsilon$ , with  $n$  as the number of iterations.

### 3. Description of Experiments

The measurements were obtained in complex terrain as shown in Fig 4. A reference mast with cup anemometers was erected nearby to serve as the ground truth for the LiDAR measurements. The data were filtered using the concept of automatic adapting parameters function to maximize the signal-to-noise ratio for various atmospheric condition [1]. The averaging period was set at 10-min intervals.



**Figure 4.** a) Field test site and device, b) site map, c) terrain profile. The measurement period is 3 months.



**Figure 5.** Comparison of coefficients of correlation. a) NW, b) SE wind. Before CFD correction: blue, after CFD correction: black.

#### 4. Results and Discussion

We first summarize the LiDAR horizontal velocity correction and validation as follows:

- We employ Eq. 1 over the course of 10-minutes to find the averaged values of  $v_x^L$ ,  $v_y^L$ .
- We use the DAL algorithm of the form described in Fig. 3. To do so, we use the LOS measurement given by LiDAR (determined in previous step) to evaluate the cost function 5. The outcome of DAL algorithm is the inlet velocity profile given by Eq. 4 such that the CFD problem of Eq. 3 is fully determined.
- We then extract the values of the vertical velocity gradient i.e.  $\frac{dw}{dx}$  and  $\frac{dw}{dy}$  at each height.
- We then use Eq. 2 to find the un-biased components of the velocity.
- For each 10-minutes interval we compare the horizontal velocity obtained by the correction method with that of the anemometer.

Fig 5 illustrates LiDAR to cup horizontal wind speed correlation at 57m altitude on a complex terrain before and after CFD-correction based on optimization. As shown for both wind directions of NW and SE, before correction a large under-estimation is observed. Such bias, which is due to homogeneous assumption, is removed significantly after the correction.

The improvement after correction for both slope and total average uncertainty, using IEC 61400-12-1, is apparent as also confirmed by results of Table 1. In the analysis, the uncertainty on the non-homogeneous flow is assumed to be neglected owing to the CFD correction but 2% uncertainty caused by complex flow is considered for the uncertainty of the cup anemometer

instead. The slope error improved from -0.09 to -0.02 for SE and from -0.09 to +0.04 for NW and the total average uncertainty decreased from 12.37% to 4.77%.

Table 1		Before correction					After correction				
Regression $y = \alpha x + \beta$		2 parameter		3 parameter			2 parameter		3 parameter		
		$\alpha$	$R^2$	$\alpha$	$\beta$	$R^2$	$\alpha$	$R^2$	$\alpha$	$\beta$	$R^2$
NW	4 - 16 m/s	0.861	0.952	0.911	-0.45	0.956	0.963	0.93	1.044	-0.734	0.937
SE	4 - 16 m/s	0.89	0.987	0.913	-0.21	0.988	0.952	0.974	0.976	-0.21	0.972
Uncertainty		12.37 %					4.77 %				

## 5. Conclusions

We demonstrated the CFD- correction based on optimization could significantly reduce the under-estimation between LiDAR and cup anemometer by accessing vertical wind variations from simulations with data assimilation. Validation is carried out for a complex terrain campaign data. Promising results were achieved in terms of the regression slope and the total average uncertainty.

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