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Terahertz Imaging of Multi-Level Pseudo-Random Reflectance

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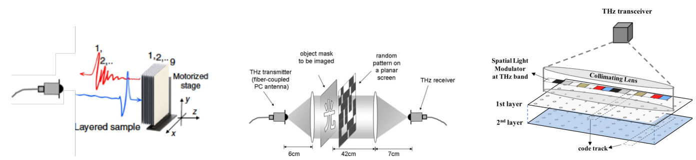
***Abstract*—This paper introduces a terahertz (THz)-based absolute positioning system with a *single* THz transceiver as the read head and a *multi-level pseudo-random* reflectance pattern (e.g., multi-level m-sequences) as the high-resolution scale in a compressed scanning mode. One of key technical challenges here is to computationally recover the multi-level pseudo-random reflectance pattern from compressed measurements. To this end, we develop a variational Bayesian approach to exploit the finite alphabet of reflectance levels and enable a pixel-wise iterative inference for fast recovery. Numerical results confirm the effectiveness of the proposed method.**

I. Introduction

Over the past years, there has been an increased interest in the use of terahertz (THz) wave for sensing, detection and imaging. THz sensing can operate in a *raster* or *compressed* scanning mode [1]–[3].

In the raster scanning mode, as shown in Fig. 1 (a), the sample under inspection is illuminated by a THz point source with a time-compact source pulse and a small spot size. The THz emitter sends a focused beam to inspect a small area of the sample, and a programmable mechanical raster moves the sample in order to measure the two-dimensional surface of the sample. In the compressed scanning mode, as shown in Fig. 1 (b), the THz pulse is first collimated to a broad beam and then spatially encoded with a random mask with the help of a spatial light modulator [3]. At the receiver side, the spatially encoded beam is re-focused by a focusing lens and received by a single-pixel photoconductive detector. The sample image can then be recovered by sparsity-driven minimization methods. Compared with the raster scanning mode, the compressed scanning mode has a much shorter acquisition period without a mechanical raster move.

Here we are particularly interested in THz-based absolute positioning systems where pseudo-random sequences (e.g., M-sequences) are used for high-resolution position encoding [4]–[6]. Fig. 1 (c) shows a THz absolute positioning system which uses a single THz transceiver, along with random masks and collimating/focusing lenses, to scan an area of the scale encoded by a multi-layer, multi-track, multi-level pseudo-random code pattern which is mapped into a unique position (hence absolute positioning). An example of the multi-level scale is shown in Fig. 3 (a), where 4 different levels are arranged into a pseudo-random code pattern in order to uniquely encode a position. The multi-level encoding at the scale can be realized by a metamaterial plate designed to reflect energy proportional to the polarization direction of the



(a) (b) (c)

Fig. 1. THz sensing with a) a raster scanning (from [2]), b) a compressed scanning (from [3]), and c) a multi-layer THz encoder system.

incident THz wave [6]. In this paper, we aim to address the remaining technical challenge: how to recover the multi-level pseudo-random code pattern with compressed measurements received at the single THz transceiver for real-time positioning.

II. Proposed Scheme

In this paper, we exploit the *non-negative, finite alphabet* features of the code pattern to recover the reflectance pattern from compressed THz measurements. To this end, we use a variational Bayesian framework to impose a hierarchical prior model for enforcing the two features and to develop a decoupled element-wise iterative algorithm to estimate the pseudo-random pattern in a computationally efficient way.

A. Compressed Measurements

Let $\boldsymbol{x} = [x_1, \cdots, x_N]^T$, $x_n \in \{\mu_1, \cdots, \mu_K\}$ denote the pseudo-random code pattern to be estimated with μ_k specifying the non-negative reflectance from a finite set of K unknown levels. The compressed scanning generates the following measurements

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{v}, \tag{1}$$

where each row of $\boldsymbol{A}$ represents a random mask at the THz band, $\boldsymbol{v} = [v_1, \cdots, v_M]^T$ is the Gaussian noise with zero mean and variance β^{-1}, and $\boldsymbol{y} = [y_1, \cdots, y_M]^T$ collects M compressed measurements.

To account for the *non-negative, finite alphabet* features of x_n, we impose a hierarchical prior model on x_n

$$\mathbb{P}\left(x_n | \boldsymbol{\alpha}_n, \boldsymbol{C}_n; \boldsymbol{u}\right) = \prod_{i=1}^{K} \mathcal{N}_+\left(x_n; \mu_i, \alpha_{n,i}^{-1}\right)^{C_{n,i}}, \tag{2}$$

where $\boldsymbol{C}_n = [C_{n,1}, \cdots, C_{n,K}]$ is a label vector with only one non-zero element assigning one of the K truncated Gaussian

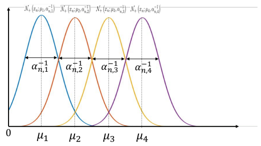


Fig. 2. The truncated Gaussian mixture distribution for the n-th reflectance x_n with 4 components.

components to x_n and

$$\mathcal{N}_+\left(x_n;\mu,\alpha^{-1}\right)=\begin{cases}\eta^{-1}\sqrt{\frac{\alpha}{2\pi}}e^{-\frac{\alpha(x-\mu)^2}{2}} & x\geq 0,\\ 0, & x<0,\end{cases} \quad (3)$$

with $\eta=1-\Phi\left(-\mu\sqrt{\alpha}\right)$ denoting the normalization factor and $\Phi\left(\cdot\right)$ denoting the cumulative distribution function of the standard normal distribution. Moreover, we assume that the label variable $\boldsymbol{C}_n$ follows the categorical distribution or generalized Bernoulli distribution $\mathbb{P}\left(\boldsymbol{C}_n;\boldsymbol{\pi}\right)=\prod_{i=1}^{K}\pi_i^{C_{n,i}}$, with event probabilities $\boldsymbol{\pi}=[\pi_1,\cdots,\pi_K]$ where $\sum_{i=1}^{K}\pi_i=1$. It is easy to see that

$$\mathbb{P}\left(x_n|\boldsymbol{\alpha}_n;\boldsymbol{u}\right)=\sum_{i=1}^{K}\pi_i\cdot\mathcal{N}_+\left(x_n;\mu_i,\alpha_{n,i}^{-1}\right), \quad (4)$$

results in the truncated Gaussian mixture distribution for x_n which is illustrated in Fig. 2 for the case of $K=4$. We further assume the Gamma distribution for $\boldsymbol{\alpha}_{n,i}$, i.e., $\mathbb{P}\left(\boldsymbol{\alpha}|a;b\right)=\prod_{i=1}^{K}\prod_{n=1}^{N}\text{Gamma}\left(\alpha_{n,i}|a,b\right)$ with $a=b=10^{-6}$.

B. Proposed Code-Pattern Recovery Algorithm

1) Decoupled element-wise likelihood function: To enable an element-wise recovery algorithm, we first decouple the original likelihood function of $\mathbf{y}$ into a decoupled approximate likelihood function of $\{x_n\}_{n=1}^{N}$

$$\mathbb{P}\left(\boldsymbol{y}|\boldsymbol{x};\beta\right)\approx\prod_{n=1}^{N}\frac{1}{\sqrt{2\pi\hat{\tau}_n}}e^{-\frac{(x_n-\hat{r}_n)^2}{2\hat{\tau}_n}}. \quad (5)$$

where the approximated element-wise mean $\hat{r}_n$ and variance $\hat{\tau}_n$ can be found in a similar way of [5].

2) Posterior distributions of hidden variables $\{\boldsymbol{x},\boldsymbol{\alpha},\boldsymbol{C}\}$: Next, we derive the posterior distributions for hidden variables $\{\boldsymbol{x},\boldsymbol{\alpha},\boldsymbol{C}\}$. *The element-wise reflectance* $\{x_n\}_{n=1}^{N}$ follows an independent truncated Gaussian posterior distribution,

$$q\left(x_n\right)=\begin{cases}\phi_n^{-1}\frac{1}{\sqrt{2\pi}\tilde{\sigma}_n}\exp\left(-\frac{(x_n-\tilde{\mu}_n)^2}{2\tilde{\sigma}_n^2}\right) & x_n>0\\ 0 & x_n\leq 0\end{cases}, \quad (6)$$

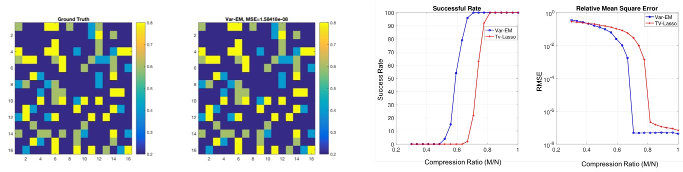
where $\phi_n=1-\Phi\left(-\tilde{\mu}_n/\tilde{\sigma}_n\right)$ is the normalization factor. *The label vector $\boldsymbol{C}$* has the categorical posterior distribution as

$$q\left(C_{n,i}\right)=\prod_{i=1}^{K}\left(\tilde{\pi}_{n,i}\right)^{C_{n,i}} \quad (7)$$

with $\tilde{\pi}_{n,i}=\exp(\gamma_{n,i}-\ln(\sum_{i=1}^{K}\exp(\gamma_{n,i})))$ and $\gamma_{n,i}=-0.5\langle\alpha_{n,i}\rangle\langle(x_n-\mu_i)^2\rangle-\langle\ln\eta_{n,i}\rangle+\ln\pi_i$. *The variable $\boldsymbol{\alpha}$* has the Gamma posterior distribution, i.e.,

$$q\left(\alpha_{n,i}\right)=\text{Gamma}\left(\alpha_{n,i}|\tilde{a}_{n,i},\tilde{b}_{n,i}\right) \quad (8)$$

with $\tilde{a}_{n,i}=a+0.5\left\langle C_{n,i}\right\rangle$, $\tilde{b}_{n,i}=b+0.5\langle C_{n,i}\rangle\langle(x_n-\mu_i)^2\rangle$.



(a) (b)

Fig. 3. Numerical validation with a 4-level pseudo-random pattern: (a) Ground truth versus recovered patterns; (b) Success rate and normalized MSE as a function of compression ratio.

3) Updating for deterministic parameters $\{\beta,\{\mu_i\}_{i=1}^{K}\}$: At the t-th iteration, the noise variance β^{-1} can be updated

$$\left(\beta^{-1}\right)^{t+1}=\sum_{m=1}^{M}\left\langle\left(y_m-w_m\right)^2\right\rangle/M, \quad (9)$$

where w_m is the m-th element of $\boldsymbol{w}=\boldsymbol{A}\boldsymbol{x}$. As we show in [5], there is no closed-form updating rule for the unknown reflectance levels μ_i for the simplest case of $K=2$, i.e., the binary reflectance. For the generalized multi-level $K\neq 2$ case, we introduce an approximate updating rule

$$\mu_i^{t+1}=\frac{\sum_{n=1}^{N}\left\langle C_{n,i}\right\rangle\left\langle\alpha_{n,i}\right\rangle\left\langle x_n\right\rangle}{\sum_{n=1}^{N}\left\langle C_{n,i}\right\rangle\left\langle\alpha_{n,i}\right\rangle} \quad (10)$$

which turns out to be the weighted average of the posterior mean of x_n (i.e., $\langle x_n\rangle$) in the corresponding class specified by $C_{n,i}$.

III. Simulation Results

The proposed method is numerically evaluated with synthetic data and the Monte-Carlo simulation on a sample with a pseudo-random reflectance pattern in Fig. 3 (a) with $K=4$ levels ($[0.2,0.4,0.6,0.8]$). The recovered reflectance patterns is almost identical to the ground truth. The results in Fig. 3 (b) from the Monte-Carlo simulation suggest that the multi-level pseudo-random pattern can be recovered reliably with compressed measurements.

IV. Conclusion

A THz-based encoder system was introduced with a single THz transceiver scanning over a *multi-level pseudo-random* code pattern for high-resolution absolute positioning. This paper proposed an efficient element-wise algorithm to recover the multi-level pseudo-random code pattern with unknown reflectance.

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