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Karl Berntorp[1] and Stefano Di Cairano[1]

Abstract— **We present a sensor-fusion approach to real-time estimation of the offsets and noise characteristics found in low-cost automotive-grade sensors. Based on recent developments in adaptive particle filtering, we develop a method for online learning of the, possibly time-varying, noise statistics in the inertial and steering-wheel sensors, where we model the offsets as Gaussian random variables. The paper contains verification against several simulation and experimental data sets compared to ground truth, which shows that our method is capable of bias-free estimation of the sensor characteristics. The results also indicate that the computational cost is feasible for implementation on computationally limited embedded hardware.**

I. INTRODUCTION

Several of the sensors found in the current generation of production vehicles are typically of low cost and as a consequence prone to time-varying offset and scale errors [1], and may have relatively low signal-to-noise ratio. For instance, the lateral acceleration and heading-rate measurements are known to have drift and large noise in the sensor measurements, leading to measurements that are only reliable for prediction over a very limited time interval. Similarly, the sensor measuring the steering-wheel angle has an offset error that, when used for dead reckoning in a vehicle model, leads to prediction errors that accumulate over time.

The active safety systems developed in the past (e.g., electronic stability control [2] and anti-lock braking systems [3]) have been focused on aiding the driver over relatively short time intervals. However, even for those safety systems, accurate state estimation is more important than advanced control algorithms [1]. The recent surge for enabling new autonomous capabilities [4]–[7] implies a need for sensor information that can be used over longer time intervals to reliably predict the vehicle motion.

Offset estimation methods for the steering wheel and yaw rate found in production vehicles are typically based on averaging to compensate for the yaw rate and steering wheel bias. However, this leads to performance that is sometimes more than one order of magnitude away from the next-generation requirements [8]. Various methods have been proposed to improve the offset compensation in the steering-wheel angle and/or inertial measurements. The method in [9] estimates the yaw-rate offset in a state-augmented Kalman filter based on a kinematic vehicle model, where the yaw-rate offset is modeled as a random walk. The approach in [8] extends this to also include estimation of the steering offset in a linear regression. Oftentimes, the bias of the inertial sensors are integrated and solved for in an estimation algorithm targeted for a specific application [1], [10]–[13]. However, this has the drawback that the same tasks are repeated in different filters, which is computationally inefficient. Also, it implies that each estimator becomes unnecessarily complex, which might have implications on observability and feasibility of the approach.

In this paper, we develop a method for real-time estimation of the offset and sensor-noise characteristics of the acceleration, gyro, and steering-wheel measurements. While our primary focus is the lateral dynamics, the method developed here can be applied to either lateral or longitudinal dynamics, or to the two combined. We model the sensor measurements as Gaussian random variables with unknown mean and covariance, and the task is to estimate these unknown quantities in real time. The vehicle dynamics and the measurements are described by a state-space model, with the noise statistics as the unknown parameters in the model. The resulting estimation problem is non-Gaussian and includes both the vehicle state trajectory and the parameters, which introduces dynamic coupling between state and parameters. Furthermore, because of the biased and unknown noise, approximate estimation methods are required.

We use particle filtering [14] for solving our nonlinear non-Gaussian estimation problem, which has previously been used in several automotive applications (see, e.g., [15], [16]). A common way to estimate slowly time-varying parameters, is to augment the state vector [10], [17]. However, this leads to an increased state dimension that is problematic for particle filters, since the number of propagated particles and hence the computational burden increases exponentially with the dimensions, and the computational capabilities of automotive micro-controllers that run the estimation algorithm are very limited. Instead, we rely on marginalization [18] and propagation of the sufficient statistics of the noise parameters, conditioned on the estimated vehicle states, by exploiting the concept of conjugate priors [19].

II. MODELING AND PROBLEM FORMULATION

Our algorithm is focused on estimating the offsets during normal driving. We therefore model the vehicle dynamics by a single-track (i.e., bicycle) model [20], in which the two wheels on each axle are lumped together, where the vehicle operates in the linear region of the tire-force curve, and where planar motion is assumed. This paper focuses on the sensors mainly related to the lateral vehicle dynamics, but our approach can also handle the combined longitudinal and lateral setting.

[1]Karl Berntorp and Stefano Di Cairano are with Mitsubishi Electric Research Laboratories (MERL), 02139 Cambridge, MA, USA. Email:{karl.o.berntorp,dicairano}@ieee.org

In the following, F^y is the lateral tire force, α is the wheel-slip angle, ψ is the yaw, δ is the steering angle at the front wheel and subscripts f, r denote front and rear, respectively. The state vector is $\boldsymbol{x} = \begin{bmatrix} v^Y & \dot{\psi} \end{bmatrix}^{\mathrm{T}}$, where v^Y is the lateral velocity of the vehicle, and $\dot{\psi}$ is the yaw rate. From the assumption of driving in the linear regime of the tire-force curve, the lateral tire force can be expressed as a linear function of the slip angle α, $F^y \approx C^y \alpha$, where C^y is the lateral stiffness. The slip angles are approximated as

$$\alpha_f \approx \delta - \frac{v^Y + l_f \dot{\psi}}{v^X}, \quad \alpha_r \approx \frac{l_r \dot{\psi} - v^Y}{v^X}, \tag{1}$$

where l_r and l_f are the distance from the center of mass to the front and rear wheel, respecitlvey. In (1), we use the velocity at the center of mass instead of the velocity at the center of the wheel. The equations of motion are [12]

$$m\dot{v}^Y = -mv^X \dot{\psi} + C_f^y \left(\delta - \frac{v^Y + l_f \dot{\psi}}{v^X} \right) + C_r^y \frac{l_r \dot{\psi} - v^Y}{v^X}, \tag{2a}$$

$$I\ddot{\psi} = l_f C_f^y \left(\delta - \frac{v^Y + l_f \dot{\psi}}{v^X} \right) - l_r C_r^y \frac{l_r \dot{\psi} - v^Y}{v^X}, \tag{2b}$$

where m is the vehicle mass and I is the inertia. Model (2) is nonlinear in v^X and there are bilinearities between states and parameters. The longitudinal velocity v^X is assumed known. This is consistent with many navigation systems, where dead reckoning is used to decrease state dimension. In this work, we determine v^X from the wheel rotation rates given by the wheel-speed sensors.

A. Estimation Model

The steering angle δ at the wheel is usually not directly measured. Furthermore, the Ackermann steering configuration causes a slight deviation between the left and right wheel. We assume a single-track model, where δ is modeled as the average between the left and right wheel angles. In general, δ can be calculated from a static map of the measured steering-wheel angle. However, the resulting measurement of δ is known to be subject to an offset, which in some cases can even be time varying. An objective with the present contribution is therefore to estimate the offset. To this end, we decompose the steering angle into one known nominal part and one unknown part,

$$\delta = \delta_m + \Delta\delta, \tag{3}$$

where δ_m is the measured value of the steering angle, and where $\Delta\delta$ is the, possibly time-varying, offset. We model

$$w_k := \Delta\delta \tag{4}$$

as random process noise acting on the otherwise deterministic vehicle dynamics. The noise term w_k is modeled as Gaussian distributed according to $w_k \sim \mathcal{N}(\mu_k, \sigma_k^2)$, where μ_k and σ_k are the unknown, usually time varying, mean and standard deviation. Inserting (3) into (2) leads to

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}(\boldsymbol{x}_k, \boldsymbol{u}_k) + \boldsymbol{g}(\boldsymbol{x}_k, \boldsymbol{u}_k) w_k, \tag{5}$$

where $\boldsymbol{u}_k = [v^X \ \delta_m]^{\mathrm{T}}$.

We are also interested in estimating the time-varying offsets in the acceleration and gyro measurements, as well as their corresponding variances. The measurement model therefore incorporates the measurements of the lateral acceleration, a_m^Y, and the yaw rate $\dot{\psi}_m$, forming the measurement vector $\boldsymbol{y}_k = [a_m^Y \ \dot{\psi}_m]^{\mathrm{T}}$. To relate $\boldsymbol{y}_k$ to the states, note that a^Y can be extracted from the right-hand side of (2a), after dividing with the vehicle mass. The yaw-rate measurement is directly related to the yaw rate.

Similar to the steering offset (4), we model the measurement noise $\boldsymbol{e}_k$ as Gaussian with unknown mean $\boldsymbol{b}_k$ (the IMU bias) and covariance $\boldsymbol{R}_k$ according to $\boldsymbol{e}_k \sim \mathcal{N}(\boldsymbol{b}_k, \boldsymbol{R}_k)$. The measurement model can be written as

$$\boldsymbol{y}_k = \boldsymbol{h}(\boldsymbol{x}_k, \boldsymbol{u}_k) + \boldsymbol{d}(\boldsymbol{x}_k, \boldsymbol{u}_k)(\delta_m + w_k) + \boldsymbol{e}_k. \tag{6}$$

The joint Gaussian distribution of the steering offset w_k and measurement noise $\bar{\boldsymbol{e}}_k$ can be written as $\bar{\boldsymbol{w}}_k = \begin{bmatrix} w_k^{\mathrm{T}} & \bar{\boldsymbol{e}}_k^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \sim \mathcal{N}(\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$, where we have introduced the short-hand notation $\bar{\boldsymbol{e}}_k = \boldsymbol{d}(\boldsymbol{x}_k, \boldsymbol{u}_k) w_k + \boldsymbol{e}_k$ and

$$\boldsymbol{\mu}_k = \begin{bmatrix} \mu_{w,k} \\ \boldsymbol{d}_k \mu_{w,k} + \boldsymbol{b}_k \end{bmatrix}, \quad \boldsymbol{\Sigma}_k = \begin{bmatrix} \sigma_k^2 & \sigma_k^2 \boldsymbol{d}_k^{\mathrm{T}} \\ \boldsymbol{d}_k \sigma^2 & \boldsymbol{d}_k \sigma_k^2 \boldsymbol{d}_k^{\mathrm{T}} + \boldsymbol{R} \end{bmatrix}. \tag{7}$$

In (7), $\boldsymbol{d}_k := \boldsymbol{d}_k(\boldsymbol{x}_k, \boldsymbol{u}_k)$. Thus, the noise sources are dependent. In this work we need to estimate the process-noise statistics μ_k and σ_k and measurement-noise statistics $\boldsymbol{b}_k$ and $\boldsymbol{R}_k$, together with the unknown state trajectory.

Observability can be analyzed by augmenting the dynamic model (5) with a random walk model of the steering offset and bias, and derive the observability Gramian by linearization. In our case, there are three states and four offset parameters to estimate, implying seven quantities to estimate in total. However, it can be shown that the Gramian has rank six, which implies that the system is not fully observable. To remedy this, we utilize the fact that the angular velocities of the rear wheels can be converted to virtual measurements of the yaw rate according to

$$\dot{\psi}_{\mathrm{virt}} = \frac{\omega_r^{(r)} r - \omega_r^{(l)} r}{l_T}, \tag{8}$$

where l_T is the distance between the rear left and rear right wheel and $\omega_r^{(l)}$, $\omega_r^{(r)}$, are the rotation rates for the rear left and rear right wheel, respectively. With the additional measurement (8), it can be shown that the Gramian is nonsingular, and hence the system is weakly observable. This work assumes that the virtual measurement (8) is Gaussian distributed with zero mean and a priori determined variance σ_{virt}^2, and we denote the full measurement vector $\bar{\boldsymbol{y}}_k = [\boldsymbol{y}_k^{\mathrm{T}} \ \dot{\psi}_{\mathrm{virt}}]^{\mathrm{T}}$. In practice, measurements using the wheel rotation speed have scale errors due to differences between the true and estimated wheel radius r. This is not considered here but we refer to [16] for one possible way to estimate the tire radii of the different wheels.

Remark 1: It is common to model the bias vector $\boldsymbol{b}_k$ as a random walk and extend the state vector, as mentioned in the introduction, and the covariance $\boldsymbol{R}_k$ of the measurement noise is typically also determined a priori. However, determining the covariance a priori can be a tedious exercise. Similarly, also the process noise of the bias random walk can be determined a priori, which can be time consuming. Unmodeled effects can lead to differences between the effective measurement noise and the sensor specifications. Hence, we include the bias and variances in the estimation problem formulation.

B. Problem Formulation

We want to recursively estimate the steering offset and the noise statistics of the inertial measurements. In a Bayesian setting, this can be expressed as learning the parameters $\boldsymbol{\theta}_k := \{\mu_{w,k}, \boldsymbol{b}_k, \sigma_k, \boldsymbol{R}_k\}$ of the Gaussian noise w_k, $\boldsymbol{e}_k$. We approach this problem in the following way. Given the system model (5)–(8), and dependent Gaussian noise between w_k and $\bar{\boldsymbol{e}}_k$ characterized by (7), where the unknown parameters $\boldsymbol{\theta}_k$ may be time varying, we recursively estimate

$$p(\boldsymbol{\theta}_k|\bar{\boldsymbol{y}}_{0:k}), \tag{9a}$$
$$p(\boldsymbol{x}_k|\bar{\boldsymbol{y}}_{0:k}). \tag{9b}$$

Eqs. (9a) and (9b) are coupled, which will be apparent in the derivation of the proposed solution, because (9a) depends on the state trajectory and the density (9b) depends on the parameter estimates.

III. Marginalized Particle Filter for Sensor Estimation

This section focuses on determining the densities in (9). We formulate the joint estimation in a Bayesian framework as approximating the joint filtering density $p(\boldsymbol{x}_{0:k}, \boldsymbol{\theta}_k|\bar{\boldsymbol{y}}_{0:k})$, that is, the joint posterior conditioned on all measurements from time index 0 to k. We decompose

$$p(\boldsymbol{x}_{0:k}, \boldsymbol{\theta}_k|\bar{\boldsymbol{y}}_{0:k}) = p(\boldsymbol{\theta}_k|\boldsymbol{x}_{0:k}\bar{\boldsymbol{y}}_{0:k})p(\boldsymbol{x}_{0:k}|\bar{\boldsymbol{y}}_{0:k}), \tag{10}$$

and recursively estimate the densities in (10).

A. State Estimation

We approximate the posterior of the state trajectory with a particle filter as

$$p(\boldsymbol{x}_{0:k}|\bar{\boldsymbol{y}}_{0:k}) \approx \sum_{i=1}^{N} q_k^i \delta(\boldsymbol{x}_{0:k} - \boldsymbol{x}_{0:k}^i), \tag{11}$$

where $\delta(\cdot)$ is the Dirac delta mass and q_k^i is the importance weight for the ith state trajectory sample $\boldsymbol{x}_{0:k}^i$. The approximate distribution (11) is propagated with a sequential importance resampling (SIR) based particle filter [14]. In general, the particles are sampled using a proposal distribution $\pi(\boldsymbol{x}_{k+1}|\boldsymbol{x}_{0:k}^i, \bar{\boldsymbol{y}}_{0:k+1})$, which starts from the particles at the previous time step. For dependent noise, the weight update is performed as [21]

$$q_k^i \propto q_{k-1}^i \frac{p(\bar{\boldsymbol{y}}_k|\boldsymbol{x}_{0:k}^i, \bar{\boldsymbol{y}}_{0:k-1})p(\boldsymbol{x}_k^i|\boldsymbol{x}_{0:k-1}^i, \bar{\boldsymbol{y}}_{0:k-1})}{\pi(\boldsymbol{x}_k^i|\boldsymbol{x}_{0:k-1}^i, \bar{\boldsymbol{y}}_{0:k})}, \tag{12}$$

where $p(\bar{\boldsymbol{y}}_k|\boldsymbol{x}_{0:k}^i, \bar{\boldsymbol{y}}_{0:k-1})$ is the likelihood. If the proposal is chosen equal to $p(\boldsymbol{x}_k^i|\boldsymbol{x}_{0:k-1}^i, \bar{\boldsymbol{y}}_{0:k-1})$, (12) simplifies to

$$q_k^i \propto q_{k-1}^i p(\bar{\boldsymbol{y}}_k|\boldsymbol{x}_{0:k}^i, \bar{\boldsymbol{y}}_{0:k-1}). \tag{13}$$

Hence, to obtain new weights, we need to evaluate

$$p(\bar{\boldsymbol{y}}_k|\boldsymbol{x}_{0:k}^i, \bar{\boldsymbol{y}}_{0:k-1}), \tag{14a}$$
$$p(\boldsymbol{x}_{k+1}^i|\boldsymbol{x}_{0:k}^i, \bar{\boldsymbol{y}}_{0:k}). \tag{14b}$$

B. Parameter Estimation

According to (5) and (6), knowing both the state and measurement trajectory leads to full knowledge about $\bar{\boldsymbol{w}}_{0:k} = [w_{0:k} \;\; \bar{\boldsymbol{e}}_{0:k}]^{\mathrm{T}}$. The posterior for the noise parameters can therefore be rewritten using Bayes' rule as

$$p(\boldsymbol{\theta}_k|\boldsymbol{x}_{0:k}, \boldsymbol{y}_{0:k}) = p(\boldsymbol{\theta}_k|\bar{\boldsymbol{w}}_{0:k}) \propto p(\bar{\boldsymbol{w}}_k|\boldsymbol{\theta}_k)p(\boldsymbol{\theta}_k|\bar{\boldsymbol{w}}_{0:k-1}). \tag{15}$$

One of the assumptions is that the noise given the noise parameters; that is, $p(\bar{\boldsymbol{w}}_k|\boldsymbol{\theta}_k)$ in (15), is Gaussian. Therefore, we can utilize the concept of conjugate priors. If a prior distribution belongs to the same family as the posterior distribution, the prior is said to be conjugate to the particular likelihood. For multivariate Normal data $\bar{\boldsymbol{w}} \in \mathbb{R}^d$ with unknown mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$, a Normal-inverse-Wishart distribution defines the conjugate prior [22], $p(\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) := \mathrm{NiW}(\gamma_{k|k}, \hat{\boldsymbol{\mu}}_{k|k}, \boldsymbol{\Lambda}_{k|k}, \nu_{k|k})$, through the model

$$\begin{aligned}
\boldsymbol{\mu}_k|\boldsymbol{\Sigma}_k &\sim \mathcal{N}(\hat{\boldsymbol{\mu}}_{k|k}, \boldsymbol{\Sigma}_k), \\
\boldsymbol{\Sigma}_k &\sim \mathrm{iW}(\nu_{k|k}, \boldsymbol{\Lambda}_{k|k}) \\
&\propto |\boldsymbol{\Sigma}_k|^{-\frac{1}{2}(\nu_{k|k}+d+1)} e^{\left(-\frac{1}{2}\mathrm{tr}(\boldsymbol{\Lambda}_{k|k}\boldsymbol{\Sigma}_k^{-1}\right)},
\end{aligned}$$

where $\mathrm{tr}(\cdot)$ is the trace operator. We compute the statistics $S_{k|k} := (\gamma_{k|k}, \hat{\boldsymbol{\mu}}_{k|k}, \boldsymbol{\Lambda}_{k|k}, \nu_{k|k})$ for each particle as (see [23])

$$\gamma_{k|k} = \frac{\gamma_{k|k-1}}{1 + \gamma_{k|k-1}}, \tag{16a}$$
$$\hat{\boldsymbol{\mu}}_{k|k} = \hat{\boldsymbol{\mu}}_{k|k-1} + \gamma_{k|k}\boldsymbol{z}_k, \tag{16b}$$
$$\nu_{k|k} = \nu_{k|k-1} + 1, \tag{16c}$$
$$\boldsymbol{\Lambda}_{k|k} = \boldsymbol{\Lambda}_{k|k-1} + \frac{1}{1 + \gamma_{k|k-1}}\boldsymbol{z}_k\boldsymbol{z}_k^{\mathrm{T}}, \tag{16d}$$
$$\boldsymbol{z}_k = \bar{\boldsymbol{w}}_k - \hat{\boldsymbol{\mu}}_{k|k-1}, \tag{16e}$$

where the data $\bar{\boldsymbol{w}}_k$ for each particle is generated by

$$\bar{\boldsymbol{w}}_k^i = \begin{bmatrix} w_k^i \\ \boldsymbol{e}_k^i \end{bmatrix} = \begin{bmatrix} \boldsymbol{g}(\boldsymbol{x}_k^i, \boldsymbol{u}_k)^{-\dagger}(\boldsymbol{x}_{k+1}^i - \boldsymbol{f}(\boldsymbol{x}_k^i, \boldsymbol{u}_k)) \\ \boldsymbol{y}_k - \boldsymbol{h}(\boldsymbol{x}_k^i, \boldsymbol{u}_k) - \boldsymbol{d}(\boldsymbol{x}_k^i, \boldsymbol{u}_k)\mu_{w,k}^i \end{bmatrix}, \tag{17}$$

and in which $\boldsymbol{g}^{-\dagger}$ is the pseudo-inverse of $\boldsymbol{g}$. Hence, a key task in this paper is how to generate the particles in (17) to update the parameters. For slowly time-varying parameters, the prediction step consists of

$$\gamma_{k|k-1} = \frac{1}{\lambda}\gamma_{k-1|k-1}, \tag{18a}$$
$$\hat{\boldsymbol{\mu}}_{k|k-1} = \hat{\boldsymbol{\mu}}_{k-1|k-1}, \tag{18b}$$
$$\nu_{k|k-1} = \lambda\nu_{k-1|k-1}, \tag{18c}$$
$$\boldsymbol{\Lambda}_{k|k-1} = \lambda\boldsymbol{\Lambda}_{k-1|k-1}, \tag{18d}$$

where $\lambda \in (0,1]$ introduces exponential forgetting. Since we know the dependence structure (7), the scale matrix $\boldsymbol{\Lambda}_k$ can be decomposed as

$$\boldsymbol{\Lambda}_k = \begin{bmatrix} \Lambda_{w,k} & \Lambda_{w,k}\boldsymbol{d}_k^{\mathrm{T}} \\ \boldsymbol{d}_k\Lambda_{w,k} & \boldsymbol{d}_k\Lambda_{w,k}\boldsymbol{d}_k^{\mathrm{T}} + \boldsymbol{\Lambda}_{\boldsymbol{e},k} \end{bmatrix}, \tag{19}$$

implying that it suffices to propagate $\Lambda_{w,k}$ and $\boldsymbol{\Lambda}_{\boldsymbol{e}}$ in (16d) and (18d). Further, for a Normal-inverse-Wishart prior, the predictive distribution of the data $\bar{\boldsymbol{w}}$ is a Student-t, $\mathrm{St}(\hat{\boldsymbol{\mu}}_{k|k-1}, \tilde{\boldsymbol{\Lambda}}_{k|k-1}, \nu_{k|k-1} - d + 1)$, with

$$\tilde{\boldsymbol{\Lambda}}_{k|k-1} = \frac{1+\gamma_{k|k-1}}{\nu_{k|k-1} - d + 1}\boldsymbol{\Lambda}_{k|k-1}.$$

If the predictive distribution $p(\boldsymbol{\theta}_k|\bar{\boldsymbol{w}}_{0:k-1})$ in (15) is a Normal-inverse-Wishart distribution, from (15), (16), also the posterior is Normal-inverse Wishart, $p(\boldsymbol{\theta}_k|\boldsymbol{x}_{0:k}, \boldsymbol{y}_{0:k}) = \mathrm{NiW}(\hat{\boldsymbol{\mu}}_{k|k}, \boldsymbol{\Lambda}_{k|k}, \nu_{k|k})$. To obtain estimates of the mean and covariance of the noise processes, we rewrite the marginal (9a) as

$$\begin{aligned} p(\boldsymbol{\theta}_k|\bar{\boldsymbol{y}}_{0:k}) &= \int p(\boldsymbol{\theta}_k|\boldsymbol{x}_{0:k}, \boldsymbol{y}_{0:k}) p(\boldsymbol{x}_{0:k}|\bar{\boldsymbol{y}}_{0:k}) \mathrm{d}\boldsymbol{x}_{0:k} \\ &\approx \sum_{i=1}^{N} q_k^i p(\boldsymbol{\theta}_k|\boldsymbol{x}_{0:k}^i, \bar{\boldsymbol{y}}_{0:k}), \end{aligned} \tag{20}$$

which has complexity $\mathcal{O}(N)$. Based on (20), the unknown parameters can be extracted; for example, the estimate of $\boldsymbol{b}_k$ and $\boldsymbol{R}_k$ can be found as

$$\hat{\boldsymbol{b}}_k = \sum_{i=1}^{N} q_k^i \hat{\boldsymbol{b}}_{k|k}^i, \tag{21a}$$

$$\hat{\boldsymbol{R}}_k = \sum_{i=1}^{N} q_k^i \left(\frac{1}{\tilde{\nu}_{k|k}} \boldsymbol{\Lambda}_{k|k}^i + (\hat{\boldsymbol{b}}_{k|k}^i - \hat{\boldsymbol{b}}_k)(\hat{\boldsymbol{b}}_{k|k}^i - \hat{\boldsymbol{b}}_k)^{\mathrm{T}} \right), \tag{21b}$$

and similarly for $\hat{\mu}_{w,k}$, $\hat{\sigma}_k$, where $\tilde{\nu}_{k|k} = \nu_{k|k} - d - 1$.

C. Noise Marginalization

Consider first the likelihood (14a) resulting in the weight update (13), and note that the noise processes of the inertial sensors and the steering-wheel angle are independent of $\dot{\psi}_{\mathrm{virt}}$. Hence, from the state-space model (5) and (6), the knowledge of $\boldsymbol{x}_{0:k}$ and $\boldsymbol{y}_{0:k}$ gives full knowledge of the unknown noise sequence $\bar{\boldsymbol{e}}_{0:k}$. The property of transformations of variables in densities [24] gives that

$$p(\boldsymbol{y}_k|\boldsymbol{x}_{0:k}, \boldsymbol{y}_{0:k-1}) \propto p(\bar{\boldsymbol{e}}_k(\boldsymbol{y}_k, \boldsymbol{x}_k)|\bar{\boldsymbol{e}}_{0:k-1}). \tag{22}$$

We marginalize out the noise parameters as

$$\begin{aligned} p(\boldsymbol{y}_k|\boldsymbol{x}_{0:k}, \boldsymbol{y}_{0:k-1}) = \int & p(\boldsymbol{y}_k|\boldsymbol{\theta}_k, \boldsymbol{x}_k) \\ & \cdot p(\boldsymbol{\theta}_k|\boldsymbol{x}_{0:k-1}, \boldsymbol{y}_{0:k-1}) \,\mathrm{d}\boldsymbol{\theta}_k. \end{aligned} \tag{23}$$

Eq. (23) is the integral of the product of a Gaussian distribution and a Normal-inverse-Wishart distribution. Hence, (23) is a Student-t distribution [22], implying that

$$p(\bar{\boldsymbol{e}}_k(\boldsymbol{y}_k, \boldsymbol{x}_k)|\bar{\boldsymbol{e}}_{0:k-1}) = \mathrm{St}(\hat{\boldsymbol{\mu}}_{\bar{\boldsymbol{e}},k|k-1}, \tilde{\boldsymbol{\Lambda}}_{\bar{\boldsymbol{e}},k|k-1}, \tilde{\nu}_{k|k-1}),$$

with $\tilde{\nu}_{k|k-1} = \nu_{k|k-1} - d + 1$, and mean and scaling

$$\begin{aligned} \hat{\boldsymbol{\mu}}_{\bar{\boldsymbol{e}},k|k-1} &= \boldsymbol{d}_k \hat{\mu}_{w,k|k-1} + \hat{\boldsymbol{b}}_{k|k-1}, \\ \tilde{\boldsymbol{\Lambda}}_{\bar{\boldsymbol{e}},k|k-1} &= \frac{1+\gamma_{k|k-1}}{\tilde{\nu}_{k|k-1}} \left(\boldsymbol{d}_k \Lambda_{w,k|k-1} \boldsymbol{d}_k^{\mathrm{T}} + \boldsymbol{\Lambda}_{\boldsymbol{e},k|k-1} \right). \end{aligned}$$

The full measurement noise also contains a scalar component $e_{\dot{\psi}}$ due to the virtual measurement (8), which is zero-mean Gaussian. However, this leads to a mixture of a Gaussian and Student-t distribution, whose density has no closed form. An approach to resolve this is to resort to moment matching; that is, we model the full measurement noise as a Student-t distribution with a common degree of freedom,

$$\begin{bmatrix} \bar{\boldsymbol{e}}_k \\ e_{\dot{\psi}} \end{bmatrix} \sim \mathrm{St}\left(\begin{bmatrix} \hat{\boldsymbol{\mu}}_{\bar{\boldsymbol{e}},k|k-1} \\ 0 \end{bmatrix}, \begin{bmatrix} \tilde{\boldsymbol{\Lambda}}_{\bar{\boldsymbol{e}},k|k-1} & \mathbf{0} \\ \mathbf{0}^{\mathrm{T}} & \Lambda_{\mathrm{virt}} \end{bmatrix}, \tilde{\nu}_{k|k-1} \right), \tag{24}$$

where

$$\Lambda_{\mathrm{virt}} = \frac{\tilde{\nu}_{k|k-1} - 2}{\tilde{\nu}_{k|k-1}} \sigma_{\mathrm{virt}}^2.$$

The Student-t converges to the Gaussian as the degrees of freedom tend to infinity. Hence, from $\lim_{\nu \to \infty} \mathrm{St}(\boldsymbol{\mu}, \boldsymbol{\Lambda}, \nu) = \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Lambda})$ and the update formulas (16c) and (18c), it follows that we recover the Gaussian measurement noise of the virtual measurement with precision determined by the forgetting factor. Hence, the measurement update (13) is done by

$$q_k^i \propto q_{k-1}^i \mathrm{St}(\boldsymbol{\mu}^*, \tilde{\boldsymbol{\Lambda}}^*, \tilde{\nu}), \tag{25}$$

which can be evaluated analytically, where

$$\begin{aligned} \boldsymbol{\mu}^* &= \boldsymbol{h}_k + \boldsymbol{d}_k \hat{\mu}_{w,k|k-1} + \boldsymbol{b}_k, \\ \tilde{\boldsymbol{\Lambda}}^* &= \frac{1+\gamma_{k|k-1}}{\tilde{\nu}_{k|k-1}} \left(\boldsymbol{d}_k \Lambda_{w,k|k-1} \boldsymbol{d}_k^{\mathrm{T}} + \boldsymbol{\Lambda}_{\boldsymbol{e},k|k-1} \right) + \\ & \quad \frac{\tilde{\nu}_{k|k-1} - 2}{\tilde{\nu}_{k|k-1}} \sigma_{\mathrm{virt}}^2. \end{aligned}$$

The prediction step (14b) is resolved in a similar way,

$$\begin{aligned} p(\boldsymbol{x}_{k+1}|\boldsymbol{x}_{0:k}, \bar{\boldsymbol{y}}_{0:k}) &\propto p(\boldsymbol{g}_k^{-\dagger}(\boldsymbol{x}_{k+1} - \boldsymbol{f}_k)|\boldsymbol{x}_{0:k}, \bar{\boldsymbol{y}}_{0:k}) \\ &= p(\boldsymbol{g}_k^{-\dagger}(\boldsymbol{x}_{k+1} - \boldsymbol{f}_k)|\bar{\boldsymbol{e}}_{0:k}) \\ &= p(w_k(\boldsymbol{x}_{k+1})|\bar{\boldsymbol{e}}_{0:k}). \end{aligned} \tag{26}$$

By integrating over the noise parameters in (14b),

$$\begin{aligned} p(\boldsymbol{x}_{k+1}|\boldsymbol{x}_{0:k}, \bar{\boldsymbol{y}}_{0:k}) = \int & p(\boldsymbol{x}_{k+1}|\boldsymbol{\theta}_k, \boldsymbol{x}_{0:k}, \bar{\boldsymbol{y}}_{0:k}) \\ & \cdot p(\boldsymbol{\theta}_k|\boldsymbol{x}_{0:k}, \bar{\boldsymbol{y}}_{0:k}) \,\mathrm{d}\boldsymbol{\theta}_k. \end{aligned} \tag{27}$$

The integrand in (27) is the product of a Gaussian and a Normal-inverse-Wishart distribution, which is a Student-t distribution. Combining with (26), we obtain a Student-t distribution for w_k as

$$p(w_k(\boldsymbol{x}_{k+1})|\bar{\boldsymbol{e}}_{0:k}) = \mathrm{St}(\hat{\mu}_k^*, \tilde{\Lambda}_k^*, \nu_k^*), \tag{28}$$

where

$$\nu_k^* = \nu_{k|k-1} - d + 1 + d_y,$$
$$\mu_k^* = \hat{\mu}_{w,k|k-1} + \boldsymbol{d}_k \Lambda_{w,k|k-1} \tilde{\boldsymbol{\Lambda}}_{\bar{\boldsymbol{e}},k|k-1}^{-1} \boldsymbol{z}_k,$$
$$\tilde{\boldsymbol{\Lambda}}_k^* = \frac{\nu_{k|k-1} - d + 1 + \boldsymbol{z}_k \tilde{\boldsymbol{\Lambda}}_{\bar{\boldsymbol{e}},k|k-1}^{-1} \boldsymbol{z}_k^{\mathrm{T}}}{\nu_{k|k-1} - d_y + 1} \Big(\Lambda_{w,k|k-1} - \boldsymbol{d}_k \Lambda_{w,k|k-1} \tilde{\boldsymbol{\Lambda}}_{\bar{\boldsymbol{e}},k|k-1}^{-1} \Lambda_{w,k|k-1}^{\mathrm{T}} \boldsymbol{d}_k^{\mathrm{T}} \Big),$$
$$\boldsymbol{z}_k = \bar{\boldsymbol{e}}_k - \hat{\boldsymbol{\mu}}_{\bar{\boldsymbol{e}},k|k-1}.$$

In the implementation, the process noise is generated from (28) and used in (5) to generate samples $\{\boldsymbol{x}_{k+1}^i\}_{i=1}^N$. The samples from (28) are used directly in the (17) for w_k. Algorithm 1 summarizes the method.

Algorithm 1 Pseudo-code of the estimation algorithm

Initialize: Set $\{\boldsymbol{x}_0^i\}_{i=1}^N \sim p_0(\boldsymbol{x}_0)$, $\{q_0^i\}_{i=1}^N = 1/N$, $\{S_0^i\}_{i=1}^N = \{\gamma_0^i, \bar{\boldsymbol{\mu}}_0^i, \boldsymbol{\Lambda}_{w,0}^i, \nu_0^i\}$

for $k \leftarrow 0$ to T **do**
  for $i \in \{1, \dots, N\}$ **do**
    Update weight $\bar{q}_k^i$ using (25).
    Update noise statistics $S_{k|k}^i$ using (16).
  end for
  Normalize weights as $q_k^i = \bar{q}_k^i / (\sum_{i=1}^N \bar{q}_k^i)$.
  Compute $N_{\text{eff}} = 1/(\sum_{i=1}^N (q_k^i)^2)$
  if $N_{\text{eff}} \leq N_{\text{thr}}$ **then**
    Resample particles and copy the corresponding statistics. Set $\{q_k^i\}_{i=1}^N = 1/N$.
  end if
  Compute state estimates $\boldsymbol{x}_k = \sum_{i=1}^N q_k^i \boldsymbol{x}_k^i$.
  Compute estimates of noise parameters using (21).
  for $i \in \{1, \dots, N\}$ **do**
    Predict noise statistics $S_{k+1|k}^i$ using (18).
    Sample $\boldsymbol{w}_k^i$ from (28).
    Predict state $\boldsymbol{x}_{k+1}^i$ using (5).
  end for
end for

IV. Results

We evaluate the algorithm on synthetic and real data. The synthetic data is generated by feeding the single-track model (2) with the measured steering-wheel angle and wheel speeds, and by adding offsets in the steering-wheel angle and inertial sensors. For experimental validation, we have used a mid-size SUV, equipped with industry-grade validation equipment to gather data, and collected several different data sets. The known parameters in the vehicle model has been extracted from data sheets and bench testing.

A. Simulation Results

The statistics of the Normal-inverse-Wishart is initialized as zero-mean with the estimated standard deviations set to be roughly twice of the true standard deviation, and the forgetting factor is set to $\lambda = 0.995$. Fig. 1 shows the estimated standard deviations for the lateral acceleration and the gyro in red and the true values in black. After the initial

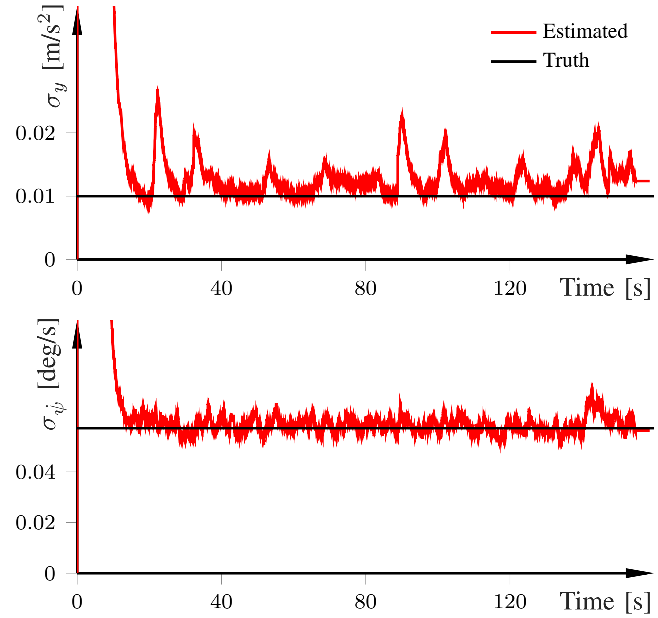

Fig. 1. Estimated standard deviations (red) and true values (black) of the lateral acceleration and gyro in simulation for $N = 100$ particles.

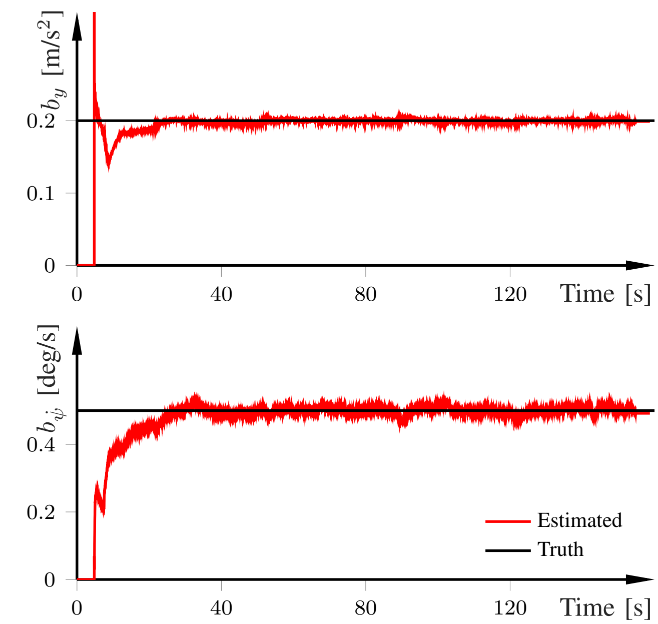

Fig. 2. The estimated bias (red) and true bias (black) in simulation for $N = 100$ particles.

transients, the estimates converge to their true values. Setting the forgetting factor to a lower value or the initial variances to larger values leads to faster convergence (at the cost of larger fluctuations in steady state). The estimated offsets are shown in Fig. 2. There is a cross-dependence between the steering offset and the measurement offsets, especially the acceleration measurement. However, the estimator is able to estimate the offsets closely.

B. Experimental Results

For the experimental results, the steering-wheel angle offset has been obtained by an offline procedure, in which the (assumed constant) steering-wheel offset that minimizes

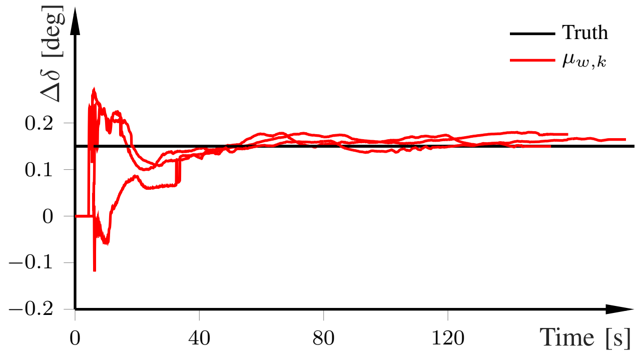

Fig. 3. Estimated steering-wheel offset (red) and the ground truth (black), as obtained by an offline optimization-based procedure, in experiments for $N = 500$ particles. The results from three data sets are overlaid in the plot.

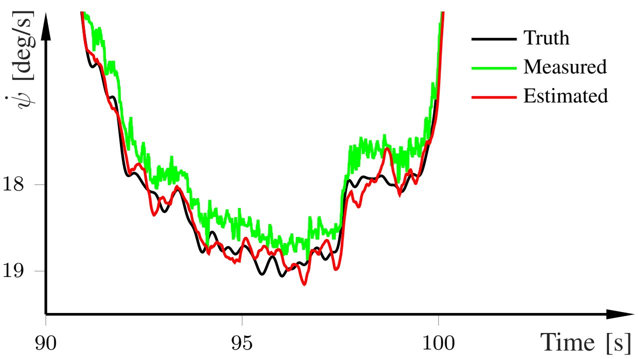

Fig. 4. The estimated yaw rate (red) for $N = 500$ particles, measured yaw rate (green), and the black line is the true yaw rate.

the errors between the ground truth inertial measurements and the predicted vehicle trajectory has been obtained.

Fig. 3 displays the estimated steering-angle offset (red) for three different data sets. The true value as obtained by the offline procedure are in black. After the initial transients, the estimated offset converges very close to the true offset. The algorithm repeatedly finds values very close to each other, even for different data sets.

In Fig. 4 we show the measured and true yaw rate, respectively, together with the estimated yaw rate. The initial values of the bias samples are set to zero. We do not have ground truth for the bias, which is a slowly time-varying process. However, by comparing the measured yaw rate with the yaw rate from the validation equipment (a very high-cost, high-precision fiber-optic gyro), we can see the instantaneous errors between them. The plot shows the results after the initial transients of the bias parameters. The estimated yaw rate follows the validation sensor quite closely.

V. Conclusion

We developed a method for estimation of the offset and noise characteristics found in low-cost automotive-grade sensors. The offset and noise of the different sensors are related through the vehicle state trajectory and the associated estimation problem is non-Gaussian. We provided a method based on marginalized particle filtering to solve the problem. Tests on simulation and experimental data sets verified that the method can accurately estimate both the offsets and sensor variances.

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