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Abstract

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MASSIVE MIMO CHANNEL ESTIMATION USING SIGNED MEASUREMENTS WITH ANTENNA-VARYING THRESHOLDS

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ABSTRACT

This paper investigates angular-domain channel estimation for massive multiple-input multiple-output (MIMO) systems using signed measurements with antenna-varying thresholds. We derive the Cramér-Rao bounds (CRBs) for estimating angles-of-arrival (AoAs), angles-of-departure (AoDs) and associated path gains and compare them with their counterparts of using time-varying and zero thresholds. We then introduce the maximum likelihood (ML) method to estimate the massive MIMO channel parameters. Since the ML estimator is computationally prohibitive, we also consider a relaxation based cyclic algorithm, referred to as one-bit RELAX, for massive MIMO channel estimation. Numerical results are provided to compare the performances of using different thresholding schemes to obtain signed measurements and to verify the effectiveness of the one-bit RELAX algorithm for channel estimation.

Index Terms— Massive MIMO, channel estimation, Cramér-Rao bound, signed measurements, antenna-varying thresholds.

1. INTRODUCTION

Massive multiple-input multiple-output (MIMO) systems have the potential to remarkably improve the communication performance in terms of link reliability, spectral efficiency, and transmit energy efficiency [1, 2]. However, such systems use a large number of antennas at base stations resulting in prohibitive cost and power consumption when a large number of high-resolution analog-to-digital converters (ADCs) are used at the antenna outputs. This problem becomes more severe in ultra-wideband [3] and millimeter wave (mmWave) communication systems [4, 5] due to the high sampling rate required by these systems. Low-resolution ADCs, such as the extreme case of one-bit ADCs, have been considered to mitigate this problem [6–12].

Using one-bit ADCs at the receive antenna outputs, the power consumption and cost of massive MIMO systems can be dramatically reduced while still achieving an acceptable performance [13]. Channel estimation and signal detection for massive MIMO with one-bit ADCs were studied in [13–16] and the achievable rate was investigated in [15, 17, 18]. However, in the aforementioned works, only zero threshold was considered for one-bit ADCs when the noise variance is assumed known. In the high SNR regime, a large capacity loss occurs [4]. It is also noticed that the performance of channel estimation degrades significantly [13, 14, 16] or suffers from a high-SNR error floor [15]. More recently, one-bit ADCs with time-varying thresholds [19–21] are considered for massive MIMO systems [22], where the Cramér-Rao bound (CRB) analysis shows that enhanced channel estimation can be obtained, especially at high SNRs. Moreover, [22] shows that one-bit ADCs with zero threshold lead to an ambiguity between the path gain and noise variance.

In this paper, we consider a simple and inexpensive thresholding scheme that uses different but fixed thresholds for one-bit ADCs for different receive antennas, referred to as one-bit sampling with *antenna-varying* thresholds. We investigate the corresponding channel estimation by deriving the CRB matrix for angles-of-arrival (AoAs), angles-of-departure (AoDs) and their associated path gains. We show that with *antenna-varying* thresholds for one-bit sampling, the noise variance can be unknown and estimated. Moreover, the antenna-varying thresholding scheme is simple for implementation and requires no further hardware controls over ADCs, while providing almost the same performance as the time-varying thresholding scheme. We then introduce the maximum likelihood (ML) estimator for channel estimation. Considering the high computational complexity of the ML estimator, we also introduce a relaxation based cyclic algorithm, referred to as one-bit RELAX algorithm [21], for channel estimation. Finally, numerical results are provided to compare the performances of different thresholding schemes for one-bit sampling and to verify the effectiveness of the one-bit RELAX algorithm for massive MIMO channel estimation.

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2. SYSTEM MODEL

2.1. Massive MIMO System Model

Consider a point-to-point flat block-fading massive MIMO system with N_t transmit antennas at the mobile station (MS) and N_r receive antennas at the base station (BS). In the training phase of each coherent processing interval, a pilot signal of length K is sent from the MS to the BS. Then the received signal $\mathbf{Y} \in \mathbb{C}^{N_r \times K}$ at the BS can be represented as

$$\mathbf{Y}=\sqrt{\rho}\mathbf{H}\mathbf{X}+\mathbf{N}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix and $\mathbf{X} \in \mathbb{C}^{N_t \times K}$ is the transmit pilot signal with $\mathrm{E}\left[\boldsymbol{x}_k^H \boldsymbol{x}_k\right] = 1, 1 \leq k \leq K$. $\boldsymbol{x}_k$ represents the kth column of $\mathbf{X}$. $\mathbf{N} \in \mathbb{C}^{N_r \times K}$ is the circularly symmetric complex-valued white Gaussian noise matrix with i.i.d. $\mathcal{CN}\left(0, \sigma^2\right)$ entries, and ρ is the average transmit power.

Consider the geometric channel model parameterized via angles and gains associated with different propagation paths. Assume that there are N_s active scatterers between the MS and the BS. Denote ξ_ℓ as the gain of the ℓth scattering path, θ_ℓ and φ_ℓ as the associated AoA and AoD, respectively. Hence, for a uniform linear array (ULA), the ℓth steering vectors for the BS and MS are, respectively,

$$\boldsymbol{\alpha}_{\mathrm{BS}}\left(\theta_\ell\right) = \left[1, e^{j2\pi \sin(\theta_\ell)\frac{d_r}{\lambda}}, \cdots, e^{j(N_r-1)2\pi \sin(\theta_\ell)\frac{d_r}{\lambda}}\right]^T, \tag{2}$$

and

$$\boldsymbol{\alpha}_{\mathrm{MS}}\left(\varphi_\ell\right) = \left[1, e^{j2\pi \sin(\varphi_\ell)\frac{d_t}{\lambda}}, \cdots, e^{j(N_t-1)2\pi \sin(\varphi_\ell)\frac{d_t}{\lambda}}\right]^T, \tag{3}$$

where $(\cdot)^T$ denotes transpose, λ is the wavelength, and d_r and d_t are the antenna spacings of the ULAs at the receiver and transmitter, respectively. Then the channel matrix $\mathbf{H}$ is given by

$$\begin{aligned}\mathbf{H}=&\sum_{\ell=1}^{N_s} \xi_\ell \boldsymbol{\alpha}_{\mathrm{BS}}\left(\theta_\ell\right)\boldsymbol{\alpha}_{\mathrm{MS}}^H\left(\varphi_\ell\right)\\ \triangleq&\mathbf{A}_{\mathrm{BS}}\mathbf{H}_\Lambda\mathbf{A}_{\mathrm{MS}}^H,\end{aligned} \tag{4}$$

where $(\cdot)^H$ denotes conjugate transpose. The ℓ-th columns of $\mathbf{A}_{\mathrm{BS}} \in \mathbb{C}^{N_r \times N_s}$ and $\mathbf{A}_{\mathrm{MS}} \in \mathbb{C}^{N_t \times N_s}$ are $\boldsymbol{\alpha}_{\mathrm{BS}}\left(\theta_\ell\right)$ and $\boldsymbol{\alpha}_{\mathrm{MS}}\left(\varphi_\ell\right)$, respectively. Also, $\mathbf{H}_\Lambda$ is a diagonal matrix with diagonal elements $\xi_n, n = 1, \cdots, N_s$. Thus the received signal at the BS has the following form:

$$\mathbf{Y}=\sqrt{\rho}\mathbf{A}_{\mathrm{BS}}\mathbf{H}_\Lambda\mathbf{A}_{\mathrm{MS}}^H\mathbf{X}+\mathbf{N}. \tag{5}$$

2.2. One-Bit Quantization

Let $\mathbf{y}_{\mathrm{v}}= \mathrm{vec}\left(\mathbf{Y}\right)$, where $\mathrm{vec}(\mathbf{Y})$ denotes the vectorization operation which stacks the columns of $\mathbf{Y}$ on top of each other. Using $\mathrm{vec}\left(\mathbf{ABC}\right) = \left(\mathbf{C}^T \otimes \mathbf{A}\right)\mathrm{vec}\left(\mathbf{B}\right)$, the received signal can be vectorized as

$$\begin{aligned}\mathbf{y}_{\mathrm{v}}=&\ \mathrm{vec}\left(\sqrt{\rho}\mathbf{A}_{\mathrm{BS}}\mathbf{H}_\Lambda\mathbf{A}_{\mathrm{MS}}^H\mathbf{X}\right)+\mathbf{n}\\ =&\left[\left(\sqrt{\rho}\mathbf{X}^T\mathbf{A}_{\mathrm{MS}}^*\right)\otimes\mathbf{A}_{\mathrm{BS}}\right]\mathbf{h}+\mathbf{n}\\ \triangleq&\boldsymbol{\Gamma}\left(\boldsymbol{\theta},\boldsymbol{\varphi}\right)\mathbf{h}+\mathbf{n},\end{aligned} \tag{6}$$

where $(\cdot)^*$ denotes the complex conjugate of a matrix, and $\mathbf{h}= \mathrm{vec}\left(\mathbf{H}_\Lambda\right) = \left[\xi_1\mathbf{e}_1^T, \cdots \xi_{N_s}\mathbf{e}_{N_s}^T\right]^T$, with $\mathbf{e}_n$ denoting an vector with 1 at the nth element and 0 elsewhere, and $\mathbf{n}= \mathrm{vec}\left(\mathbf{N}\right)$. We can rewrite (6) in a real-valued form as follows:

$$\mathbf{y}_{\mathrm{r}}=\begin{bmatrix}\mathrm{Re}\left(\mathbf{y}_{\mathrm{v}}\right)\\ \mathrm{Im}\left(\mathbf{y}_{\mathrm{v}}\right)\end{bmatrix}=\boldsymbol{\Gamma}_{\mathrm{r}}\left(\boldsymbol{\theta},\boldsymbol{\varphi}\right)\mathbf{h}_{\mathrm{r}}+\mathbf{n}_{\mathrm{r}}, \tag{7}$$

where $\boldsymbol{\Gamma}_{\mathrm{r}}\left(\boldsymbol{\theta},\boldsymbol{\varphi}\right)=\begin{bmatrix}\mathrm{Re}\left(\boldsymbol{\Gamma}\right) & -\mathrm{Im}\left(\boldsymbol{\Gamma}\right)\\ \mathrm{Im}\left(\boldsymbol{\Gamma}\right) & \mathrm{Re}\left(\boldsymbol{\Gamma}\right)\end{bmatrix} \in \mathbb{R}^{2KN_r \times 2N_s^2}$, $\mathbf{h}_{\mathrm{r}} = \begin{bmatrix}\mathrm{Re}\left(\mathbf{h}\right)\\ \mathrm{Im}\left(\mathbf{h}\right)\end{bmatrix} \in \mathbb{R}^{2N_s^2 \times 1}$, and $\mathbf{n}_{\mathrm{r}}=\begin{bmatrix}\mathrm{Re}\left(\mathbf{n}\right)\\ \mathrm{Im}\left(\mathbf{n}\right)\end{bmatrix} \in \mathbb{R}^{2KN_r \times 1}$. Also, $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ represent the operation of taking the real and imaginary part of a matrix, respectively.

In one-bit massive MIMO systems, the signed measurements of the in-phase and quadrature components of the signal are from a pair of one-bit ADCs equipped at each receive antenna. The quantized output can be written as:

$$\mathbf{z}= \mathrm{sign}\left(\mathbf{y}_{\mathrm{r}}-\boldsymbol{\eta}\right), \tag{8}$$

where $\boldsymbol{\eta} \in \mathbb{R}^{2KN_r \times 1}$ is the threshold vector and $\mathrm{sign}\left(x\right)$ is the element-wise one-bit quantization function, which returns -1 if $x \leq 0$ and 1 else. Let $\boldsymbol{\chi} = \left[\boldsymbol{\theta}^T, \boldsymbol{\varphi}^T, \boldsymbol{\xi}_R^T, \boldsymbol{\xi}_I^T\right]^T \in \mathbb{R}^{4N_s \times 1}$ be the unknown channel parameter vector, where $\boldsymbol{\xi}_R = \mathrm{Re}(\boldsymbol{\xi})$ and $\boldsymbol{\xi}_I = \mathrm{Im}(\boldsymbol{\xi})$ with $\boldsymbol{\xi} = [\xi_1, \cdots, \xi_{N_s}]^T$. The problem of interest here is to estimate $\boldsymbol{\chi}$ with the quantized output $\mathbf{z}$ and the known threshold vector $\boldsymbol{\eta}$.

3. CRAMÉR-RAO BOUNDS

As the real-valued noise $\mathbf{n}_{\mathrm{r}}$ has variance $\frac{\sigma^2}{2}$, the log-likelihood function of $\mathbf{z}$ is given by:

$$\mathcal{L}\left(\boldsymbol{\chi}\right) = \ln p\left(\boldsymbol{z}|\boldsymbol{\chi}\right) = \sum_{m=1}^{2KN_r} \ln\Phi\left(z_m\frac{\boldsymbol{\gamma}_{\mathrm{r}m}\mathbf{h}_{\mathrm{r}} - \eta_m}{\sigma/\sqrt{2}}\right), \tag{9}$$

where $\Phi\left(x\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}dt$ is the standard normal cumulative distribution function (cdf) and $\boldsymbol{\gamma}_{\mathrm{r}m}$ is the mth row of $\boldsymbol{\Gamma}_{\mathrm{r}}\left(\boldsymbol{\theta},\boldsymbol{\varphi}\right)$. When the noise variance σ^2 is known, the general expression of the (i,j)th element of the Fisher information matrix (FIM) is given by:

$$\mathbf{FIM}_{i,j} = -\mathrm{E}\left[\frac{\partial^2\mathcal{L}\left(\boldsymbol{\chi}\right)}{\partial\chi_i\partial\chi_j}\right], 1 \leq i,j \leq 4N_s, \tag{10}$$

where $\mathrm{E}\left[\cdot\right]$ denotes the expectation operator and $\frac{\partial}{\partial\chi_i}$ represents the partial derivative with respect to the ith element of

$\boldsymbol{\chi}$. When σ^2 is unknown, it should also be included into the unknown parameter vector $\boldsymbol{\chi}$, and thus the FIM becomes a $(4N_s+1)\times(4N_s+1)$ matrix. The specific expressions of $\mathrm{FIM}_{i,j}$ are given in [22]. The CRB matrix can be calculated as $\mathbf{CRB}=\mathbf{FIM^{-1}}$. For the parameter χ_i, the lower bound of the mean-squared error (MSE) of any unbiased estimator is given by the (i,i)th element of the CRB matrix, i.e.,

$$\mathrm{E}\left[(\hat{\chi}_i-\chi_i)^2\right]\geq\mathbf{CRB}_{i,i}. \tag{11}$$

Note that for one-bit sampling with zero thresholding scheme, FIM is singular due to the ambiguity between the path gain vector $\mathbf{h}_{\mathrm{r}}$ and the noise variance σ^2 when σ^2 is unknown [22]. This ambiguity can be removed by using known antenna-varying or time-varying thresholds. For both of these thresholding schemes, the thresholds are selected randomly from a finite discrete set. For the time-varying thresholding scheme, the thresholds change at each sampling instant. In the antenna-varying thresholding case, the thresholds for all one-bit ADCs at the antenna outputs are chose randomly once and fixed at all times. Note that $\boldsymbol{\eta}$ is a space-time threshold vector. For the time-varying thresholding scheme, the thresholds are vary in both space and time. In antenna-varying case, the thresholds are fixed in time and are only spatially-varying, which makes it more attractive due to the simpler and less expensive system and reduced power consumption.

4. CHANNEL ESTIMATION

4.1. Maximum Likelihood Estimation

The maximum likelihood (ML) method is considered herein for channel estimation. For ML estimation with known noise variance σ^2, the problem to be solved is to find the parameter estimates by minimizing the negative log-likelihood function,

$$\hat{\boldsymbol{\chi}}=\arg\min_{\boldsymbol{\chi}}-\sum_{m=1}^{2KN_r}\ln\Phi\left(z_m\frac{\boldsymbol{\gamma}_{\mathrm{r}m}\mathbf{h}_{\mathrm{r}}-\eta_m}{\sigma/\sqrt{2}}\right). \tag{12}$$

Recall the formula (6) and note that $\boldsymbol{\Gamma}_{\mathrm{r}}(\boldsymbol{\theta},\boldsymbol{\varphi})$ is characterized only by $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$ corresponding to N_s scatterers. For given $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$, the problem of finding the optimal $\hat{\boldsymbol{\xi}}_R$, $\hat{\boldsymbol{\xi}}_I$ that minimize the cost function in (12) is an unconstrained convex optimization problem [23].

When the noise variance σ^2 is unknown, we can reparameterize (12) by defining $\varepsilon=\frac{1}{\sigma}$, $\boldsymbol{\beta}=\frac{\boldsymbol{\xi}_R}{\sigma}$, and $\boldsymbol{\mu}=\frac{\boldsymbol{\xi}_I}{\sigma}$. Let $\bar{\boldsymbol{\chi}}=\left[\boldsymbol{\theta}^T,\boldsymbol{\varphi}^T,\boldsymbol{\beta}^T,\boldsymbol{\mu}^T,\varepsilon\right]^T$. Thus, the problem with unknown σ^2 can be cast as

$$\hat{\bar{\boldsymbol{\chi}}}=\arg\min_{\bar{\boldsymbol{\chi}}}-\sum_{m=1}^{2KN_r}\ln\Phi\left(\sqrt{2}z_m\left(\boldsymbol{\gamma}_{\mathrm{r}m}\bar{\mathbf{h}}_{\mathrm{r}}-\varepsilon\eta_m\right)\right), \tag{13}$$

where $\bar{\mathbf{h}}_{\mathrm{r}}=\mathbf{h}_{\mathrm{r}}/\sigma$. The problem is again convex for given $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$.

We give herein the detailed steps of the ML method when σ^2 is known, which can be easily extended to the σ^2 unknown case. Specifically, we can first establish a discrete set of L points which forms a grid in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right]$. For the $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$ selected from the discrete set, we can solve the problem (12) using, e.g., the Newton's method. Then we obtain the optimal $\hat{\boldsymbol{\xi}}_R$, $\hat{\boldsymbol{\xi}}_I$ for the given $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$. We can repeat the previous step over all possible combinations of $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$, and then select the ML parameter estimates corresponding to the minimum negative log-likelihood function value.

The above ML method requires a $2N_s$-dimensional search, with L points to be searched for each dimension. Therefore the aforementioned convex optimization problem must be solved $\mathcal{O}\left(L^{2N_s}\right)$ times. As the number of scatterers increases, the ML estimator becomes computationally prohibitive.

4.2. One-Bit RELAX

In [24], a relaxation based cyclic algorithm, referred to as RELAX, is proposed for sinusoidal parameter estimation in the infinite precision quantization case. RELAX is conceptually and computationally simple. In this work, we extend it to massive MIMO channel estimation.

Denote $\boldsymbol{\chi}_n=[\theta_n,\varphi_n,\xi_{Rn},\xi_{In}]$ as the parameter vector corresponding to the nth scatterer. The received signal can be decomposed into the sum of the signals scattered from the N_s scatterers, i.e., $\boldsymbol{\Gamma}_{\mathrm{r}}(\boldsymbol{\theta},\boldsymbol{\varphi})\mathbf{h}_{\mathrm{r}}=\sum_{n=1}^{N_s}\boldsymbol{\Gamma}_{\mathrm{r},n}\mathbf{h}_{\mathrm{r},n}$, where $\boldsymbol{\Gamma}_{\mathrm{r},n}\in\mathbb{R}^{2KN_r\times 2}$ and $\mathbf{h}_{\mathrm{r},n}\in\mathbb{R}^{2\times 1}$.

When the noise variance σ^2 is known, the one-bit RELAX algorithm begins, in Step 1, by solving (14) below, assuming that there is only one dominate scattering path between the BS and MS:

$$\hat{\boldsymbol{\chi}}_1=\arg\min_{\boldsymbol{\chi}_1}-\sum_{m=1}^{2KN_r}\ln\Phi\left(z_m\frac{\boldsymbol{\gamma}_{r,1}^m\mathbf{h}_{\mathrm{r},1}-\eta_m}{\sigma/\sqrt{2}}\right), \tag{14}$$

where $\boldsymbol{\gamma}_{r,1}^m$ represents the mth row of the matrix $\boldsymbol{\Gamma}_{\mathrm{r},1}$. The aforementioned ML method can be used for parameter estimation for this path. Then in Step 2, assume that there are two scattering paths. The one-bit RELAX algorithm solves the problem (15) below to estimate the parameter vector $\boldsymbol{\chi}_2$ of the second strongest channel path:

$$\hat{\boldsymbol{\chi}}_2=\arg\min_{\boldsymbol{\chi}_2}-\sum_{m=1}^{2KN_r}\ln\Phi\left(z_m\frac{\hat{\boldsymbol{\gamma}}_{\mathrm{r},1}^m\hat{\mathbf{h}}_{\mathrm{r},1}+\boldsymbol{\gamma}_{\mathrm{r},2}^m\mathbf{h}_{\mathrm{r},2}-\eta_m}{\sigma/\sqrt{2}}\right), \tag{15}$$

which uses the $\hat{\boldsymbol{\chi}}_1$ obtained from Step 1. With $\hat{\boldsymbol{\chi}}_2$, $\hat{\boldsymbol{\chi}}_1$ is refined using the $\hat{\boldsymbol{\chi}}_2$ in a similar fashion to solve the problem (16) below.

$$\hat{\boldsymbol{\chi}}_1=\arg\min_{\boldsymbol{\chi}_1}-\sum_{m=1}^{2KN_r}\ln\Phi\left(z_m\frac{\hat{\boldsymbol{\gamma}}_{\mathrm{r},2}^m\hat{\mathbf{h}}_{\mathrm{r},2}+\boldsymbol{\gamma}_{\mathrm{r},1}^m\mathbf{h}_{\mathrm{r},1}-\eta_m}{\sigma/\sqrt{2}}\right). \tag{16}$$

This cyclic procedure continues until practical convergence. In Step 3, one-bit RELAX assumes that $N_s = 3$. The algorithm moves on to estimate the parameter vector $\hat{\chi}_3$ of the third strongest channel path using the $\hat{\chi}_1$ and $\hat{\chi}_2$ obtained from Step 2. Then the $\hat{\chi}_1$, $\hat{\chi}_2$, and $\hat{\chi}_3$ are cyclically refined. The algorithm continues in this fashion until the parameter vectors of all paths are estimated.

We note that the one-bit RELAX can also be implemented in the unknown noise variance case with a slight modification. In contrast to the original ML method, one-bit RELAX requires only two-dimensional searches over the parameter space of AoA and AoD for each path, resulting in significantly reduced computational complexities.

5. NUMERICAL EXAMPLES

Numerical examples are given below to evaluate the channel estimation performance. We consider a system model with $N_r = 16$, $N_t = 4$ and $N_s = 2$. Since the channel matrix is normalized as $\|\mathbf{H}\|_F^2 = N_r N_t$, where $\|\cdot\|_F^2$ represents the Frobenius norm of a matrix. And with $\rho = 1$, the average SNR at each receive antenna is $\frac{1}{\sigma^2}$. The angular-domain parameters corresponding to the 2 scatterers, which are listed in Table 1, are drawn randomly once and fixed at all trials. For one-bit sampling with time-varying thresholding (TT) and antenna-varying thresholding (AT) schemes, we use 8 discrete thresholds uniformly distributed in $[-\mathrm{th}_{\max}, \mathrm{th}_{\max}]$ with $\mathrm{th}_{\max} = 0.5$.

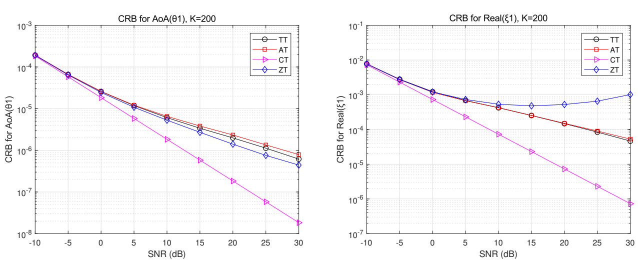

Fig. 1. CRB comparison as a function of SNR with $K = 200$ when σ^2 is known.

Path index	AoA ($\boldsymbol{\theta}$)	AoD ($\boldsymbol{\varphi}$)	Real($\boldsymbol{\xi}$)	Imag($\boldsymbol{\xi}$)
1	$69.50°$	$-13.92°$	0.6139	-0.4554
2	$58.33°$	$-44.92°$	-0.5638	0.3129

Table 1. Angular-domain channel parameters used in the simulations.

We compare the CRBs as a function of SNR when the noise variance σ^2 is known in Fig. 1. Except for the aforementioned TT and AT schemes, we also consider the zero thresholding (ZT) scheme with $\boldsymbol{\eta} = \mathbf{0}$ and a clairvoyant thresholding (CT) scheme with $\boldsymbol{\eta} = \mathbf{y}_r$. As the CRB curves for estimating the AoAs ($\boldsymbol{\theta}$) and AoDs ($\boldsymbol{\varphi}$) are similar, which also holds ture for estimating Re($\boldsymbol{\xi}$) and Im($\boldsymbol{\xi}$), we only plot CRBs for the AoA(θ_1) and Re(ξ_1) corresponding to the first scattering path here for clarity. It is shown in Fig. 1 that the CT curve is log-linear with respect to the SNR and it provides a lower bound for channel estimation using signed measurements. For angular estimation, one-bit sampling with the ZT, AT, and TT schemes offers similar performances. However, for the channel gain estimation, the performance of the ZT scheme degrades significantly in the high SNR regime.

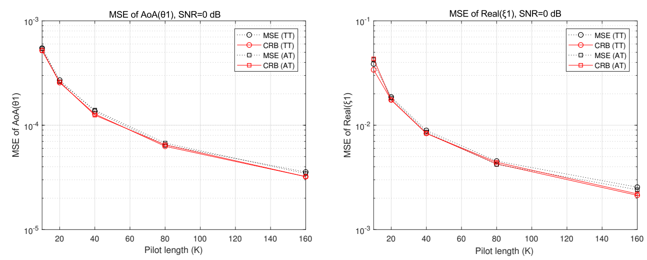

Fig. 2. MSE of one-bit RELAX as a function of K with SNR = 0 dB when σ^2 is unknown.

Next, we evaluate the mean-squared error (MSE) performance of the proposed one-bit RELAX algorithm for channel estimation as a function of pilot length K when SNR = 0 dB as shown in Fig. 2. When the noise variance σ^2 is unknown, the FIMs corresponding to the ZT and CT schemes are singular, which means they fail to work properly in this case and, hence, they are not included here. The MSEs are obtained by averaging over 1000 Monte Carlo trials with random realizations of the noise in each trial. It is seen from Fig. 2 that the AT scheme yields almost identical performance compared with its TT counterpart, but at a much lower cost. Note also that the MSEs obtained using one-bit RELAX algorithm approach the corresponding CRBs closely even when the pilot length K is as small as 10.

6. CONCLUSION

In this paper, we have investigated massive MIMO channel estimation using signed measurements obtained with one-bit sampling with antenna-varying thresholds. We have derived the performance bound, the CRB, for unbiased channel estimators. In contrast to the zero thresholding scheme, time-varying and antenna-varying thresholding schemes allow for the noise variance to be unknown. Moreover, they are shown to outperform their zero thresholding counterpart, especially at high SNRs. We have also shown that the antenna-varying thresholding scheme offers a similar performance compared to its time-varying thresholding counterpart, but allows a much simpler practical implementation. We also proposed a computationally efficient algorithm, referred to as one-bit RELAX, to mitigate the high computational complexities of the ML approach. Numerical examples have been provided to show that one-bit RELAX can be used to obtain accurate channel estimates for massive MIMO systems.

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