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Abstract

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ESTIMATION OF FREQUENCY UNSYNCHRONIZED MILLIMETER-WAVE CHANNELS

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ABSTRACT

We propose an algorithm for mmWave channel estimation in the presence of a non-negligible carrier frequency offset (CFO) impairment. The algorithm exploits inherent sparsity of the mmWave channel in the angle of departure (AoD) and arrival (AoA) domain, employs sparse channel sensing protocol, and performs blind CFO compensation and estimation of channel paths in the angular domain. The simulation tests show that the proposed algorithm considerably outperforms the conventional orthogonal matching pursuit (OMP)-based mmWave channel estimation algorithm under different CFO and signal-to-noise ratio (SNR) regimes. Aside from the mmWave channel estimation problem, the algorithm is more generally applicable for greedy sparse recovery problems associated with estimating a sparse vector from measurements impaired by frequency offset.

***Index Terms*—** mmWave channel estimation, carrier frequency offset (CFO), frequency synchronization, sparse recovery, OMP.

1. INTRODUCTION

The proliferation of wireless services and continuous increase in the data throughput demand have sparked the interest and research in communications over unlicensed millimeter-wave (mmWave) frequency ranges [1]. The mmWave channel modeling and estimation have received a considerable interest in the past few years. A variety of reported channel measurements suggest that the mmWave channel is sparse in the angular domain, indicating that there are a few dominant paths between transmitter and receiver [2].

In general, two types of approaches have been used for channel estimation, both relying on the sparse nature of the channel. In a more traditional approach, the transmitter and receiver effectively probe each possible channel path, i.e., each combination of the angle of departure (AoD) and angle of arrival (AoA), and delineate the paths that deliver most of the transmitted energy [1]. To reduce the computational burden and time required to probe all AoD-AoA pairs (note that each pair is probed separately and in serial), different schemes have been proposed. A common approach in those schemes is to sense the channel initially with wider probing angles, coarsely detect sectors containing the dominant paths and then sense the sectors with narrower probing angles until the dominant paths are precisely located in the angular domain. The currently existing IEEE 802.11ad and IEEE 802.15.3c standards operating over 60 GHz frequency range employ this approach for channel estimation [3, 4]. A particularly interesting solution with overlapping probing angles has been proposed in [5].

The other mmWave channel estimation approach directly exploits the sparse nature of the channel in the angular domain. In particular, it senses the channel by transmitting a certain number of randomly precoded unit symbols and applying random combiners at the receiver side, thereby sensing all AoD and AoA directions randomly with low energy [6]. A sparse recovery algorithm, such as the Orthogonal Matching Pursuit (OMP) [7], is then employed to estimate the paths and their gains. The application of compressive sensing/sparse recovery methods for general communication systems is surveyed in [8].

This approach has shown benefits with respect to the traditional angle probing approach in several aspects. First, it has been shown that this approach requires a considerably smaller number of transmitted pilots, thus reducing the duration of the training stage [9]. Second, the sparse channel sensing protocol is suitable for channel estimation in multi-user system, where multiple users perform downlink channel estimation in parallel using the same randomly precoded pilots transmitted from a base station [9]. Finally, a standard sparse recovery formulation, stemming from sparse channel sensing protocol, enables the development of channel estimation algorithms for mmWave channels exhibiting not only sparse, but also spatially spread departures and arrivals [10, 11]. Despite advantages, the outstanding issue of the sparse recovery-based mmWave channel estimation is its lack of robustness to carrier frequency offset (CFO). Namely, the sensing protocol and the sparse recovery formulation do not account for CFO. Consequently, the inability to estimate and compensate the CFO from channel measurements precludes the sparse recovery-based approach from practical usage [12].

Numerous methods have been proposed for CFO estimation and compensation [13]. Of those, most contemporary algorithms have been developed for Orthogonal Frequency Division Multiplexing (OFDM) systems, e.g., [14, 15, 16]. However, to the extent of our knowledge, the CFO issue has not been addressed in the context of sparse recovery-based mmWave channel estimation. To bridge that gap, we propose an algorithm which blindly compensates the CFO and estimates the mmWave channel by directly exploiting the sparsity in the angular domain and random channel sensing protocol [6], thereby preserving all above elaborated advantages of the sparse recovery-based mmWave channel estimation. The performance of the proposed algorithm is validated using Monte-Carlo simulations and a significant performance improvement over the conventional OMP-based mmWave channel estimation method is demonstrated.

2. CHANNEL AND SENSING MODELS

This section first presents received signal and channel model and then details channel sensing protocol.

2.1. Model without CFO

The signal vector in the complex baseband representation, transmitted at discrete time n, is denoted $\mathbf{x}_n \in \mathbb{C}^{N_t \times 1}$, where N_t is the number of transmit antennas. The received signal in the complex baseband representation is denoted by $\mathbf{y}_n \in \mathbb{C}^{N_r \times 1}$, where N_r is the number of receive antennas. It is given by

$$\mathbf{y}_n = \mathbf{H}_n \mathbf{x}_n + \mathbf{v}_n, \tag{1}$$

where $\mathbf{H}_n \in \mathbb{C}^{N_r \times N_t}$ is a realization of channel matrix at discrete time n and $\mathbf{v}_n$ is circularly symmetric additive white Gaussian noise (AWGN), $\mathbf{v}_n \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$. As implied by the model (1), we assume the communication system is narrowband. In addition, as in most research works on mmWave channel estimation, we assume time-invariant channel over some fixed time interval, whose one segment is devoted to channel estimation and the remaining segment to communications.

A variety of field measurements [1, 2] have shown that mmWave channel is sparse in the angular domain and this fact has been well accepted in the research community. This essentially means that the transmitted signal propagates over a small number of paths (direct path and reflected and scattered paths) before reaching the receiver. Each path is characterized with its own AoD and AoA. The channel matrix is thus represented as

$$\mathbf{H} = \mathbf{A}_r \mathbf{G} \mathbf{A}_t^H. \tag{2}$$

Above, $\mathbf{A}_r \in \mathbb{C}^{N_r \times G_r}$ and $\mathbf{A}_t \in \mathbb{C}^{N_t \times G_t}$ are, respectively, receiver and transmitter array manifold matrices, while G_r and G_t are the number of discretization points of the receiver and transmitter angular domains. The channel representation in the virtual angular domain is $\mathbf{G} \in \mathbb{C}^{G_r \times G_t}$.

Without loss of generality, we assume vertical line arrays and use them in the simulations. Discretizing the AoA domain $\theta_r \in [-\pi/2, \pi/2]$ into $G_r = N_r$ points, the i-th column in $\mathbf{A}_r$, corresponding to AoA $\theta_{r,i}$, is given by

$$\mathbf{a}_i = \begin{bmatrix} 1 & e^{-j\pi \sin\theta_{r,i}} & \dots & e^{-j\pi(N_r-1)\sin\theta_{r,i}} \end{bmatrix}, \tag{3}$$

where $i = 1, 2, \dots, G_r$ and we assume the antennas are separated by half the wavelength corresponding to the carrier frequency, while the angle is measured with respect to the line perpendicular to the array such that $\theta_r = 90^o$ corresponds to an arrival coming from the array endfire, while $\theta_r = 0^o$ corresponds to an arrival coming from the broadside of the array. The array manifold vectors in $\mathbf{A}_t$, corresponding to AoDs $\theta_{t,j}$, are obtained analogously.

Since the mmWave channel is sparse in the angular domain, the representation $\mathbf{G}$ is a sparse matrix. A non-zero entry (i, j) in $\mathbf{G}$ implies that a mmWave path between transmitter and receiver has AoD $\theta_{t,j}$, and AoA $\theta_{r,i}$. The mmWave channel estimation problem consists of estimating AoDs and AoAs of non-zero paths and the corresponding coefficients. The following part describes a sparse channel sensing strategy.

2.2. Channel Sensing

We assume that both transmitter and receiver employ a single radio frequency (RF) chain. That means that only one stream of data symbols is communicated between the transmitter and receiver. In the considered sensing strategy, the transmitter applies precoding vector $\mathbf{p}_n \in \mathbb{C}^{N_t \times 1}$ onto the transmitted symbol s_n at time n to exploit the channel spatial degrees of freedom. Due to hardware related constraints, the precoding is, up to a constant magnitude, implemented by performing phase shifting in each transmit antenna. Therefore, the transmitted signal is

$$\mathbf{x}_n = \mathbf{p}_n s_n, \tag{4}$$

The receiver collects signals arriving from different directions by applying a combining vector $\mathbf{q}_n \in \mathbb{C}^{N_r \times 1}$. Similar to precoding vectors, the combining vector is up to a constant magnitude realized analogously with phase shifters in each receiver antenna. The signal obtained after analog combining is

$$z_n = \mathbf{q}_n^H \mathbf{y}_n. \tag{5}$$

The channel sensing strategy proposed in [6] is utilized here. Intuitively, since the mmWave channel is sparse in the angular domain, each measurement needs to randomly and uniformly insonify all channel directions. This is achieved by applying random precoding and combining vectors whose entries are sampled uniformly at random from the corresponding alphabets.

Assuming the applied precoding and combining vectors are $\mathbf{p}_n$ and $\mathbf{q}_n$, the observation at time n is from (1), (4) and (5) evaluated as

$$z_n = \mathbf{q}_n^H \mathbf{H} \mathbf{p}_n s_n + \mathbf{q}_n^H \mathbf{v}_n. \tag{6}$$

The following steps are standard manipulations used in mmWave channel estimation framework. Namely, using that $\text{vec}\{\mathbf{ABC}\} = \left(\mathbf{C}^T \otimes \mathbf{A}\right) \text{vec}\{\mathbf{B}\}$, where $\text{vec}\{\mathbf{X}\}$ vectorizes matrix $\mathbf{X}$, and $\otimes$ denotes the Kroneker product [17, Theorem 13.26], the observation z_n in (6) is expressed as

$$z_n = \left(\mathbf{p}_n^T \otimes \mathbf{q}_n^H\right) \text{vec}\{\mathbf{H}\} + \tilde{v}_n, \tag{7}$$

where $\tilde{v}_n \sim \mathcal{CN}(0, \sigma^2)$ because $\|\mathbf{q}_n\| = 1$ by assumption, and $s_n = 1$ without loss of generality. Using the representation of $\mathbf{H}_n$ in (2), and applying the Kroneker property on $\text{vec}\{\mathbf{H}\}$ in (7) yields

$$z_n = \left(\mathbf{p}_n^T \otimes \mathbf{q}_n^H\right) \left(\mathbf{A}_t^* \otimes \mathbf{A}_r\right) \text{vec}\{\mathbf{G}\} + \tilde{v}_n. \tag{8}$$

To simplify the observation model in (8), we denote $\mathbf{b}_n^H \triangleq \left(\mathbf{p}_n^T \otimes \mathbf{q}_n^H\right) \left(\mathbf{A}_t^* \otimes \mathbf{A}_r\right)$ and $\mathbf{g} \triangleq \text{vec}\{\mathbf{G}\}$ so that

$$z_n = \mathbf{b}_n^H \mathbf{g} + \tilde{v}_n. \tag{9}$$

We note that there is one-to-one correspondence between the locations of non-zero entries in $\mathbf{g}$ and AoA and AoD of the existing paths. Therefore, the goal is to recover the support of $\mathbf{g}$ and the corresponding coefficients, based on the observations z_n and known $\mathbf{b}_n^H$, $n = 1, \dots, K$, where K is the training overhead.

Stacking up the measurements z_n in a vector, the resulting observation vector $\mathbf{z}$ is expressed as

$$\mathbf{z} = \mathbf{B}\mathbf{g} + \mathbf{v}, \tag{10}$$

where $\mathbf{v}$ is a vector of noise samples $\tilde{v}_n$ and $\mathbf{B}$ is a measurement/sensing matrix whose k-th row is $\mathbf{b}_n^H$. Given that $\mathbf{g}$ is a long and sparse vector, it can be recovered from (10) using a sparse recovery method. The OMP [7] is conventionally used in this setting.

As an aside remark, we note that generalization of the above sensing protocol to the case where more RF chains are employed is relatively straightforward. In addition, such a scenario essentially yields more measurements, e.g., transmitting and receiving one pilot symbol with random precoding and combining over 2 RF chains on each channel end, yields two channel measurements.

3. PROPOSED ALGORITHM

3.1. Model with CFO

The lack of synchronization between the transmitter and receiver results in incorrect estimates of mmWave path locations and the corresponding coefficients. The sensing scheme described above does not naturally take the CFO into account, let alone estimate it.

$$\gamma_{n,m} = y_{n+m} y_n^* = e^{j2\pi m \Delta f} \left[\sum_{i=1}^{N} \sum_{l=1}^{N} a_{n+m,i} a_{n,l}^* \tilde{g}_i \tilde{g}_l^* + v_k'^* \sum_{i=1}^{N} a_{n+m,i} \tilde{g}_i + v_{n+m}' \sum_{l=1}^{N} a_{n,l}^* \tilde{g}_l^* + v_{n+m}' v_n'^* \right]$$
$$= e^{j2\pi m \Delta f} \sum_{i=1}^{N} \sum_{l=1}^{N} a_{n+m,i} a_{n,l}^* \tilde{g}_i \tilde{g}_l^* + q_{n,m} = e^{j2\pi m \Delta f} \mathbf{c}_{n,m}^T \mathbf{h} + q_{n,m} \quad (15)$$

In the presence of CFO, the received signal at discrete time n is using (9) given by

$$y_n = e^{j(n-1)2\pi \Delta f} z_n = e^{j(n-1)2\pi \Delta f} \mathbf{b}_n^H \mathbf{g} + v_n', \quad (11)$$

where $n = 1, \ldots, K$, Δf is the normalized CFO, and $v_n' \sim \mathcal{CN}(0, \sigma^2)$. The overall signal $\mathbf{z}$ is thus modeled in the vector form as

$$\mathbf{z} = \mathbf{FBg} + \mathbf{v}', \quad (12)$$

where $\mathbf{F} = \text{diag}\{1, e^{j2\pi \Delta f}, \ldots, e^{j2\pi(K-1)\Delta f}\}$.

Since model (12) better represents the reality, applying OMP or some other sparse recovery method, tailored for model (10) results in wrong channel estimates. The numerical examples demonstrate that even relatively small CFO leads to considerable degradation in channel estimation mean square error (MSE) performance. In the following, we present an algorithm which jointly estimates the CFO Δf and mmWave channel $\mathbf{g}$. The proposed algorithm iteratively updates the estimate of Δf and recovers the support of $\mathbf{g}$ along with the corresponding coefficients. To understand how the algorithm works, we first consider a simpler problem where the support of $\mathbf{g}$ (i.e, the indices of its non-zero entries) is known. In such a case, the problem is to jointly estimate the CFO and channel coefficients.

3.2. Joint CFO and Channel Estimation

Assuming the support $\mathcal{S}$ of the channel vector $\mathbf{g}$ is known, the received signal at time n is expressed as

$$y_n = e^{j(n-1)2\pi \Delta f} \mathbf{a}_n^T \tilde{\mathbf{g}} + v_n', \quad (13)$$

where $\mathbf{a}_n^T$ and $\tilde{\mathbf{g}}$ are obtained by taking the entries from $\mathbf{b}_n^H$ and $\mathbf{g}$ corresponding to the support $\mathcal{S}$. The vector-matrix representation of the above model is given by

$$\mathbf{y} = \mathbf{FA}\tilde{\mathbf{g}} + \mathbf{v}', \quad (14)$$

where the n-th row in $\mathbf{A}$ is $\mathbf{a}_n^T$.

The autocorrelation of the received signal y_n at lag m is given by (15) (top of this page), where $q_{n,m}$ is the cumulative effect of all noise-like terms, $n = 1, \ldots, K - m$, and

$$\mathbf{h} = \begin{bmatrix} |\tilde{g}|_1^2 & \tilde{g}_1 \tilde{g}_2^* & \ldots & |\tilde{g}|_2^2 & \ldots & |\tilde{g}|_N^2 \end{bmatrix}^T \quad (16)$$

contains N entries solely depending on the magnitude of the channel $\tilde{\mathbf{g}}$ coefficients. Note that the vector $\mathbf{c}_{n,m}$ is defined in a similar way. The vector of the autocorrelations $\gamma_{n,m}$, $n = 1, \ldots, K - m$, is using (15) expressed as

$$\boldsymbol{\gamma}_m = e^{j2\pi m \Delta f} \mathbf{Ch} + \mathbf{q}_m$$
$$= \mathbf{C}_m \mathbf{h}_m' + \mathbf{q}_m, \quad (17)$$

where the n-th row of $\mathbf{C}_m$ is $\mathbf{c}_{n,m}^T$, $\mathbf{h}_m' = e^{j2\pi m \Delta f} \mathbf{h}$ and $\mathbf{q}_m$ is the vector of noise-like samples $q_{n,m}$.

The crux of the algorithm is in the observation that the phases of N entries in $\mathbf{h}_m'$ solely depend on the frequency offset Δf. Thus, the estimate of the CFO Δf can be obtained from phase arguments of the corresponding entries in the estimate of $\mathbf{h}_m'$.

The least-squares (LS) estimate of $\mathbf{h}_m'$ is obtained from (17) and given by

$$\hat{\mathbf{h}}_m' = (\mathbf{C}_m^H \mathbf{C}_m)^{-1} \mathbf{C}_m \boldsymbol{\gamma}_m. \quad (18)$$

Denoting with $\{I\}$ the set of indices of entries in $\mathbf{h}_m$ whose phase is zero, the CFO is estimated as

$$\Delta \hat{f}_m = \frac{1}{2\pi m} \arg \sum_{j \in \{I\}} [\hat{\mathbf{h}}_m']_j, \quad (19)$$

where $[\mathbf{x}]_j$ denotes the jth entry in the vector $\mathbf{x}$. The final estimate of the CFO is given as an average over estimates $\Delta \hat{f}_m$, obtained for different lags m. Thus,

$$\Delta \hat{f} = \frac{1}{M} \sum_{m=1}^{M} \Delta \hat{f}_m. \quad (20)$$

The estimate of the unknown channel $\tilde{\mathbf{g}}$ is finally obtained from (14) by substituting $\Delta \hat{f}$ into $\mathbf{F}$ and using the LS,

$$\hat{\tilde{\mathbf{g}}} = \left(\mathbf{A^H A} \right)^{-1} \mathbf{A^H F^H y}. \quad (21)$$

3.3. Iterative Method

Now we are ready to describe the iterative estimation of the CFO Δf and sparse channel vector $\mathbf{g}$.

The recovered support at the beginning of iteration p is $\mathcal{S}_{p-1}$. Its complement, i.e., the set of indices of zero entries in the current estimate of the channel vector $\mathbf{g}$, is denoted as $\mathcal{S}_{p-1}^c$. All entries in $\mathcal{S}_{p-1}^c$ are separately considered to be included in the support during iteration p. Note that $\mathcal{S}_0$ is an empty set in the first iteration.

The CFO and channel vector corresponding to the support $\mathcal{S}_{p-1}^s = \mathcal{S}_{p-1} \cup \{s\}$ are estimated using the procedure outlined in Section 3.2, for each $s \in \mathcal{S}_{p-1}^c$. The objective function corresponding to $\mathcal{S}_{p-1}^s$ is the power of the residual signal resulting from the LS fit (21). The index s' which minimizes the objective function is declared as new support element such that $\mathcal{S}_p = \mathcal{S}_{p-1} \cup \{s'\}$ is the recovered support fed back to the next iteration $p + 1$.

The iterative procedure is terminated after a predefined number of iterations or after a certain convergence criterion is met. Assuming the procedure is terminated after P iterations, the recovered support is $\mathcal{S}_P$ and the estimates of the CFO Δf and channel vector $\mathbf{g}$ are those corresponding to the index s' which minimizes the objective function in the last iteration.

4. NUMERICAL STUDY

The numerical study has been performed using Monte-Carlo simulations. In the considered scenario, the number of transmit and receive

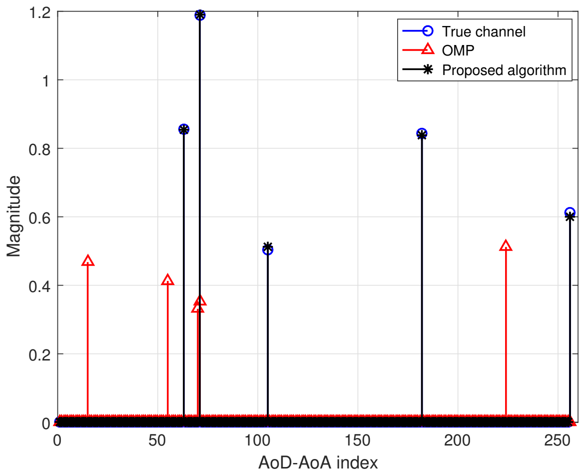

Fig. 1: True and estimated channel magnitudes for the normalized CFO of 1% and SNR of 10 dB.

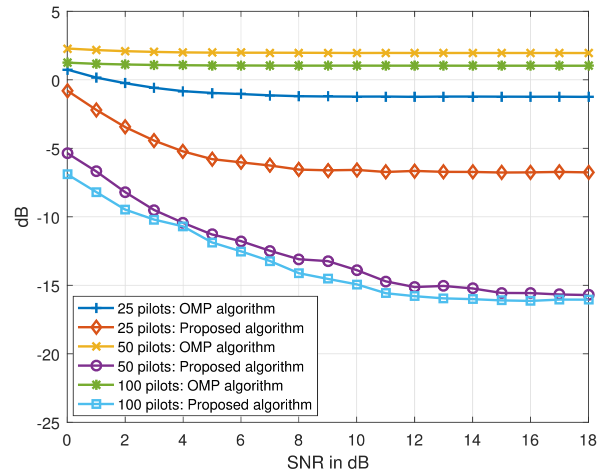

Fig. 2: Average NMSE of the OMP and proposed algorithm for fixed normalized CFO of 1% and different number of pilots K.

antennas is $N_t = N_r = 16$ and the mmWave channel has 5 paths. A particular realization of the AoDs and AoAs and the corresponding gains are uniformly at randomly generated from the corresponding sets in each Monte-Carlo run. In addition, each entry in the precoding and combining vectors is uniformly at random sampled from the set $\{\pm 1, \pm j\}$, meaning that the phase shifters in each antenna are quantized with 4 levels. Finally, the AWGN is also random in each run. Effectively, the reported estimation performance is averaged over AoAs, AoDs, path gains, precoding and combining vectors, as well as the additive noise.

The performance metric used for evaluation is the average normalized channel estimation mean square error (NMSE), obtained from averaging the NMSEs computed in each Monte-Carlo run. The number of averaging terms M in (20) is 10 and kept constant in all simulations. As an aside note, the algorithm's performance can be optimized with respect to M and, in fact, we have observed that larger M is more suitable for smaller CFOs, while smaller M is more suitable for larger CFOs.

As a benchmark, we simulate the performance of the conventional OMP algorithm with the aim to study how the channel estimation performance deteriorates with frequency offset. Since we are primarily interested in the robustness of the channel estimation with respect to the CFO, we assume the algorithms are supplied with the correct number of iterations to be run (i.e, the channel paths). In addition, and for the same reason, all AoDs and AoAs are assumed to be on the discrete Fourier transform (DFT) grid.

The true channel and its estimates obtained from the OMP and proposed algorithm in one simulation run for K of 100, SNR of 10 dB and normalized CFO of 1% are shown in Fig. 1. Although the OMP correctly identifies the index (i.e., AoD and AoA) of the strongest path in this example (which is, in general, not necessarily the case), it fails to correctly estimate the corresponding gain due to the CFO and, consequently, the following iterations produce completely incorrect estimates. On the other hand, the proposed algorithm correctly recovers the indices of all paths, with possibly small errors in the gain estimates.

The NMSEs of the OMP and proposed algorithm for a fixed normalized CFO of 1% and varying number of pilots K are shown in Fig. 2. As can be seen, the proposed algorithm outperforms the OMP. Note that a seemingly improved OMP performance as K decreases from 100 (or 50) to 25 is because smaller number of pilots cause smaller phase rotation in the received signal due to the CFO.

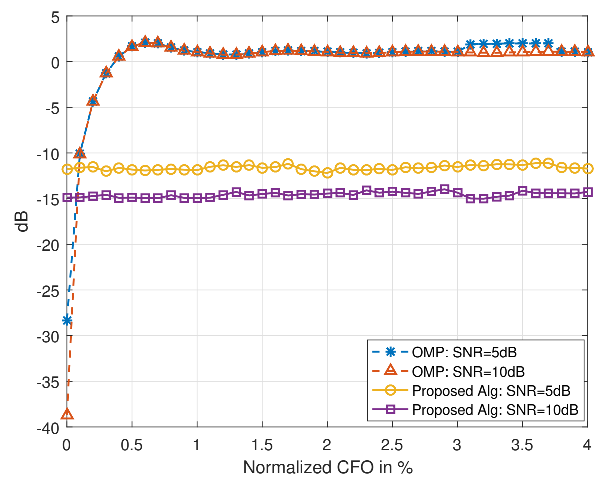

Fig. 3: Average NMSE of the OMP and proposed algorithm for fixed number of pilots $K = 100$ and different SNRs.

The NSMEs of the OMP and proposed algorithm for $K = 100$ and different SNRs are shown in Fig. 3. As can be seen, the NMSE performance of the proposed algorithm is almost insensitive to the considered range of CFO values and uniformly outperforms the OMP. The only exception occurring when the CFO is not present is due to the approximations made in the algorithm development. Further investigation of this case is left as our future research.

5. CONCLUSION

We have proposed a sparse recovery-based algorithm which blindly compensates frequency offset and estimates mmWave channel. The proposed algorithm can also be used for greedy sparse recovery from frequency offset impaired measurements. The algorithm's performance is validated using numerical simulations and uniform performance over a broad range of CFOs has been demonstrated.

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