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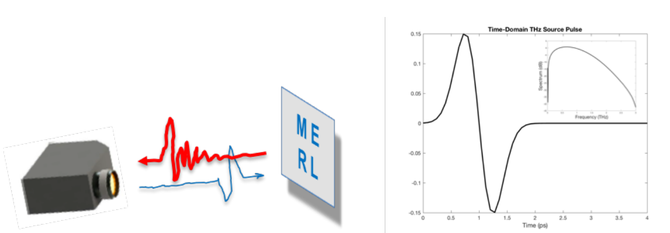
***Abstract*—This paper considers Terahertz (THz) sensing of contents on a single-layer object with time-domain spectroscopy (TDS) in a reflection mode. With an extremely small pulse width, the THz-TDS sensing is highly sensitive to non-ideal configurations such as a tilted or uneven sample, depth variation due to mechanical scanning operations, and intra-layer reflections. One consequence is sweep distortion resulting from slightly different time delay from different scanning points. In this paper, we follow an existing alternative minimization scheme and propose a simple modification to take into account, in addition to the binary nature of reflection coefficients, the two-dimensional block feature of the content (i.e., letters and shapes) in the sample. Preliminary results with synthetic datasets show the effectiveness of the proposed method.**

I. Introduction

Over the past several years, there has been an increased interest in the potential of terahertz (THz) sensing, detection and imaging, due to its application in gas sensing, moisture analysis, package inspection, biomedical diagnosis, and security screening [1]. In many of these imaging applications it is valuable to generate an image of samples under inspection.

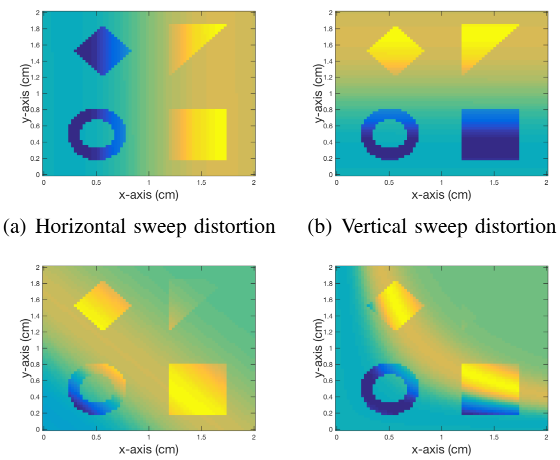
In this paper, we are particularly interested in extracting content on a single-layer object from a reflection mode. As shown in Fig. 1 (a), the sample under inspection is illuminated by a THz-TDS source with a time-compact source pulse of Fig. 1 (b). The content is often obtained by a two-dimensional scanning process [2], [3]. Specifically, the THz-TDS source first sends a focused beam with the normal incident angle to illuminate a small area (e.g., a spot or pixel) on the sample, the THz-TDS detector then samples corresponding reflected time-domain waveforms, and a programmable mechanical raster moves the sample in the plane perpendicular to the incidental waveform to illuminate other pixels on the sample.

One challenge here is that the scanning process is subject to different depth variations from one pixel to another, either due to irregular sample surfaces or the mechanical scanning itself [4]. Consequently, the depth variation results in phase distortions and causes sweep distortion in the reflected waveform. Fig. 2 shows several types of sweep distortion due to systematically introduced depth variations during the two-dimensional raster scanning of the sample.



(a) (b)

Fig. 1. THz-TDS sensing (a) with a reflection mode and (b) using a time-compact source pulse.



(a) Horizontal sweep distortion (b) Vertical sweep distortion

(c) Diagonal sweep distortion (d) Spherical sweep distortion

Fig. 2. Various types of sweep distortions due to irregular sample surfaces or the mechanical raster scanning.

II. Problem Formulation

To address the issue of sweep distortion, [4] first formulated the phase distortions induced by depth variations a bilinear inverse problem:

$$\mathbf{y}_m(\mathbf{x}) = \boldsymbol{\rho}(\mathbf{x}) \odot \mathbf{u}_m(\mathbf{x}) + \mathbf{v}_m(\mathbf{x}), \quad m = 1, 2, \cdots, M, \quad (1)$$

where $\mathbf{x} = (x, y)$ denotes the compact spatial coordinate of a given pixel, $\mathbf{y}_m(\mathbf{x})$ is the reflected waveform at the time m for the pixel $\mathbf{x} = (x, y)$, $\boldsymbol{\rho}(\mathbf{x})$ is the reflection coefficient, $\mathbf{u}_m(\mathbf{x})$ is the distortional profile at time m, $\mathbf{v}_m(\mathbf{x})$ is the noise, and $\odot$ denotes the point-wise product. All vectors here are of length P where P is the total number of pixels.

Given the distortion profile can be represented by a subspace and the binary reflection vector can be expressed by another subspace, i.e., $\mathbf{u}_m = \mathbf{S}^m \boldsymbol{\alpha}_m$ and $\boldsymbol{\rho} = \mathbf{Q}\boldsymbol{\beta}$, the reflection coefficients can be recovery by a low-rank recovery problem

$$\min_{\mathbf{X}} \|\mathbf{X}\|_*, \quad \text{s.t. } \|\mathbf{Y} - \mathcal{A}(\mathbf{X})\|_F \leq \epsilon, \quad (2)$$

where $\|\cdot\|_*$ denotes the nuclear norm, $\|\cdot\|_F$ denotes the Frobenius norm, and $\mathcal{A}(X)$ defines a linear operator $\mathcal{A}(\mathbf{X})_{i,j} = \left\langle \mathbf{X}, \mathbf{Q}_{i,:}^T \mathbf{S}_{i,:}^j \mathbf{P}_j \right\rangle$. However, this formulation of (2) may not capture the binary nature of the reflection vector $\boldsymbol{\rho}$. Instead, [4] solved it with an alternating minimization scheme [6] which, by fixing one of two unknown parameters in the bilinear term in (1), i.e., either $\boldsymbol{\rho}$ or $\mathbf{u}_m$, the other parameter can be estimated by a standard least squares problem. Specifically, it used a universal Gaussian mixture distribution as the prior distribution for the reflection vector $\boldsymbol{\rho}$ to exploit its binary nature. Then the maximum *a posteriori* (MAP) estimate of $\boldsymbol{\rho}$ can be obtained by exploiting the binary

III. Proposed Method

In this paper, we follow the alternative minimization scheme and propose a simple modification to take into account, in addition to the binary nature of the reflection vector $\boldsymbol{\rho}$, the two-dimensional block feature of the content (i.e., letters and shapes) in the sample. Specifically, we modify the MAP updating procedure of [4] into a *coupled* updating procedure: for each individual pixel, $p = 1, 2, \cdots, P$,

1) Compute

$$w_0 = \max\left(\frac{(N_p\zeta+1)\sigma^2\rho^0 + \sigma_0^2\sum_{p\in\mathcal{N}(p)}\zeta_p\sum_{m=1}^M (u_{m,p}^{(k)} y_{m,p})}{(N_p\zeta+1)\sigma^2 + \sigma_0^2\sum_{p\in\mathcal{N}(p)}\zeta_p\sum_{m=1}^M u_{m,p}^{(k)2}}, 0\right)$$

2) Compute

$$w_1 = \max\left(\frac{(N_p\zeta+1)\sigma^2\rho^1 + \sigma_1^2\sum_{p\in\mathcal{N}(p)}\zeta_p\sum_{m=1}^M (u_{m,p}^{(k)} y_{m,p})}{(N_p\zeta+1)\sigma^2 + \sigma_1^2\sum_{p\in\mathcal{N}(p)}\zeta_p\sum_{m=1}^M u_{m,p}^{(k)2}}, 0\right)$$

3) Compare $g(w_0, 0)$ and $g(w_1, 1)$:
   - if $g(w_0, 0) \leq g(w_1, 1)$, $\hat{\rho}_p = w_0$
   - if $g(w_0, 0) > g(w_1, 1)$, $\hat{\rho}_p = w_1$

where (ρ^0, σ_0) and (ρ^1, σ_1) are the means and standard deviations of the Gaussian mixture distributions, respectively, σ^2 is the noise variance, $\mathcal{N}(p)$ is an index set including the p-th pixel and its neighboring pixels for the p-th pixel, e.g., $\mathcal{N}(p) = \{p, p-1, p+1\}$ or $\mathcal{N}(p) = \{p, p-1, p+1, p-N_r, p+N_r\}$ with N_r denoting the number of pixels in the x-axis, $\zeta_i = \zeta$ if $i \neq p$ and $\zeta = 1$ if $i = p$, and $g(\cdot,\cdot)$ is defined as follows:

$$g(\rho_p, C_p) = \sum_{p=\mathcal{N}(p)} \zeta_p \left[\log\frac{\sigma_{C_p}}{p_{C_p}} + \frac{(\rho_p - \rho^{C_p})^2}{2\sigma_{C_p}^2} + \sum_{m=1}^M \frac{(y_{m,p} - \rho_p u_{m,p}^{(k)})^2}{2\sigma^2}\right] \quad (3)$$

In general, we use the joint posterior likelihood over local neighboring pixels, instead of a single pixel, to determine the MAP estimate of $\boldsymbol{\rho}$.

IV. Simulation Results

In the following, we consider a dielectric slab with the pattern shown in Fig. 2, where 4 different shapes. For the binary dielectric slab, the reflection coefficients are 0.8 and 0.3, respectively. The sample is probed with a bipolar THz pulse as the same as [4]. To simulate the sweep distortion, the depth variation is modeled as $z = z_0 + \alpha_1 x + \alpha_2 y$, where $\alpha_1 = 3\cdot 10^{-5}$ and $\alpha_2 = 3\cdot 10^{-5}$, similar to Fig. 2.C. The dielectric thickness is 100μm. $M = 10$ uniform samples of the reflected wave in a time period of 0.8 ps were used for the bilinear inverse problem.

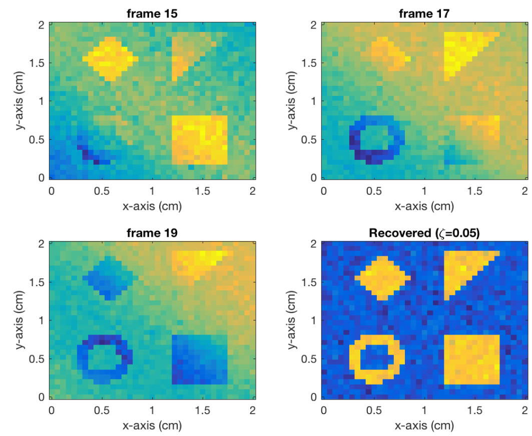


Fig. 3. The observed sweep distortion at different time frames and the recovered THz image with $\zeta = 0.05$.

As shown in Fig. 3, the sweep distortion profiles were observed at different time instants when the SNR is about 10 dB. To apply the coupled-pixel recovery algorithm, we use the neighboring pixel set as $\mathcal{N}(p) = \{p, p-1, p+1\}$ and the coupling coefficient $\zeta = 0.05$. The recovered THz image of the binary dielectric slab is shown in the lower right plot of Fig. 3. It is shown that the recovery image can effectively remove the sweep distortion and relatively keep the binary feature of the reflection coefficients.

V. Conclusion

We proposed to use the joint posterior likelihood over several neighboring pixels to update the reflection coefficients. We will leave the automatic selection of prior distribution parameters and experimental verification for future studies.

References

[1] N. Horiuchi, "Terahertz technology: Endless applications," *Nature Photonics*, vol. 4, no. 4, pp. 140, September 2010.

[2] G. C. Walker, J. W. Bowen, J. Labaune, J-B. Jackson, S. Hadjiloucas, J. Roberts, G. Mourou, and M. Menu, "Terahertz deconvolution," *Optics Express*, vol. 20, no. 25, pp. 27230–27241, December 2012.

[3] A. Redo-Sanchez, B. Heshmat, A. Aghasi, S. Naqvi, M. Zhang, J. Romberg, and R. Raskar, "Terahertz time-gated spectral imaging for content extraction through layered structures," *Nature Communications*, vol. 7, pp. 1–7, September 2016.

[4] A. Aghasi, B. Heshmat, A. Redo-Sanchez, J. Romberg, and R. Raskar, "Sweep distortion removal from terahertz images via blind demodulation," *Optica*, vol. 3, no. 7, pp. 754–762, Jul 2016.

[5] B. Recht, M. Fazel, and P. A. Parrilo, "Guaranteed minimum-rank solutions of linear matrix equations via nuclear norm minimization," *SIAM Rev.*, vol. 52, no. 4, pp. 471–501, 2010.

[6] P. Jain, P. Netrapalli, and S. Sanghavi, "Low-rank matrix completion using alternating minimization," *45th Annual ACM Symposium on Theory of Computing*, pp. 665–674, 2013.