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Abstract: We show that polar codes with list+CRC decoding can outperform state-of-the-art LDPC codes in short block lengths. In addition, we introduce an efficient interleaver for polar-coded high-order modulations, achieving greater than 0.5 dB gain for 256QAM.

OCIS codes: (060.4510) Optical communications, (060.1660) Coherent communications, (060.4080) Modulation.

1. Introduction

We investigate polar codes [1–5] and compare with other capacity-approaching forward error correction (FEC) codes, specifically, low-density parity-check (LDPC) codes [6–12] and block turbo codes (BTC) [13, 14]. In particular, we evaluate performance at short block lengths for latency-constrained applications because longer block lengths increase encoding and decoding latency in general. Polar codes have drawn significant attention in the coding theory community since their ability to achieve capacity over any arbitrary discrete-input memoryless channel when paired with low-complexity successive-cancellation (SC) decoding was proven in 2009 by Arıkan [1]. However, in spite of theoretical strength, polar codes have not yet been adopted in practical systems due to their poor performance at short block lengths in comparison to LDPC codes. A major breakthrough was made in 2015 when Tal and Vardy [2] introduced list decoding plus cyclic redundancy check (CRC) to make polar codes competitive with LDPC codes. Due to the resulting excellent performance and simple decoding algorithm, polar codes are now strong candidates for future FEC codes in wireless standards. In contrast, in the context of optical communications, to date there have been a few studies [7] that compared LDPC codes with polar codes under SC decoding. However, there have been no studies on the impact of block length and list+CRC decoding in the optical research community. We verify that the polar codes with list+CRC decoding can outperform the recent LDPC codes [11] for latency-constrained lightwave systems when the block length is shorter than 3000 bits. We then show that an additional gain greater than 0.5 dB can be achieved when we properly design an interleaver for polar-coded high-order quadrature-amplitude modulation (QAM).

2. Polar Codes with List+CRC Decoding vs. LDPC Codes with Finite-Iteration LDA

For comparison, we use the recently proposed Pareto-optimal LDPC codes [11], which show the best tradeoff between threshold and decoding complexity. We first modify the design method for a layered decoding algorithm (LDA) [10] to improve performance further. Through extrinsic information transfer (EXIT) trajectory analysis, we found that $2I$-iteration optimal degrees for flooding can be near optimal for I-iteration LDA. Fig. 1 shows the bit-error rate (BER) performance of optimized LDPC codes with block length $N = 38{,}400$ and code rate $R = 0.8$ for 4-iteration LDA. We observe that a 0.12 dB gain is achieved by the modified design method for LDA.

We compare the optimized LDPC codes with systematic polar codes, whose frozen bit locations are optimized with the method proposed in [3]. Instead of using conventional SC decoding, we employ the recently proposed list+CRC decoder [2], using CRC-8 given by polynomial 0xD5. Besides frozen bit optimization, there is an additional room to design a mapping pattern of the polar-coded bit into modulation bit for high-order QAMs, which have non-uniform reliability, as suggested in [4]. We discuss an interleaver design for such bit-interleaved polar-coded modulation (Fig. 2).

3. BER Comparison of LDPC Codes, Polar Codes, and BTC

We now compare the BER performance of several short-block FEC codes (code rate $R = 0.8$) for 4QAM. Note that state-of-the-art LDPC codes typically use block lengths greater than 30,000 bits for long-haul systems, whereas shorter FEC codes are preferred for latency-stringent systems such as short-reach optical interconnects and front/back-hauls.

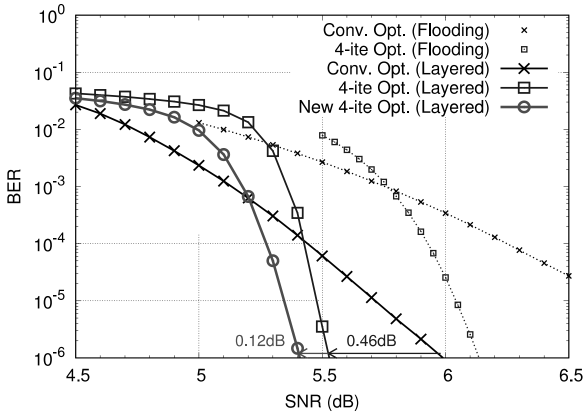


Fig. 1: Optimized LDPC code with $N = 38{,}400$ and $R = 0.8$ for 4-iteration LDA.

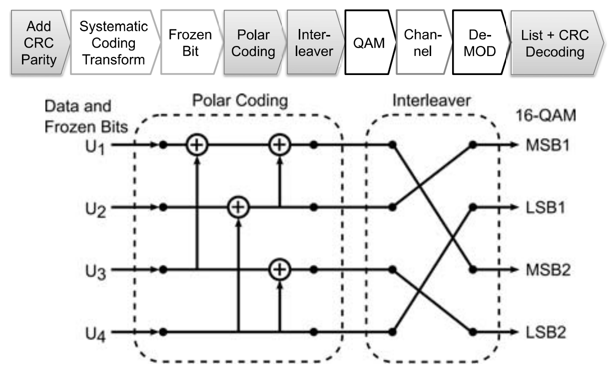
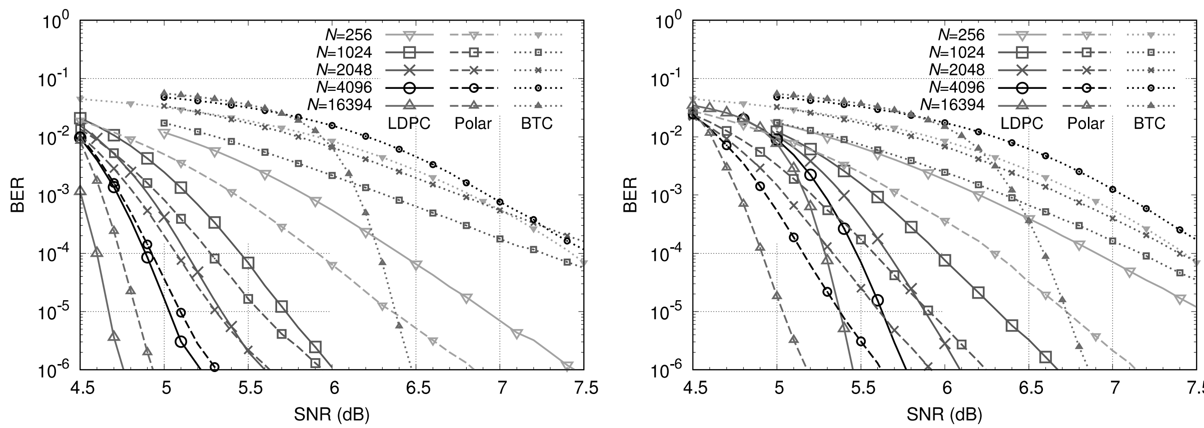


Fig. 2: Bit-interleaved polar-coded modulation. Frozen bit and interleaving are designed for high-order QAMs, where most- to least-significant bits (MSB/LSB) have non-identical reliability.



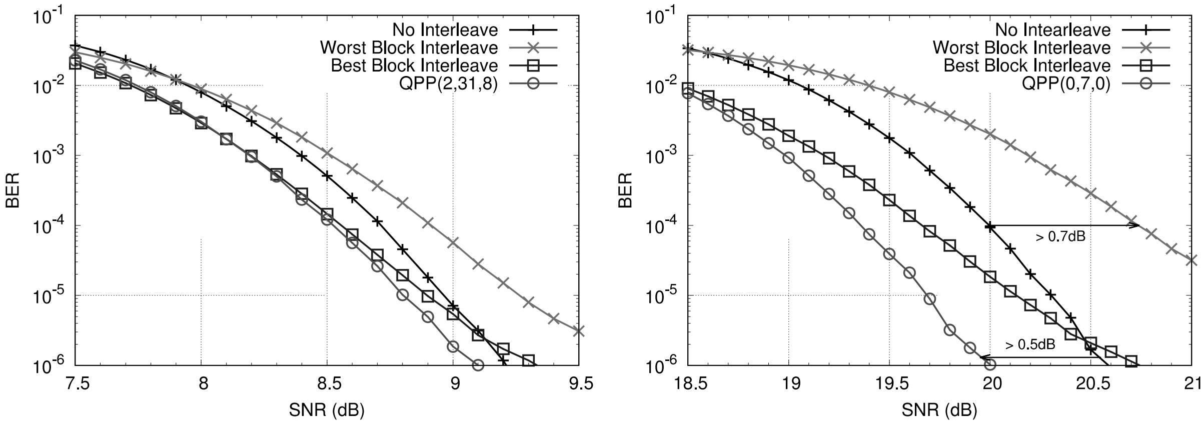
(a) $I = L = 32$

(b) $I = L = 4$

Fig. 3: BER comparison of LDPC codes with LDA and polar codes with CRC-8 for various short block lengths.

Fig. 3(a) compares the BER performance of optimized LDPC codes, systematic polar codes, and BTCs. Here, we use 32-layer LDA with a relatively large number of iterations of $I = 32$ for LDPC decoding, and a large list size of $L = 32$ for polar decoding. We consider five short block lengths, $N \in \{256, 1024, 2048, 4096, 16394\}$. For BTC, we use turbo decoding with $I = 32$ iterations, and comparable block lengths for code rates near 0.8, chosen under the constraint of available BCH component codes. For example, we use $\text{BCH}[127,113]^2$ for BTC of block length $N \simeq 16{,}394$. As shown in Fig. 3(a), the BER performance can significantly degrade when the block lengths are limited. Hence, the longest possible block lengths that achieve the latency requirements should be used. It is seen in Fig. 3(a) that polar codes with list+CRC decoding can outperform optimized LDPC codes with LDA at block lengths shorter than 3000 bits. Note that BTC suffers more than a 1.5 dB loss relative to LDPC codes and polar codes.

The BER performance depends greatly on the decoding complexity (the available number of iterations I for LDPC codes and the list size L for polar codes). In Fig. 3(b), we compare BER curves for $I = L = 4$. Compared to the case of $I = L = 32$ in Fig. 3(a), most curves shift by approximately 0.5 dB due to the reduced decoder complexity. However, the performance loss of polar codes is relatively small compared to that suffered by LDPC codes. Consequently, LDPC codes perform worse than polar codes for all block lengths we considered. These results suggest that polar codes are better candidates than LDPC codes for latency- and power-stringent lightwave systems. Detailed complexity analysis and further comparisons remain as future work.



(a) 16QAM (b) 256QAM

Fig. 4: Bit-interleaved polar-coded modulation for $N = 1024$, $L = 32$, and CRC-8.

4. Interleaver Design for Polar-Coded High-Order Modulation

In the previous performance evaluations, we considered 4QAM, whose BER does not depend on interleavers because of uniform reliability. By using higher-order modulation schemes, we can reduce the transmission latency of coded blocks. In [4], it was shown that an additional gain can be obtained by designing interleavers to map codewords into high-order modulations. The authors of [4] considered very short mapping patterns, up to 16 bits in length, and did not use list-CRC decoding. We propose to use a hardware efficient interleaver, called quadratic polynomial permutation (QPP) [15], which is used for turbo coding in wireless standards. The n-th coded bit is interleaved by QPP(f_0, f_1, f_2) as follows: $\pi(n) = (f_0 + f_1 n + f_2 n^2) \bmod N$, where f_0, f_1, and f_2 are interleaver coefficients to be optimized under the constraints that f_1 must be co-prime to N and f_2 must contain all prime factors of N. Fig. 4 shows the BER improvement that results from optimizing the interleaver for polar codes of length $N = 1024$ for 16QAM and 256QAM. We observe that the interleaver does not always improve the performance, i.e., more than a 0.7 dB penalty is incurred by the worst block interleaver for 256QAM. Nevertheless, the QPP interleaver with optimized coefficients can perform better than the best block interleaver, and achieve significant gain (more than 0.5 dB for 256QAM).

5. Conclusions

In this paper, we evaluated capacity-approaching codes: LDPC codes, polar codes, and BTCs. We verified that polar codes with list+CRC decoding are promising candidates for latency-constrained systems with short block lengths. In addition, we introduced QPP interleaving to achieve additional gains greater than 0.5 dB for high-order 256QAM.

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