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Sequential Estimation of State of Charge and Equivalent Circuit Parameters for Lithium-Ion Batteries

Toshihiro Wada[1], Tomoki Takegami[1] and Yebin Wang[2]

Abstract— **We propose a method to estimate the state of charge (SoC) and the equivalent circuit parameters for lithium-ion batteries. Model-based approaches for SoC estimation, such as Kalman filter, achieve better accuracy than Coulomb counting or open circuit voltage method, albeit requiring accurate model parameters of the battery. We analyze bias errors in the Kalman filter-based SoC estimation induced by errors of the battery model parameters, and develop a simultaneous recursive least squares filter to produce unbiased estimation of the battery parameters.**

I. INTRODUCTION

Lithium-ion batteries (LiBs) have been widely used in electric appliances and electric cars; the application of LiBs becomes wider because of its high energy density, high power density and long life [1]. While it is important to control and manage the battery systems, the internal states of LiBs are however difficult to obtain because of the limited measurability relative to the complexity of the internal processes.

Physical quantities which are able to measure directly are only the terminal voltage, the current and the temperature of the battery. Quantities which should be estimated are state of charge (SoC), internal resistances, full charge capacity (FCC) and other parameters.

A basic approach to SoC estimation is the open circuit voltage method, with which the SoC is estimated precisely in exchange for a long time resting of the battery [2]. For online SoC estimation, Coulomb counting and several model-based approaches are proposed, in which battery parameters, including the internal resistances and the FCC, are assumed [2], [3]. The estimation accuracy is limited in these approaches, because the battery parameters have individual variability, depend on the temperature and vary along with the degradation.

Although simultaneous estimation methods of the SoC and the battery parameters are also proposed in [4], [5], the application of the methods are limited because of the local unobservability (see [6]) about battery parameters depending on the input current. For example, the FCC of the battery is inherently unobservable with a constantly zero input current.

Authors [7]–[9] proposed another approaches, where the battery parameters are estimated without estimating the SoC, after that the SoC is optionally estimated. These methods are based on a adaptive filter using the time derivative of the measured signals. Therefore, these methods are intrinsically sensitive to the measurement noises.

We propose yet another approach, where the SoC is estimated by a simple model-based method with predefined battery parameters, then the differences between the true and the predefined parameters are estimated by a simultaneous recursive least squares (RLS) filter, where the bias errors on the estimated SoC driven by the errors of the predefined parameters are taken into consideration.

II. MODEL-BASED SOC ESTIMATION

A. Lithium-ion battery

An LiB consists mainly of a positive electrode, a negative electrode, current collectors and a separator; all components are soaked with electrolyte solution (Fig. 1). In typical design, the positive electrode is made of a porous material composed of metal oxide particles such as $\mathrm{LiCoO_2}$, $\mathrm{LiMn_2O_4}$ and so on. The negative electrode is also made of a porous material composed of graphite ($\mathrm{C_6}$). The electrolyte solution is an organic solvent with electrolyte such as $\mathrm{LiPF_6}$ [10].

A charge-discharge reaction in the positive electrode is expressed by:

$$\mathrm{LiCoO_2} \underset{\text{Discharge}}{\overset{\text{Charge}}{\rightleftharpoons}} \mathrm{Li}_{1-x_\mathrm{p}}\mathrm{CoO_2} + \mathrm{Li}^+_{x_\mathrm{p}} + \mathrm{e}^-_{x_\mathrm{p}} \tag{1}$$

where x_p denotes the number of reaction electrons. In the negative electrode, the reaction is expressed by:

$$\mathrm{C_6} + \mathrm{Li}^+_{x_\mathrm{n}} + \mathrm{e}^-_{x_\mathrm{n}} \underset{\text{Discharge}}{\overset{\text{Charge}}{\rightleftharpoons}} \mathrm{Li}_{x_\mathrm{n}}\mathrm{C_6} \tag{2}$$

where x_n denotes the number of reaction electrons [11].

In a charge process, Li^+s are emitted from the positive electrode, then absorbed into the negative electrode from the

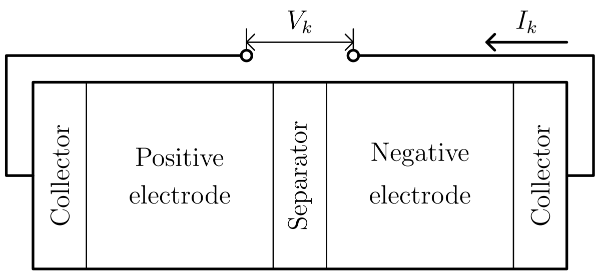

Fig. 1. A typical structure of lithium-ion battery cells. Lithium-ions pass through the separator, while electrons conduct via the electric circuit.

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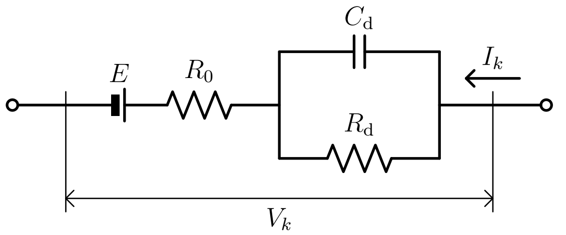

Fig. 2. An equivalent circuit expression of a simplified lithium-ion battery model. The voltage source E depends on the state of charge of the battery.

electrolyte solution. The electrons flow along the external electric circuit via the current collectors, because the electrodes are electrically isolated by the separator.

SoC s is defined by

$$s := \frac{x_\mathrm{p} - x_\mathrm{p}^-}{x_\mathrm{p}^+ - x_\mathrm{p}^-} = \frac{x_\mathrm{n} - x_\mathrm{n}^-}{x_\mathrm{n}^+ - x_\mathrm{n}^-} \tag{3}$$

where $[x_\mathrm{p}^-, x_\mathrm{p}^+]$ and $[x_\mathrm{n}^-, x_\mathrm{n}^+]$ denote the rated range of use of positive electrode and negative electrode respectively.

B. Electric characteristics

Although the general model of LiBs is complex [**?**], we consider a simplified battery model (see [8]) to describe our method. The equivalent circuit model, which includes two resistors R_d, R_0 and a capacitor C_d, is shown in Fig. 2. Let q_b be the electric quantity charged in the battery, that is, $q_\mathrm{b} = F_\mathrm{cc} s$ where F_cc is the FCC of the battery. The voltage source E expresses the open circuit voltage (OCV) of the battery. and q_d be the electric quantity charged in the capacitor. Although the OCV is a nonlinear function of SoC in actual batteries [12], we approximate the function by a linear relationship $E = E_0 + E_1 s$ for simplicity. The terminal voltage of the battery V is described as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{F}_\mathrm{c}\,\boldsymbol{x} + \boldsymbol{G}_\mathrm{c}\,I, \quad V = E_0 + \boldsymbol{H}\boldsymbol{x} + R_0 I \tag{4}$$

where

$$\boldsymbol{F}_\mathrm{c} := \begin{bmatrix} -\frac{1}{C_\mathrm{d} R_\mathrm{d}} & \\ & 0 \end{bmatrix}, \quad \boldsymbol{G}_\mathrm{c} := \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$\boldsymbol{H} := \begin{bmatrix} \frac{1}{C_\mathrm{d}} & \frac{E_1}{F_\mathrm{cc}} \end{bmatrix}, \qquad \boldsymbol{x} := \begin{bmatrix} q_\mathrm{d} & q_\mathrm{b} \end{bmatrix}^\top.$$

C. SoC estimation

Let t_s be the sampling period, a discretized system of (4) is written as follows:

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}\boldsymbol{x}_k + \boldsymbol{G} I_k, \quad V_k = E_0 + \boldsymbol{H}\boldsymbol{x}_k + R_0 I_k \tag{5}$$

where $t_k := t_\mathrm{s} k$, $\boldsymbol{x}_k := \boldsymbol{x}(t_k)$, $I_k := I(t_k)$, $V_k := V(t_k)$ and

$$\boldsymbol{F} := \begin{bmatrix} e^{-\frac{t_\mathrm{s}}{R_\mathrm{d} C_\mathrm{d}}} & \\ & 1 \end{bmatrix}, \quad \boldsymbol{G} := \begin{bmatrix} R_\mathrm{d} C_\mathrm{d} (1 - e^{-\frac{t_\mathrm{s}}{C_\mathrm{d} R_\mathrm{d}}}) \\ t_\mathrm{s} \end{bmatrix}.$$

For estimating SoC of a battery, we have to estimate the state $\boldsymbol{x}_k$ from the measurements of the current I_k and the voltage V_k. If the measurement noises are additive Gaussian and all parameters C_d, R_d, R_0 and F_cc are accurate, it is known that the Kalman filter is the optimal estimator in the sense of minimizing the error covariance of $\boldsymbol{x}_k$ [13].

Let σ_I^2, σ_V^2 be variances of noises on the current and the voltage measurements, respectively. The prediction step of the Kalman filter is written as follows:

$$\hat{\boldsymbol{x}}_{k+1|k} = \boldsymbol{F}\hat{\boldsymbol{x}}_{k|k} + \boldsymbol{G} I_k, \tag{6}$$
$$\hat{\boldsymbol{P}}_{k+1|k} = \boldsymbol{F}\hat{\boldsymbol{P}}_{k|k}\boldsymbol{F}^\top + \boldsymbol{Q} \tag{7}$$

and the update step is written as follows:

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k (V_k - \boldsymbol{z}_k), \tag{8}$$
$$\hat{\boldsymbol{P}}_{k|k} = (I - \boldsymbol{K}_k \boldsymbol{H})\hat{\boldsymbol{P}}_{k|k-1} \tag{9}$$

where

$$\boldsymbol{z}_k := E_0 + \boldsymbol{H}\hat{\boldsymbol{x}}_{k|k-1} + R_0 I_k, \tag{10}$$
$$\boldsymbol{S}_k := \sigma_V^2 + \boldsymbol{H}\hat{\boldsymbol{P}}_{k|k-1}\boldsymbol{H}^\top, \tag{11}$$
$$\boldsymbol{K}_k := \hat{\boldsymbol{P}}_{k|k-1}\boldsymbol{H}^\top \boldsymbol{S}_k^{-1} \tag{12}$$

and $\boldsymbol{Q} := \Phi_{12}\Phi_{22}^{-1}$ where

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} := \exp\left(\begin{bmatrix} \boldsymbol{F}_\mathrm{c} & \sigma_I^2 \boldsymbol{G}_\mathrm{c} \boldsymbol{G}_\mathrm{c}^\top \\ 0 & -\boldsymbol{F}_\mathrm{c}^\top \end{bmatrix} t_s \right).$$

The SoC estimation result is calculated by $\hat{s}_{k|k} = \hat{q}_{\mathrm{b},k|k}/F_\mathrm{cc}$, where $\begin{bmatrix} \hat{q}_{\mathrm{d},k|k} & \hat{q}_{\mathrm{b},k|k} \end{bmatrix} := \hat{\boldsymbol{x}}_{k|k}^\top$.

III. Battery parameter estimation

A. Bias error analysis in SoC estimation

The state of the Kalman filter is estimated from a time series of the measured current and the terminal voltage. The estimation is obviously depends on the battery parameters such as $R_\mathrm{d}, C_\mathrm{d}, R_0$ and F_cc. Let $\boldsymbol{\theta}$ denote the battery parameters, the dependency is expressed by:

$$\hat{\boldsymbol{x}}_{l|k} = \mathcal{F}_{l|k}((I_k, V_k), \ldots, (I_0, V_0) \mid \boldsymbol{\theta}), \tag{13}$$

where $\mathcal{F}_{l|k}$ is a map from a set of the measured values to an estimated value of the state.

If a typical value of the parameters used in the Kalman filter are slightly different from the true value, the estimated value of the state is biased. Actually, the resistors R_d, R_0 depend strongly on the temperature [14], [15]. The FCC has a initial variation, and decreases due to the degradation. Additionally an offset error of the current measurements, which is inevitable in widely used Hall effect sensor [16], causes significant estimation error of the state. This is because the current is integrated in q_b, therefore the offset error is also integrated in the state. Let I_off denote the offset error of the current measurements, and $\tilde{\boldsymbol{\theta}}$ denote an inaccurate battery parameters, the biased estimation of the state is expressed by:

$$\tilde{\boldsymbol{x}}_{l|k} := \mathcal{F}_{l|k}((I_k + I_\mathrm{off}, V_k), \ldots, (I_0 + I_\mathrm{off}, V_0) \mid \tilde{\boldsymbol{\theta}}). \tag{14}$$

Let $(\tilde{R}_{\mathrm{d}}, \tilde{C}_{\mathrm{d}}, \tilde{R}_0, \tilde{F}_{\mathrm{cc}}) := \tilde{\boldsymbol{\theta}}$, we parameterize the difference between $\boldsymbol{\theta}$ and $\tilde{\boldsymbol{\theta}}$ as follows:

$$
\begin{aligned}
\tilde{R}_{\mathrm{d}}\tilde{C}_{\mathrm{d}} &= R_{\mathrm{d}}C_{\mathrm{d}}(1+p_1), && (15)\\
\frac{1}{\tilde{C}_{\mathrm{d}}} &= \frac{1}{C_{\mathrm{d}}}(1+p_2), && (16)\\
\tilde{R}_0 &= R_0(1+p_3), && (17)\\
\tilde{F}_{\mathrm{cc}} &= F_{\mathrm{cc}}(1+p_4), && (18)\\
I_{\mathrm{off}} &= I_{\mathrm{typ}}p_5 && (19)
\end{aligned}
$$

where I_{typ} is a constant introduced to scale p_5 as $|p_5| \ll 1$. The biased estimation $\tilde{\boldsymbol{x}}_{k|k}$ is approximated as:

$$
\tilde{\boldsymbol{x}}_{k|k} \approx \hat{\boldsymbol{x}}_{k|k} + \sum_{j=1}^{5} \frac{\partial \mathcal{F}_k}{\partial p_j} p_j, \tag{20}
$$

by a Taylor series expansion (see [17] for matrix derivatives).

For simplicity, we define $\partial \boldsymbol{x}_{l|k}/\partial p_j := \partial \mathcal{F}_{l|k}/\partial p_j$, $\begin{bmatrix}\partial q_{\mathrm{d},l|k}/\partial p_j & \partial q_{\mathrm{b},l|k}/\partial p_j\end{bmatrix} := \partial \boldsymbol{x}_{l|k}/\partial p_j{}^{\top}$ and

$$
\frac{\partial s_{l|k}}{\partial p_j} := \begin{cases} \frac{1}{\tilde{F}_{\mathrm{cc}}}\left(\frac{\partial q_{\mathrm{b},l|k}}{\partial p_j} - \hat{q}_{\mathrm{b},l|k}\right) & \text{for} \quad j=4, \\ \frac{1}{\tilde{F}_{\mathrm{cc}}}\frac{\partial q_{\mathrm{b},l|k}}{\partial p_j} & \text{otherwise} \end{cases}. \tag{21}
$$

Then we get the following relationships:

$$
\begin{aligned}
q_{\mathrm{cc},k} &= \frac{\tilde{F}_{\mathrm{cc}}}{1+p_4}\left(\tilde{s}_{k|k} - \sum_{j=1}^{5}\frac{\partial s_{k|k}}{\partial p_j}p_j\right) \\
&\quad + t_k I_{\mathrm{typ}} p_5 - q_0, && (22)\\
\tilde{q}_{\mathrm{d},k|k} &- \sum_{j=1}^{5}\frac{\partial q_{\mathrm{d},k|k}}{\partial p_j}p_j \\
&= \left(\tilde{\boldsymbol{F}}_{11} - \frac{\partial \boldsymbol{F}_{11}}{\partial p_1}p_1\right)\left(\tilde{q}_{\mathrm{d},k-1|k} - \sum_{j=1}^{5}\frac{\partial q_{\mathrm{d},k-1|k}}{\partial p_j}p_j\right) \\
&\quad + \left(\tilde{\boldsymbol{G}}_1 - \frac{\partial \boldsymbol{G}_1}{\partial p_1}p_1\right)(I_{k-1} - I_{\mathrm{typ}}p_5), && (23)\\
V_k &= E_0 + E_1\left(\tilde{s}_{k|k} - \sum_{j=1}^{5}\frac{\partial s_{k|k}}{\partial p_j}p_j\right) \\
&\quad + \frac{1}{\tilde{C}_{\mathrm{d}}(1+p_2)}\left(\tilde{q}_{\mathrm{d},k|k} - \sum_{j=1}^{5}\frac{\partial q_{\mathrm{d},k|k}}{\partial p_j}p_j\right) \\
&\quad + \frac{\tilde{R}_0}{1+p_3}(I_k - I_{\mathrm{typ}}p_5), && (24)
\end{aligned}
$$

where $\boldsymbol{F}_{11}$ and $\boldsymbol{G}_1$ are $(1,1)$-element of $\boldsymbol{F}$ and the first element of $\boldsymbol{G}$, respectively. The backward estimation $\hat{\boldsymbol{x}}_{k-1|k}$ is calculated by 1-step Kalman smoother algorithm as follows:

$$
\hat{\boldsymbol{x}}_{k-1|k} = \hat{\boldsymbol{P}}_{k-1|k-1}\boldsymbol{F}^{\top}\hat{\boldsymbol{P}}_{k|k-1}^{-1}(\hat{\boldsymbol{x}}_{k|k} - \hat{\boldsymbol{x}}_{k|k-1}). \tag{25}
$$

B. Unbiased parameter estimation

Now it is able to estimate $p_1, \ldots, p_5$ by a simultaneous RLS filter. Let $\boldsymbol{p} := \begin{bmatrix} p_1 & \cdots & p_5 & q_0 \end{bmatrix}$,

$$
\boldsymbol{y}_k := \begin{bmatrix} q_{\mathrm{cc},k} - \tilde{q}_{\mathrm{b},k|k} \\ \tilde{q}_{\mathrm{d},k|k} - \tilde{\boldsymbol{F}}_{11}\tilde{q}_{\mathrm{d},k-1|k} - \tilde{\boldsymbol{G}}_1 I_{k-1} \\ V_k - E_0 - \frac{E_1}{\tilde{F}_{\mathrm{cc}}}\tilde{q}_{\mathrm{b},k|k} - \frac{1}{\tilde{C}_{\mathrm{d}}}\tilde{q}_{\mathrm{d},k|k} - \tilde{R}_0 I_k \end{bmatrix}, \tag{26}
$$

$$
U_k := \begin{bmatrix} \boldsymbol{u}_{1,k} & \boldsymbol{u}_{2,k} & \boldsymbol{u}_{3,k} \end{bmatrix}, \tag{27}
$$

where

$$
\boldsymbol{u}_{1,k} := \begin{bmatrix} -\frac{\partial q_{\mathrm{b},k|k}}{\partial p_1} \\ -\frac{\partial q_{\mathrm{b},k|k}}{\partial p_2} \\ -\frac{\partial q_{\mathrm{b},k|k}}{\partial p_3} \\ -\frac{\partial q_{\mathrm{b},k|k}}{\partial p_4} \\ -\frac{\partial q_{\mathrm{b},k|k}}{\partial p_5} + I_{\mathrm{typ}}t_k \\ -1 \end{bmatrix}, \tag{28}
$$

$$
\boldsymbol{u}_{2,k} := \begin{bmatrix} \frac{\partial q_{\mathrm{d},k|k}}{\partial p_1} - \tilde{\boldsymbol{F}}_{11}\frac{\partial q_{\mathrm{d},k-1|k}}{\partial p_1} - \kappa_k \\ \frac{\partial q_{\mathrm{d},k|k}}{\partial p_2} - \tilde{\boldsymbol{F}}_{11}\frac{\partial q_{\mathrm{d},k-1|k}}{\partial p_2} \\ \frac{\partial q_{\mathrm{d},k|k}}{\partial p_3} - \tilde{\boldsymbol{F}}_{11}\frac{\partial q_{\mathrm{d},k-1|k}}{\partial p_3} \\ \frac{\partial q_{\mathrm{d},k|k}}{\partial p_4} - \tilde{\boldsymbol{F}}_{11}\frac{\partial q_{\mathrm{d},k-1|k}}{\partial p_4} \\ \frac{\partial q_{\mathrm{d},k|k}}{\partial p_5} - \tilde{\boldsymbol{F}}_{11}\frac{\partial q_{\mathrm{d},k-1|k}}{\partial p_5} - \tilde{\boldsymbol{G}}_1 I_{\mathrm{typ}} \\ 0 \end{bmatrix}, \tag{29}
$$

$$
\boldsymbol{u}_{3,k} := \begin{bmatrix} -\frac{E_1}{\tilde{F}_{\mathrm{cc}}}\frac{\partial q_{\mathrm{b},k|k}}{\partial p_1} - \frac{1}{\tilde{C}_{\mathrm{d}}}\frac{\partial q_{\mathrm{d},k|k}}{\partial p_1} \\ -\frac{E_1}{\tilde{F}_{\mathrm{cc}}}\frac{\partial q_{\mathrm{b},k|k}}{\partial p_2} - \frac{1}{\tilde{C}_{\mathrm{d}}}\left(\frac{\partial q_{\mathrm{d},k|k}}{\partial p_2} + \tilde{q}_{\mathrm{d},k|k}\right) \\ -\frac{E_1}{\tilde{F}_{\mathrm{cc}}}\frac{\partial q_{\mathrm{b},k|k}}{\partial p_3} - \frac{1}{\tilde{C}_{\mathrm{d}}}\frac{\partial q_{\mathrm{d},k|k}}{\partial p_3} - \tilde{R}_0 I_k \\ -\frac{E_1}{\tilde{F}_{\mathrm{cc}}}\left(\frac{\partial q_{\mathrm{b},k|k}}{\partial p_4} - \tilde{q}_{\mathrm{b},k|k}\right) - \frac{1}{\tilde{C}_{\mathrm{d}}}\frac{\partial q_{\mathrm{d},k|k}}{\partial p_4} \\ -\frac{E_1}{\tilde{F}_{\mathrm{cc}}}\frac{\partial q_{\mathrm{b},k|k}}{\partial p_5} - \frac{1}{\tilde{C}_{\mathrm{d}}}\frac{\partial q_{\mathrm{d},k|k}}{\partial p_5} - \tilde{R}_0 I_{\mathrm{typ}} \\ 0 \end{bmatrix}, \tag{30}
$$

$$
\kappa_k := \frac{\partial \boldsymbol{F}_{11}}{\partial p_1}\tilde{q}_{\mathrm{d},k-1|k} + \tilde{\boldsymbol{G}}_1 I_{k-1}, \tag{31}
$$

the equation (22)–(24) are rewritten as follows:

$$
\boldsymbol{y}_k = U_k^{\top}\boldsymbol{p} \tag{32}
$$

by omitting higher order terms about $\boldsymbol{p}$. An RLS estimator which minimizes following cost function:

$$
J(\hat{\boldsymbol{p}}_k) := \frac{1}{2}\sum_{l=0}^{k}\lambda^{k-l}(\hat{\boldsymbol{y}}_{l|k} - \boldsymbol{y}_k)^{\top}\Sigma^{-1}(\hat{\boldsymbol{y}}_{l|k} - \boldsymbol{y}_k), \tag{33}
$$

where $\hat{\boldsymbol{y}}_{l|k} := U_l^{\top}\hat{\boldsymbol{p}}_k$, is calculated recursively by the following scheme:

$$
\begin{aligned}
\boldsymbol{Y}_k &= \lambda^{-1}(\boldsymbol{I} - \boldsymbol{L}_k U_k^{\top})\boldsymbol{Y}_{k-1}, && (34)\\
\hat{\boldsymbol{p}}_k &= \hat{\boldsymbol{p}}_{k-1} + \boldsymbol{L}_k(\boldsymbol{y}_k - U_k^{\top}\hat{\boldsymbol{p}}_{k-1}), && (35)
\end{aligned}
$$

where λ is a forgetting factor in $(0,1]$, Σ is a symmetric positive definite weighting matrix and

$$
\boldsymbol{L}_k := \boldsymbol{Y}_{k-1}U_k(\lambda\Sigma^{-1} + U_k^{\top}\boldsymbol{Y}_{k-1}U_k)^{-1}. \tag{36}
$$

In our approach, the SoC is estimated based on the system (5), which is clearly observable as long as $E_1 > 0$. Although

TABLE I
BATTERY PARAMETERS USED IN THE SIMULATION

Parameter	True	Biased
R_d	$5\,\mathrm{m\Omega}$	$5.1\,\mathrm{m\Omega}$
C_d	$2\,\mathrm{kF}$	$2.06\,\mathrm{kF}$
R_0	$5\,\mathrm{m\Omega}$	$5.25\,\mathrm{m\Omega}$
F_cc	$2\,\mathrm{Ah}$	$1.95\,\mathrm{m\Omega}$
I_off	$0\,\mathrm{mA}$	$50\,\mathrm{mA}$
E_0	$2.6\,\mathrm{V}$	-
E_1	$1.6\,\mathrm{V}$	-

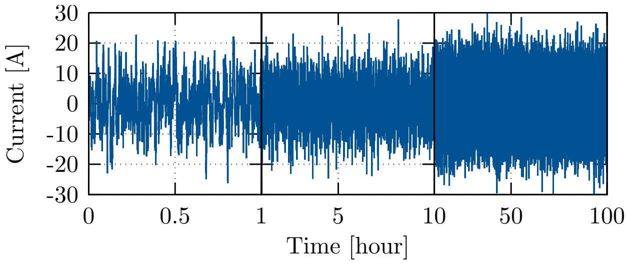

Fig. 3. Input current for the simulation. The time axis is zoomed in during the first 1 hour and the first 10 hour.

the OCV of the actual battery is a nonlinear function of the SoC, the extended Kalman filter (EKF) or unscented Kalman filter (UKF) (see [18]) is applicable as long as the OCV is an injective function of the SoC. Therefore the SoC estimation is more stable relative to simultaneous estimation approaches [4], [5].

Another advantage of our approach is that adaptive forgetting factor methods (for example [19]) are easily applicable to our parameter estimation filter. The battery parameters are intrinsically impossible to estimate if the measurements do not have enough information about the parameters, for example, F_cc could not be estimated with $I(t) \equiv 0$. The parameters could be better estimated by ignoring measurements when $I(t) \approx 0$ by adjusting the forgetting factor.

IV. NUMERICAL EXAMPLE

In this section, we illustrate the validity of our method by a numerical simulation. In our simulation, we employed a simplified battery model shown in Fig. 2 with battery parameters in Table. I

First, we calculated the terminal voltage of the battery using the input current shown in Fig. 3 and the true values of the parameters. The sampling period $t_s = 100\,\mathrm{ms}$. We also show the true SoC of the battery in Fig. 4 for visibility.

Next, we estimated the SoC of the battery and calculated the derivative thereof from the input current and the simulated terminal voltage using a Kalman filter with the biased values of the parameters, where we offset the current by $50\,\mathrm{mA}$, and added Gaussian noises of variances $\sigma_I^2 = 10^{-4}$ and $\sigma_V^2 = 10^{-5}$ to the current and voltage, respectively.

After that, we estimated the battery parameters from the input current, the simulated terminal voltage, estimated state of the Kalman filter and Coulomb counting of the input current by the RLS filter proposed in the Section III.

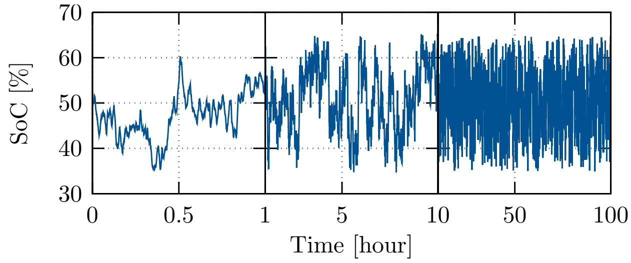

Fig. 4. State of charge of the battery in the simulation.

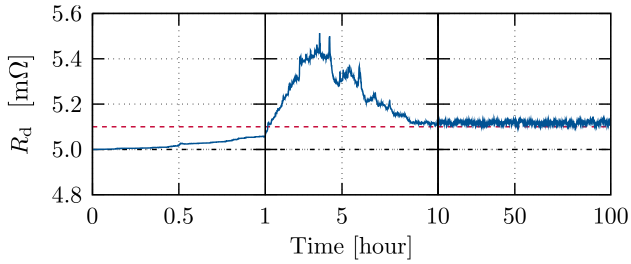

Fig. 5. Estimated R_d. The dashed line indicates the true value, the long dashed short dashed line indicates the biased value.

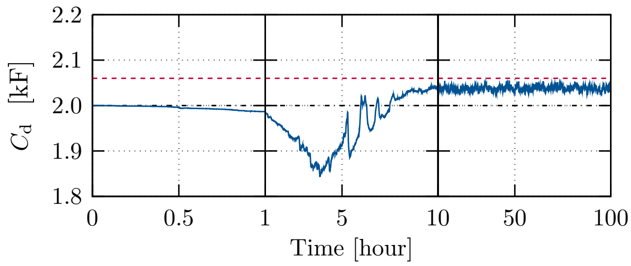

Fig. 6. Estimated C_d. The dashed line indicates the true value, the long dashed short dashed line indicates the biased value.

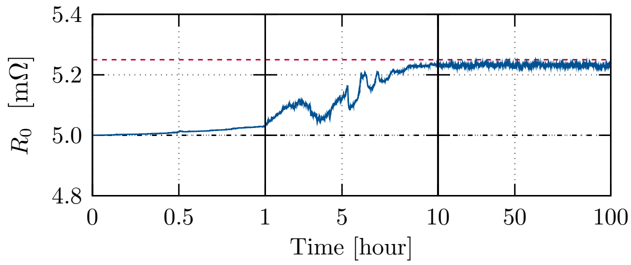

Fig. 7. Estimated R_0. The dashed line indicates the true value, the long dashed short dashed line indicates the biased value.

We employed $\lambda = e^{-t_s/\tau}$ where $\tau = 30\,\mathrm{min.}$, $I_\mathrm{typ} = 100\,\mathrm{A}$, $\Sigma = \mathrm{diag}\{10^{-7}, 10^{-3}, 1\}$. The results of the parameter estimation are shown in Fig 5–9. All parameters are estimated correctly while $t_k \geq 10\,\mathrm{hour}$.

V. CONCLUSION

We have proposed a method for SoC and battery parameters estimation for LiBs, in which the SoC is estimated by a Kalman filter based on an equivalent circuit model, and parameters in the model are estimated from the estimated SoC by an RLS filter.

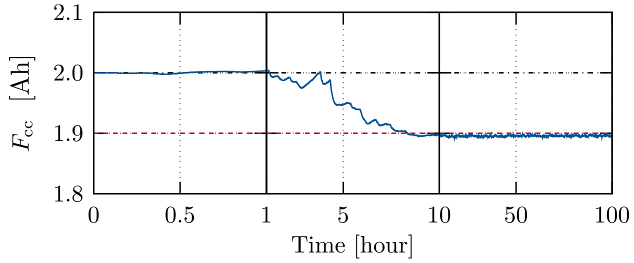

Fig. 8. Estimated $F_{\rm cc}$. The dashed line indicates the true value, the long dashed short dashed line indicates the biased value.

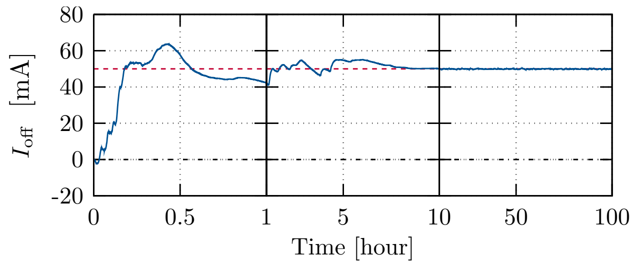

Fig. 9. Estimated $I_{\rm off}$. The dashed line indicates the true value, the long dashed short dashed line indicates the biased value.

The parameters in the model are predefined in the SoC estimation, therefore the SoC estimation has bias errors induced by the inaccuracy of the predefined parameters. We have analyzed the bias errors of the SoC estimation, then developed an RLS filter to produce unbiased estimation of the battery parameters.

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