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# A Rayleigh Quotient-Based Recursive Total-Least-Square Online Maximum Capacity Estimation for Lithium-ion Batteries

Taesic Kim, *Student Member, IEEE,* Yebin Wang, *Member, IEEE*, Zafer Sahinoglu, *Senior Member, IEEE,* Toshihiro Wada, Satoshi Hara, and Wei Qiao, *Senior Member, IEEE*

***Abstract*—The maximum capacity, the amount of maximal electric charge that a battery can store, not only indicates the state of health, but also is assumed in numerous methods for state of charge estimation. This paper proposes an alternative approach to perform the online estimation of the maximum capacity by solving the recursive total least square (RTLS) problem. Different from prior art, the proposed approach poses and solves the RTLS problem as a Rayleigh quotient optimization problem. The Rayleigh quotient-based approach can be readily generalized to other parameter estimation problems including impedance estimation. Compared to other capacity estimation methods, the proposed algorithm enjoys the advantages of other existing RTLS-based algorithms for instance, low computational cost, simple implementation, and high accuracy. The proposed method is compared with existing methods via simulations and experiments. The proposed method is suitable for use in real-time embedded battery management systems.**

***Index Terms*—Lithium-ion battery, online capacity estimation, Rayleigh quotient, recursive total least square (RTLS), state of charge, state of health**

## I. Introduction

LITHIUM-ION batteries have gained increasingly pervasive use in numerous applications due to the high energy and power densities, and longer cycle life [1]. While viewed as a promising rechargeable battery technology, lithium-ion batteries carry limited thermal stability and performance degradation caused by the aging process. A battery management system (BMS), which monitors and controls the operation of a battery system, is generally required to ensure the safety and efficiency [2]. A key function of the BMS is to monitor the state of charge (SOC), state of health (SOH), instantaneous available power (i.e., the state of power SOP), internal impedance, maximum capacity, etc. [3]. For example, the SOC is required by optimal control of EVs and PHEVs. Meanwhile, control strategies may take the SOH into consideration for long-term targets [4].

The maximum capacity (i.e., capacity fade) and the internal impedance (i.e., power fade) have been commonly used to quantify the SOH, which is used to prevent catastrophic failures of the batteries [5]. The maximum capacity is generally difficult to measure, and needs to be estimated from measurements.

The maximum capacity can be estimated by using a full discharge test, where a fully charged battery is discharged with a small current until the battery terminal voltage reaches a cutoff threshold [6]. The delivered charge is measured to obtain the maximum capacity. The full discharge test is a simple but time-consuming approach, and thus is not an online solution.

Online maximum capacity estimation methods fall into three categories: analytical, computational intelligence-based, and model-based. The analytical methods perform the maximum capacity estimation based on the two-point SOC (TP SOC). For example, Texas Instrument developed a battery management integrated circuit chip, which estimates the maximum capacity using the SOC values obtained from the measured open-circuit voltage (OCV) at two operating points and the delivered charge between the two operating points [7]. Similar TP SOC methods have been presented in [4], [8], and [9]. The TP SOC methods are simple and easy to implement in real-time systems, but is prone to the SOC estimation and current measurement errors [10]. Recently, a total least square (TLS)-based capacity estimation method [10], [11] has been introduced to reduce the capacity estimation error caused by the SOC estimation and current measurement errors.

The computational intelligence-based methods produce the capacity estimation based on learning the nonlinear relationship between the capacity and measurable battery parameters, such as voltage, current, and temperature [12], [13]. The learning is usually carried out on the basis of neural network models, e.g. artificial neural networks [12], adaptive recurrent neural networks [13] and structured neural networks [14]. The capacity estimation accuracy is significantly affected by training data and training methods, which are usually chosen heuristically. Along with the learning process is the prohibitive computational complexity.

T. Kim is with the Department of Computer Science and Computer Engineering and W. Qiao is with the Department of Electrical and Computer Engineering, University of Nebraska–Lincoln, Lincoln, NE 68588-0511 USA (e-mail: taesickim@ huskers.unl.edu; wqiao@engr.unl.edu). This work was done while T. Kim was an intern with Mitsubishi Electronic Research Laboratories, 201 Broadway, Cambridge, MA 02139 USA.

Y. Wang and Z. Sahinoglu are with Mitsubishi Electric Research Laboratories, 201 Broadway, Cambridge, MA 02139 USA (e-mail: yebinwang@ieee.org, zafer@merl.com).

T. Wada and S. Hara are with the Advanced Technology R&D Center Mitsubishi Electric Corporation, 8-1-1, Tsukaguchi-honmachi, Amagasaki City, 661-8661, Japan (e-mail: Wada.Toshihiro@bx.MitsubishiElectric.co.jp, Hara.Satoshi@cb.MitsubishiElectric.co.jp).

The model-based methods estimate both the SOC and the maximum capacity based on battery models. Kalman filter (KF)-based approaches, such as dual extended Kalman filter (DEKF) [15], [16], [18], and dual sigma point Kalman filter (SPKF) [17], constitute a large portion of this category. KF-based approaches have common disadvantages: prone to linearization errors; assuming Gaussian noises; requiring accurate models uncertainties; and lack of the stability. Particle filter-based identification methods [20] take into consideration the nonlinearity and non-Gaussian noise, and the filter performance depends on the number of particles. Normally, particle filter-based methods involve higher computational cost than KF-based methods [19]. Apart from aforementioned filters, work [21], [22] consider a dual sliding-mode observer (SMO), which is computationally efficient as well as robust with respect to measurement noises and modeling errors. Recent contribution [24] follows the linear adaptive method and introduces two linear observers to perform joint estimation of the SOC and capacity.

The goal of this paper is to develop an alternative capacity estimation algorithm which is computationally effective, accurate, as well as suitable for real-time embedded BMS applications. This paper proposes a Rayleigh quotient-based online capacity estimation algorithm which solves the recursive TLS (RTLS) problem in [11]. Simulations and experiments are performed to verify that the proposed method improves the robustness of the TP SOC method, outperforms TP-based analytical methods, and requires less computational costs than model-based DEKF methods.

## II. Related Work

The maximum battery capacity represents the maximum amount of energy that can be drawn from a fully charged battery until its terminal voltage reaches a cutoff value without any nonlinear capacity effect [6]. The maximum capacity is only temperature-dependent. On the other hand, the available battery capacity is the amount of electric charge stored in the battery. The available capacity is dependent on temperature and the current rate due to the nonlinear capacity effects, such as the rate capacity effect and recovery effect [6]. We need to estimate the maximum capacity not only because it changes due to aging and temperature fluctuation but also because of the inconsistency during the battery manufacturing process [25].

### A. Least Square Problem

Denote $C_{\max}$ the maximum capacity of the battery cell. A number of maximum capacity estimation methods rely on the following Coulomb counting formula

$$SOC(t_{j+1}) = SOC(t_j) - \int_{t_j}^{t_{j+1}} \frac{\eta i_B(t)}{3600 C_{\max}} dt \tag{1}$$

where $i_B$ denote the current of the battery cell ($i_B$ is positive if the battery is operated in the discharge mode and negative if operated in the charge mode); $t_j, j \in Z^+$ are the time instants when the SOC is sampled; and $\eta$ is Coulomb efficiency. It usually assumes that $\eta$ = 1. As noted in [4], [7], and [8], the maximum capacity can be simply calculated from

$$C_{\max} = \frac{\int_{t_j}^{t_{j+1}} \frac{\eta i_B(t)}{3600} dt}{SOC(t_{j+1}) - SOC(t_j)} \tag{2}$$

Assume that the SOC is uniformly sampled at time instants $t_j, j \in Z^+$ with $t_1 = 0$, and the cell current is uniformly sampled with a sample period $T_s$. Discretize (2) over $[t_j, t_{j+1}]$ and rearrange it as follows

$$T_s \sum_{n=k_j}^{k_{j+1}} \frac{\eta i_B(n)}{3600} = C_{\max}(SOC(t_{j+1}) - SOC(t_j)) \tag{3}$$

where $k_j, k_{j+1}$ are the time indices corresponding to $t_j, t_{j+1}$ respectively. Define

$$y(j) = T_s \sum_{n=k_j}^{k_{j+1}} \frac{\eta i_B(n)}{3600} \text{ and } x(j) = SOC(t_{j+1}) - SOC(t_j))$$

Then (3) can be written as

$$y(j) = C_{\max} x \tag{4}$$

A least square (LS) problem formulation, assuming a noisy $y$ but accurate $x$, allows us to estimate the maximum capacity $C_{\max}$ by minimizing the following cost function [26]:

$$J(C_{\max}) = \sum_{j=1}^{N} [y(j) - C_{\max} x(j)]^2 \tag{5}$$

The matrix form of the LS solution has been widely used and is written as follows:

$$C_{\max}(k) = (X^T X)^{-1} X^T Y \tag{6}$$

where $X = [x(1), \cdots, x(N)]^T$ and $Y = [y(1), \cdots, y(N)]^T$. Fig. 1(a) illustrates the concept of the LS method, where the dots represent the data points, the solid line represents the fitting line, and the dashed lines represent the vertical distances from the data points to the fitting line. The standard LS method provides unbiased capacity estimation only if the estimated SOC (i.e., $X$) are correct [10].

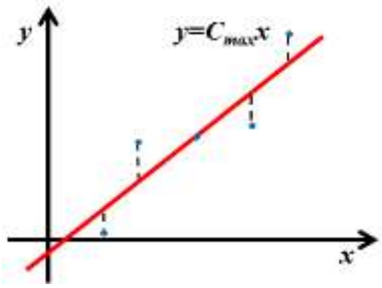
### B. Total Least Square Problem

Total least squares problem is not new and is also known as orthogonal regression, errors-in-variables, and measurement errors in statistics. Formulating the capacity estimation as a TLS problem introduces extra freedom by allowing inaccuracy in the observation matrix $X$ and the measurement matrix $Y$. That is the TLS problem assumes the following Coulomb counting model

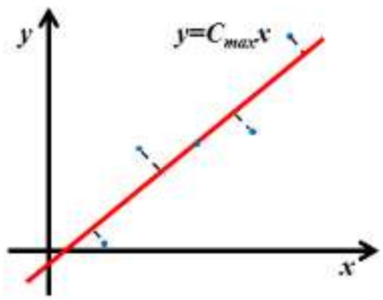
$$\underbrace{y(k) - \Delta y}_{\tilde{y}(k)} = C_{\max} \underbrace{(x(k) - \Delta x)}_{\tilde{x}(k)} \tag{7}$$

where $x(k)$ and $y(k)$ are error free input and output, respectively; $\Delta y$ is the output measurement error ; $\Delta x$ is the SOC estimation error. Both $\Delta y$ and $\Delta x$ are assumed to be zero mean, normally distributed random variables with covariance matrices that are a multiple of the identity,

As illustrated in Fig. 1(b), where the dashed lines represent the orthogonal distances from the data points to the fitting line, the TLS performs the orthogonal regression which minimizes the sum of the squared orthogonal distances from the data points to the fitting line [27]. In another words, the TLS method is to solve the following optimization problem




(a)




(b)

Fig. 1. Comparison of linear regression methods: (a) LS method and (b) TLS method.

$$\{C_{\max}, \Delta X, \Delta Y\} = \arg\min \|[\Delta X \ \Delta Y]\|_F \qquad (8)$$

where $\|\cdot\|_F$ denotes the Frobenius norm. Typical approaches to solve the TLS include the singular value decomposition (SVD) algorithm [27], recursive Rayleigh quotient algorithms in adaptive signal filtering [28], [28], and recursive approximate weighted TLS (RAWTLS) in the battery capacity estimation [10]. The SVD-based batch TLS algorithm however suffers high computational complexity and requires a large memory. Recursive TLS algorithms, which generally have lower computational complexity, are preferable in embedded systems. Although the TLS problem has been studied for decades by the signal processing community, its application to solve the battery capacity estimation was fairly new [10], [10]. Specifically, the RAWTLS in [10] performs the maximum capacity estimation, but is only validated by using simulation results.

## III. The Rayleigh Quotient-Based Algorithm

This paper proposes a constrained Rayleigh quotient-based RTLS algorithm for the maximum capacity estimation for lithium-ion batteries. The proposed algorithm can be viewed as an alternative to algorithms in [10], [10]. While with a significantly reduced computational complexity than RAWTLS [10], the proposed algorithm can achieve good accuracy that is comparable with the batch TLS algorithm.

### A. *The Proposed RTLS Algorithm*

Assume that $\Delta y$ and $\Delta x$ are zero-mean Gaussian random processes with known variances $\sigma_y^2$ and $\sigma_x^2$, respectively. Denote the autocorrelation matrix of the noisy input $\tilde{x}(k)$ as follows

$$\tilde{R}(k) = E\left[\tilde{x}(k)\tilde{x}^T(k)\right] \qquad (9)$$

Introduce the augmented data $\bar{x}(k) = \left[\tilde{x}(k), \tilde{y}(k)\right]^T$ and express its autocorrelation matrix as follows

$$\bar{R}(k) = E\left[\bar{x}(k)\bar{x}^T(k)\right] = \begin{bmatrix} R(k) & b(k) \\ b^T(k) & c(k) \end{bmatrix} \qquad (10)$$

where $b(k) = E\left[\tilde{x}(k)\tilde{y}^T(k)\right]$, $c(k) = E\left[\tilde{y}(k)\tilde{y}^T(k)\right]$. If considering a forgetting factor $\mu$ ($0.95 \le \mu < 1$), the autocorrelation matrix is updated as follows

$$\bar{R}(k) = \mu\bar{R}(k-1) + \bar{x}(k)\bar{x}^T(k)$$

Similar to [28], the stochastic quantities $R(k)$, $b(k)$, and $c(k)$ can be expressed as follows

$$\begin{cases} R(k) = \mu R(k-1) + \tilde{x}(k)\tilde{x}^T(k) \\ b(k) = \mu b(k-1) + \tilde{x}(k)\tilde{y}^T(k) \\ c(k) = \mu c(k-1) + \tilde{y}(k)\tilde{y}^T(k) \end{cases} \qquad (11)$$

It has been shown in [29] that the optimization problem (8) is equivalent to minimizing the Rayleigh quotient $F(q)$:

$$F(q) = \frac{q^T \cdot A^T A \cdot q}{q^T q} \qquad (12)$$

where $q$ is the eigenvector associated with the smallest eigenvalue of the symmetric positive-definite matrix $A^TA$ in the TLS solution. Interested readers are referred to [30] and references therein for details.

For the maximum capacity estimation, a constrained Rayleigh quotient is used as a cost function where $q^T$ and $A^TA$ in (12) are replaced with $[C_{\max}, -1]$ and $\bar{R}$, respectively. The maximum capacity estimation problem therefore has the following constrained Rayleigh quotient cost function:

$$J(C_{\max}) = \frac{[C_{\max}, -1] \cdot \bar{R} \cdot [C_{\max}, -1]^T}{[C_{\max}, -1] \cdot \bar{D} \cdot [C_{\max}, -1]^T} = \frac{RC_{\max}{}^2 - 2bC_{\max} + c}{C_{\max}{}^2 + \beta} \qquad (13)$$

where $\bar{D} = diag(1, \beta)$ is a diagonal weighting matrix with $\beta = \sigma_y^2/\sigma_x^2$. Denote two eigenvectors of $\bar{R}$ as $q_1 = [C^1_{\max}, -1]^T$, $q_2 = [C^2_{\max}, -1]^T$, and the corresponding two eigenvalues $\lambda_1$, $\lambda_2$, respectively. We cite the following conclusion about the existence and uniqueness of the global minimizer of (13).

*Theorem 3.1* [Thm. 3.1, 28] If $\lambda_1 > \lambda_2$, then $C^2_{\max}$ is the global minimizer of (13).

Proof of Theorem 3.1 is omitted, and interested readers are referred to [28] for details.

Instead of solving a TLS problem at each new data arrival, which is time-consuming, one would like to develop a closed-form update law of the minimizer. The $C_{\max}$ is updated by successive approximation as:

$$C_{\max}(k) = C_{\max}(k-1) + \alpha(k)\tilde{x}(k) \qquad (14)$$

where $\alpha(k)$ is an adaptive gain chosen to minimize $J(C_{\max}(k-1) + \alpha(k)\tilde{x}(k))$ in the direction of $\tilde{x}(k)$. Let the gradient of $J(C_{\max}(k-1) + \alpha(k)\tilde{x}(k))$ be equal to zero, i.e.,

$$\frac{\partial J(C_{\max}(k-1) + \alpha(k)\tilde{x}(k))}{\partial \alpha(k)} = \frac{c_1\alpha^2(k) + c_2\alpha(k) + c_3}{c_4\alpha^2(k) + c_5\alpha(k) + c_6} = 0 \qquad (15)$$

where

$$\begin{cases} c_1 = 2\tilde{x}^3 b(k), \\ c_2 = 2\tilde{x}^2 (2b(k)C_{\max}(k-1) + \beta R(k) - c(k)) \\ c_3 = 2\tilde{x}(b(k){C_{\max}}^2(k-1) - (\beta R(k) + c(k))C_{\max}(k-1) + \beta b(k)) \end{cases} \tag{16}$$

Then, $\alpha(k)$ can be obtained by solving the following quadratic equation formed by the numerator term of (15).

$$c_1\alpha^2(k) + c_2\alpha(k) + c_3 = 0 \tag{17}$$

The quadratic equation (17) has two roots, from which the solution of $\alpha(k)$ can be obtained as follows:

$$\alpha(k) = \frac{-c_2 + \sqrt{{c_2}^2 - 4c_1c_3}}{2c_1} \tag{18}$$

In the proposed RTLS method, the three running time-averaged estimations $c_1$, $c_2$, and $c_3$ need to be calculated by (16) to obtain $\alpha(k)$. The maximum capacity estimate is updated with a time interval of $T_l$ ($T_l = k_2 - k_1$). In the next algorithm update index, the SOC($k_2$) of the battery cell becomes SOC($k_1$).

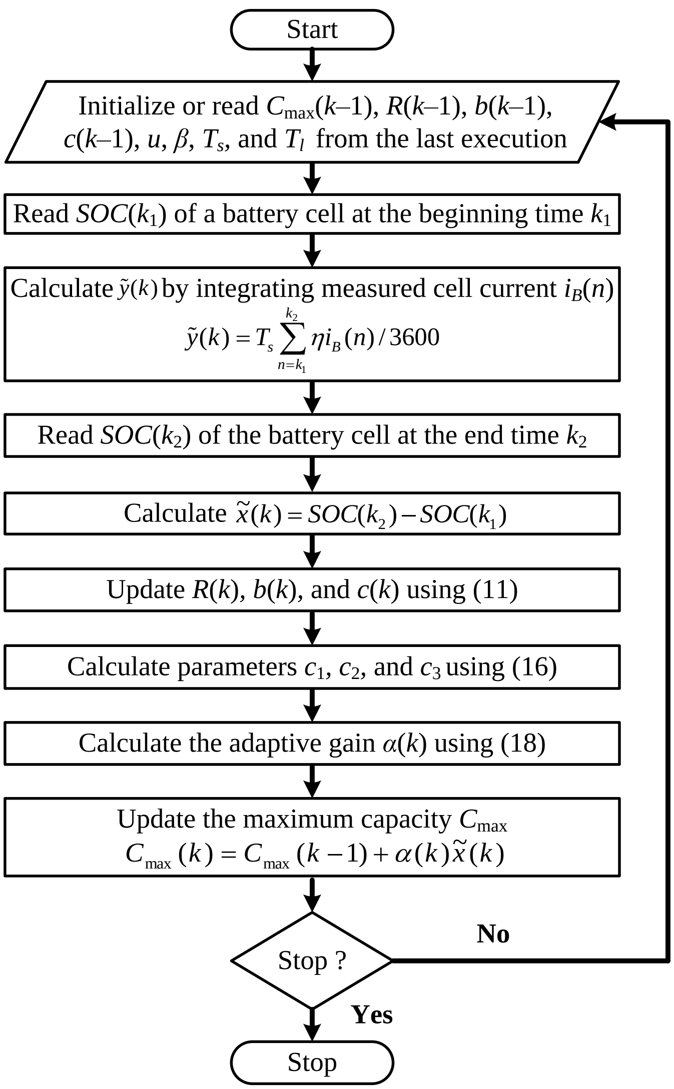



Fig. 2. The overall flow diagram of the proposed Rayleigh quotient-based online RTLS maximum capacity estimation method.

The overall flow diagram of the proposed algorithm is shown in Fig. 2. As a comparison, the RAWTLS method in [10] has six running time-averaged estimations and finds the optimal solution from four roots using the Ferrari method.

### *B. Convergence, Stability, Robustness, and Tuning*

Like many other capacity estimation algorithms, the proposed RTLS algorithm takes two SOC estimates as inputs: one is constructed from the Coulomb counting, and the other is typically obtained from a model-based SOC estimator. The effectiveness of the proposed RTLS algorithm is similarly contingent on two fundamental assumptions:

(1) The model (7) captures the battery's charge/discharge behavior, i.e. the capacity estimation problem can be formulated as a TLS problem

(2) Both inputs are perturbed by zero-mean Gaussian noises. It is worth pointing out that Assumption (2) is always not satisfied. Thus the resulted capacity estimate is necessarily biased.

As shown in [Thm. 5.1, 28], the proposed RTLS algorithm converges to the optimal solution under certain conditions. We include the results as follows for completeness.

*Theorem 3.2* If $\bar{R}(k) \to \bar{R}^*$ as $k \to \infty$, then the sequence of $\{C_{\max}(k)\}$ generated according to (14)-(18) converges to $C^*_{\max}$ which is the unique global minimizer of (13).

Here we provide a sketch of the proof. Readers are referred to [28] for detailed proof.

*Proof*: The steps to determine $\alpha(k)$ ensure that the resultant sequence $\{J(C_{\max}(k))\}$ is monotonically decreasing, and the cost function $J(C_{\max}(k))$ is a Lyapunov function of the dynamics (14). Meanwhile, the sequence $\{J(C_{\max}(k))\}$ clearly has a lower bound, and thus is bounded, which further implies the boundedness of the sequence $\{C_{\max}(k)\}$, i.e. all solutions of (14) lies in a compact set $\Omega$. Let $F$ be the set of all points in $\Omega$ where $J(C_{\max}(k)) - J(C_{\max}(k\text{-}1)) = 0$, i.e.

$F = \{w(k) | J(C_{\max}(k)) - J(C_{\max}(k\text{-}1)) = 0, \text{ for all } k\}$.

One can verify that the set $F$ is a subset of all stationary points of $J(C_{\max}(k))$. From Theorem.3.1, the largest invariant set in $F$ is the global minimizer of (13). Applying the LaSalle's invariance principle, all solutions of (14) converge to the maximal invariant set. This completes the proof.

It is likely that the minimizer of (13) with $\bar{R}$ replaced by $\bar{R}^*$ might still give biased capacity estimation.

*Remark 3.3* Although the proposed RTLS algorithm converges to the global optimum, no convergent rate result has been established. The convergent rate is however essential to establish stability results for the entire estimation error dynamics, including the SOC estimation and the capacity estimation errors.

*Remark 3.4* A convergent capacity estimation algorithm and exponentially convergent SOC estimation error dynamics do not guarantee the stability of the entire system. One needs to redesign the SOC estimator and the capacity estimation algorithm, for instance Lyapunov redesign, to ensure stability.

*Remark 3.5* If the capacity estimation algorithm yields exponentially convergent estimation error dynamics, then combined with the arbitrarily fast exponentially convergent

SOC estimation error dynamics and the Lipschitz condition, one can establish the exponentially convergent stability for the entire system.

*Remark 3.6* The proposed algorithm is not related to well-known recursive least square or least mean square algorithms by any means. Although it takes a similar form of gradient-based algorithms, the proposed algorithm does not update the decision variable (capacity estimate) along the gradient of the cost function. Instead, defining the search direction $\tilde{x}$ is for computation simplicity. The key part of the algorithm is the analytical expression of the optimal step length along the search direction. In terms of the numerical stability, taking the $\beta = 1$ case, the proposed RTLS problem formulation is unlikely ill-conditioned. This is because the diagonal components of $\bar{R}(k)$ is at the same scale and much larger than off-diagonal components, which implies the eigenvalue spread is not remarkable.

From Theorem 3.2, the proposed RTLS algorithm converges to the global minimizer of (13) as long as $\bar{R}(k) \to \bar{R}^*$ as $k \to \infty$. It is equivalent to study the robustness of the minimizer of (13) with $\bar{R}$ replaced by $\bar{R}^*$. Qualitative robustness analysis is possible by looking into two aspects: How good does $\bar{R}^*$ characterize capacity-related factors such as model mismatch, temperature fluctuation, aging, etc.? Is the global minimizer sensitive to perturbations in $\bar{R}^*$? For the first aspect, the proposed algorithm provides a design freedom: the forgetting factor $\mu$ during the computation of $\bar{R}$. The second aspect is essentially concerned about the robustness of the optimal solution, and can resort to numerous works in the perturbation analysis for SVD, e.g. [35], [36].

It is well-known that the maximum capacity changes due to aging and temperature fluctuation. The proposed algorithm can be readily applied to estimate the capacity as a time-varying parameter, as long as the temperature variation and aging process do not invalidate the aforementioned two fundamental assumptions. One way to ensure the satisfaction of the fundamental assumptions is to design the SOC estimation algorithm which can compensate the aging and temperature effects and provide accurate estimation. After the capacity is properly estimated, one can perform post-analysis to differentiate the causes of capacity variations. A number of techniques can be candidates for this prognosis purpose: pattern recognition, frequency analysis, or even simple threshold mechanism.

Estimating the capacity as a time-varying parameter requires appropriate tuning of the forgetting factor in (11). The forgetting factor $\mu$ reflects the design tradeoff between robustness and tracking capability. Specifically, algorithms with smaller values of $\mu$ weight more on tracking the time-varying capacity at the expense of more sensitivity to measurement noises and the SOC estimation error; on the contrary, large values of $\mu$ improve robustness but compromise the tracking capability. The maximum capacity varies due to a number of reasons, for instance aging and the environmental temperature [32]. With the environmental temperature regulated to a constant set point, the maximum capacity variation is largely due to the slow aging process, and thus a large value of $\mu$ (e.g., $0.98 \le \mu < 1$) can be chosen. To address the environmental temperature fluctuation which generally happens in a faster time scale than other factors (e.g., aging), $\mu$ can be taken a small value (e.g., $0.95 \le \mu < 0.98$). More advanced optimal or adaptive $\mu$ should be further investigated, which, however, is out of scope of this paper.

## IV. Method Validation

Simulation and experimental studies validate the proposed maximum capacity estimation algorithm by comparing with several existing methods. Specifically, in Section IV.A, the proposed RTLS algorithm is compared to the TP method [8], a batch LS method, and a SVD-based batch TLS method [27], and simulation results verify the effectiveness of the RTLS problem formulation for both constant and time-varying capacity cases; Section IV.B makes comparison between the proposed RTLS algorithm and an existing DEKF, and shows that the proposed RTLS results in a lower computation cost but comparable capacity estimation accuracy. Section IV.C makes comparison between the proposed RTLS, the TP, and the DEKF methods using experimental data.

Across this section, the EKF-based SOC estimation is implemented on the basis of the electrical battery model [33], which is given in Appendix for completeness. All algorithms are implemented in MATLAB on a desktop computer. It is worth pointing out that the accumulation error along with the Coulomb counting can be effectively mitigated by good calibration of the initial SOC, and use of accurate current sensors. We therefore use the SOC from the Coulomb counting as the reference (true) SOC.

### *A. Simulation Study #1*

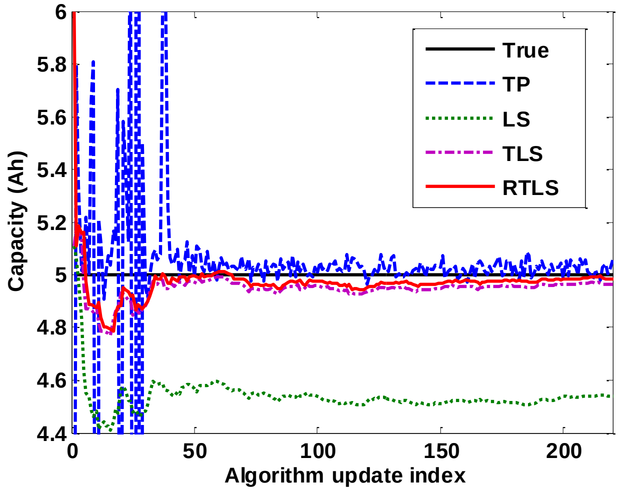
In simulation, the nominal capacity, nominal voltage, and cutoff voltage of a single battery cell are 5 Ah, 3.7 V, and 2.5 V, respectively. All methods use the same current profiles and SOC as shown in Fig. 3(b) and Fig. 3(c), respectively. Particularly, both the SOC and the current are corrupted by zero-mean Gaussian random noises with variances $\sigma_x^2 = (0.01)^2$ and $\sigma_y^2 = (0.001)^2$, respectively. Each capacity estimation algorithm runs for ten times to suppress the effects of random noises, and the average value of $C_{\max}$ over ten times is used as the estimated $C_{\max}$. All capacity estimation algorithms are executed with an interval of 200 seconds (i.e., $T_l = 200$ seconds).

For the proposed RTLS, the initial maximum capacity for is 6 Ah, the forgetting factor $\mu = 0.999$. For the TP method, $k_1$ is fixed to be the initial time (i.e., $k_1 = 0$). Fig. 3(a) compares the true and the estimated $C_{\max}$. Simulation shows that the proposed RTLS method provides the best estimation accuracy and converges to the true maximum capacity value; the TLS method is slightly worse than the proposed method; the LS method gives biased estimation; and the TP method takes longer time to converge. Note that the TP method is sensitive to the SOC error and thus the estimated $C_{\max}$ oscillates. The simulation results show that given the two fundamental assumptions satisfied, the proposed RTLS algorithm can provide unbiased capacity estimation, and the LS or the TP methods fail.

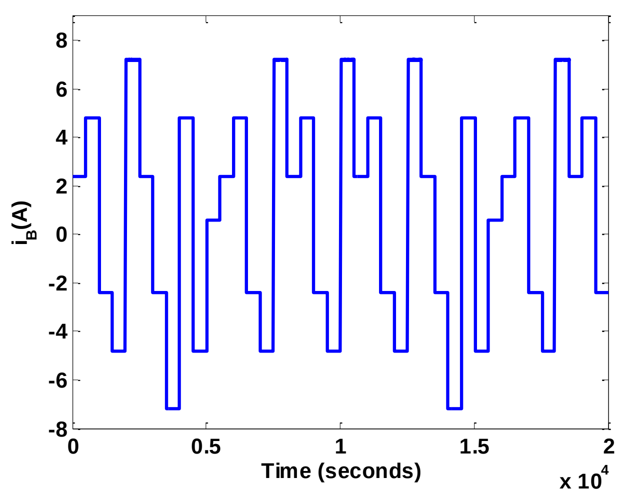
Next, we verify that the proposed RTLS algorithm works effectively in the time-varying maximum capacity case. In this test, the true maximum capacity changes linearly over time with a slope of -2.5 mAh per algorithm execution time. The same current input, as shown in Fig. 3(b), is used and all algorithms are executed ten times as well. Both the true and initial maximum capacities are set to be 10 Ah for the proposed RTLS algorithm, and the forgetting factor $\mu$ is 0.98. The true and the estimated average time-varying maximum capacities are shown in Fig. 4. The TP method does not converge to the true value and the estimation error of the TLS method increases over time. However, the proposed RTLS method can track the time-varying maximum capacity accurately, and thus outperforms TP-based analytical methods.

### *B. Simulation Study #2*

We compare the following three setups: an EKF-based SOC estimator cascaded by the proposed RTLS algorithm; an EKF-based SOC estimator cascaded by the TP algorithm; a simple DEKF estimating both the SOC and the capacity simultaneously [15]. The simple DEKF includes two EKFs: one for SOC estimation and another for the maximum capacity estimation. All three setups use the same EKF-based SOC estimator, and the only difference lies in the capacity estimation algorithm. In the EKF design for SOC estimation, the system's process noise covariance matrix and initial state error covariance matrix are diagonal matrices with each element equals to 0.16 and 1, respectively. For the capacity estimation in the simple DEKF, the system's noise covariance matrix and initial state error covariance matrix are defined as $10000^2$ and 100000, respectively. The measurement noise covariance matrix for all EKFs is 0.25. The covariance matrices of all EKFs are chosen by trial-and-error in an effort to reduce estimation errors. In the SOC and capacity estimation algorithms, the initial SOC and maximum capacity are set to




(a)

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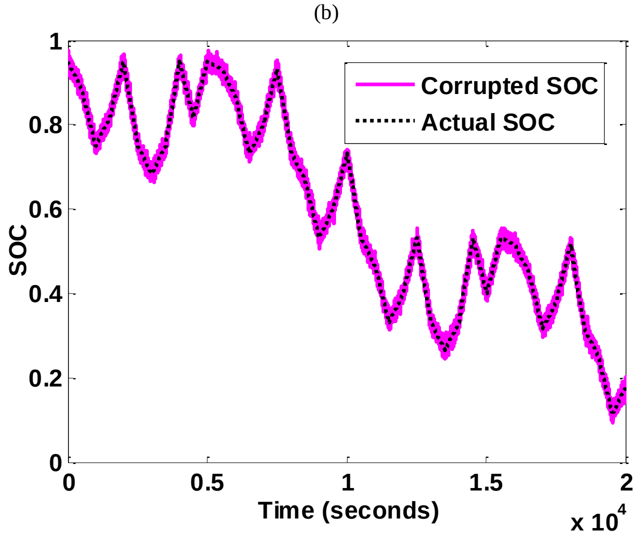


(c)

Fig. 3. Comparison of true and estimated average values of the maximum capacity using the TP, LS, TLS, and proposed RTLS algorithms: (a) the maximum capacity, (b) the noisy pulse current cycle, and (c) the corrupted SOC applied to the algorithms and the actual SOC.

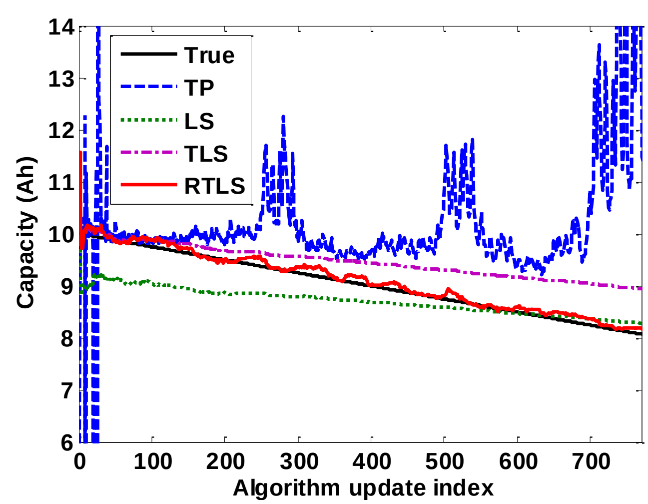


Fig. 4. Comparison of true the estimated average values of the time-varying maximum capacity using the TP, LS, TLS, and proposed RTLS algorithms.

0.8 and 6 Ah, respectively, while the true initial SOC and maximum capacity are 0.95 and 5Ah, respectively. The EKF is executed with a small time-scale (e.g., $T_s$ = 1 second) to keep track of the SOC, while a large time-scale (e.g., $T_l$ = 200 seconds) is used in the cascaded RTLS and TP algorithms for the capacity estimation. The simple DEKF is executed with a small time-scale (e.g., $T_s$ = 1 second) for both SOC and capacity estimation. The value of $\mu$ is 0.98 in the proposed RTLS.

Fig. 5(a) compares the true and the estimated $C_{max}$ using the TP, the simple DEKF, and the proposed RTLS for the dynamic noisy current cycle shown in Fig. 3(b). The estimated SOC from the EKF applied to the RTLS algorithm is shown in Fig. 5(b). Table I summarizes the comparison results where the accuracy is measured by root mean square error (RMSE) calculated from algorithm update index 11 to 86, the computational cost is evaluated by the computation time taken on an Intel® Core™2 Duo CPU T6600@2.2GHz, 64-bit OS. The results indicate that the TP method has the best performance due to relatively smooth and accurate SOC estimate from the EKF; while the DEKF and RTLS methods have similar estimation accuracy and convergence. We remark that both the RTLS and DEKF performance might be compromised by non-Gaussian SOC estimation error, and the proposed RTLS however requires lower computational cost and easier implementation than the simple DEKF.

TABLE I
THE SOC AND THE CAPACITY ESTIMATION ALGORITHMS IN SIMULATION STUDY

| Method | EKF | Simple DEKF | | TP+EKF | RTLS+EKF |
|---|---|---|---|---|---|
| Estimation | SOC | SOC | Capacity | Capacity | Capacity |
| RMSE | 0.011 | 0.011 | 0.2237 | 0.0006 | 0.2151 |
| Computational time (seconds) | 10.69 | 21.4711 | | 10.6917 | 10.6977 |
| Convergence time | N/A | Very Fast | | Very Fast | Very Fast |

## C. Experimental Study

In the experiment, the data of the cell voltage and current are collected from a battery tester under the ambient temperature at 21.6°C. The SOC estimation algorithm consists of a fast upper-triangular and diagonal recursive LS (FUDRLS) block for impedance estimation and an EKF block for the SOC estimation [33]. The estimated SOC and the measured cell current are used as the inputs of the TP algorithm and the proposed RTLS algorithm. The simple DEKF also uses the electrical parameters estimated by the FUDRLS to estimate the SOC. In the EKF-based SOC estimation, the initial SOC and maximum capacity are set to 0.5 and 5 Ah, respectively; while the true initial SOC and maximum capacity are 0.31 and 4.732 Ah, respectively. In order to set the test battery cell with the desired initial SOC, the battery was first fully charged and rests

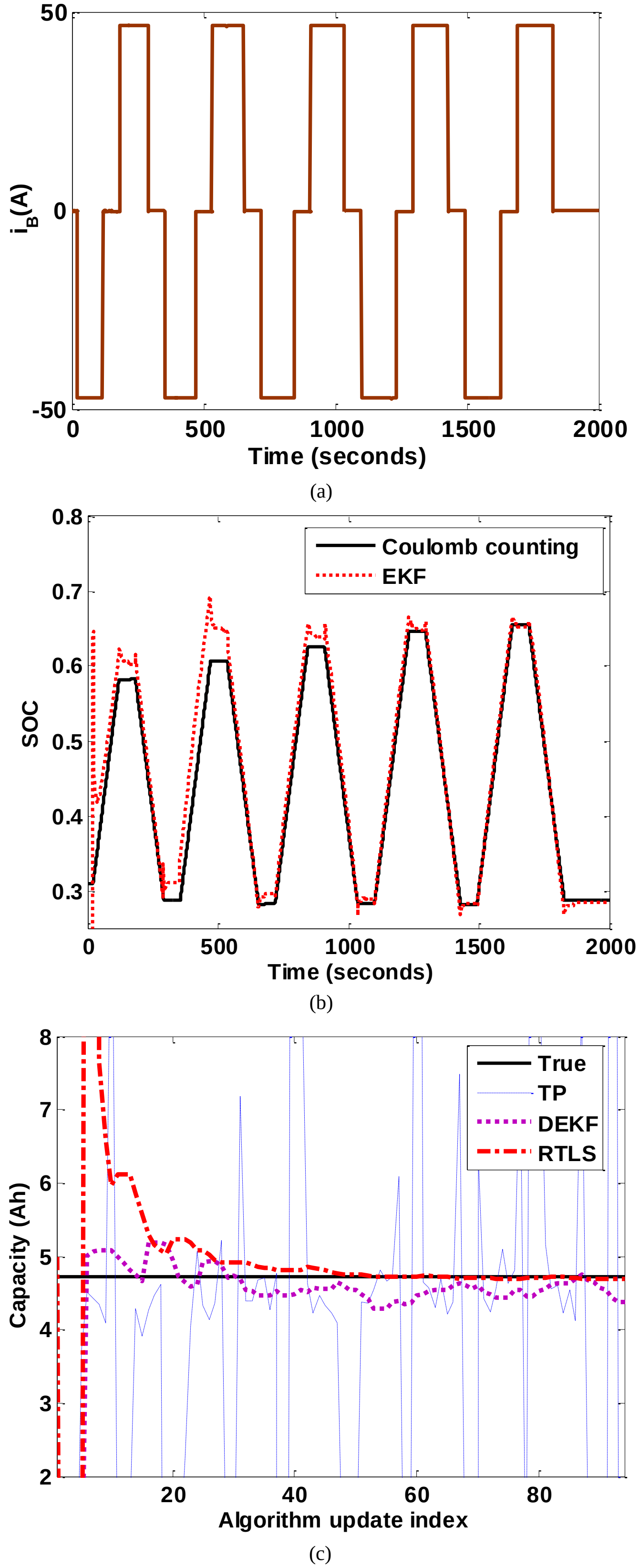


Fig. 6. Comparison of true and estimated values of the maximum capacity of the battery cell model by using the TP, simple DEKF, and proposed RTLS algorithms: (a) the pulse current cycle ($i_B$ =10C), (b) the estimated SOC applied to the TP and RTLS algorithms, and (c) the maximum capacity.

for one hour. Then the cell is discharged using a small current (e.g., 0.2 A) to the desired initial SOC value. The true maximum capacity was extracted offline using the full discharge test with a small current (e.g., 0.05C = 0.25 A) at the ambient temperature. The FUDRLS and EKF are executed with a small time-scale (e.g., $T_s$ = 1 second) to keep track of the fast time varying electrical parameters and the SOC; while a large time-scale (e.g., $T_l$ = 20 seconds) is used in the maximum capacity algorithms. The simple DEKF is executed with a small time-scale (e.g., $T_s$ = 1 second). The value of $\mu$ is taken 0.98 in the proposed RTLS.

During experiment, the battery cell was operated by a dynamic high-pulse current cycle ($i_B$ = 10C) shown in Fig. 6(a). Fig. 6(b) compares the SOC values estimated by the Coulomb counting and the EKF algorithm. The error between the EKF-estimated SOC and the Coulomb counting-estimated SOC decreases from 10% at 100 seconds to 2% at 1000 seconds. Fig. 6(c) compares the true and estimated $C_{\max}$ and shows that the proposed RTLS algorithm has the best tracking performance and converges to the true maximum capacity value quickly. Due to the relatively large oscillation error, the simple DEKF performs worse than the proposed method. The TP method is sensitive to the accuracy of the SOC input and does not converge to the true value.

Table II compares different algorithms in terms of the RMSE from algorithm update index 21 to 94, computational time, and convergence time. The results clearly show that the proposed method outperforms the simple DEKF in terms of higher accuracy and lower computational cost. Furthermore, the implementation of the proposed method is simple.

## V. Conclusion

This paper has presented a Rayleigh quotient-based online RTLS maximum capacity estimation algorithm for lithium-ion batteries. The proposed RTLS method has been implemented in MATLAB and validated by simulation and experimental results for a lithium-ion battery cell. Owing low complexity and high accuracy, the proposed method is suitable for use in the real-time embedded BMSs in various applications. The estimated maximum capacity can be used for condition monitoring (e.g., SOC and SOH estimation), diagnosis, and prognosis of lithium-ion batteries. Future work includes investigations of the temperature effect on the battery capacity and adaptive forgetting factor, developing a capacity estimation algorithm which is robust to the color noise inputs, and more realistic experimental validation.

TABLE II
The SOC and the Capacity Estimation Algorithms in Experimental Study

| Method | EKF | Simple DEKF | | TP+EKF | RTLS+EKF |
|---|---|---|---|---|---|
| Estimation | SOC | SOC | Capacity | Capacity | Capacity |
| RMSE | 0.027 | 0.027 | 0.208 | 35.058 | 0.141 |
| Computational time (seconds) | 1.734 | 3.128 | | 1.736 | 1.740 |
| Convergence time | N/A | Very Fast | | No | Fast |

## VI. Appendix

The battery model [33] used in this paper is shown in Fig. 7, where $VOC$ (i.e., the open-circuit voltage OCV) includes two parts. The first part, denoted by $V_{oc}(SOC)$, represents the equilibrium OCV, which is used to bridge the SOC to the cell open-circuit voltage. The second part $V_h$ is the hysteresis voltage capturing the nonlinearity of OCV. The RC circuit models the I-V characteristics and the transient response of the battery cell. Particularly, the series resistance, $R_s$, characterizes the charge/discharge energy losses of the cell; the charge transfer resistance, $R_c$, and the double layer capacitance, $C_d$, are used to characterize the short-term diffusion voltage, $V_d$, of the cell; $V_{cell}$ represents the terminal voltage of the cell. Defining $H(i_B) = \exp(-\rho|i_B(n)|T_s)$, a discrete-time state-space version of the real-time battery model is expressed as follows:

$$\begin{bmatrix} SOC(n+1) \\ V_d(n+1) \\ V_h(n+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \exp(\frac{-T_s}{R_c \cdot C_d}) & 0 \\ 0 & 0 & H \end{bmatrix} \cdot \begin{bmatrix} SOC(n) \\ V_d(n) \\ V_h(n) \end{bmatrix} + \begin{bmatrix} -\eta T_s / C_{\max} & 0 \\ R_c(1-\exp(\frac{-T_s}{R_c \cdot C_d})) & 0 \\ 0 & (H-1)sign(i_B) \end{bmatrix} \cdot \begin{bmatrix} i_B(n) \\ V_{h\max} \end{bmatrix} \quad (19)$$

$$V_{cell}(n) = V_{oc}(SOC) - V_d(n) - R_s \cdot i_B(n) + V_h(n) \quad (20)$$

$$V_{oc}(SOC) = a_0 \exp(-a_1 SOC) + a_2 + a_3 SOC - a_4 SOC^2 + a_5 SOC^3 \quad (21)$$

where $V_{hmax}$ is the maximum hysteresis voltage, and $\rho$ is the hysteresis parameter representing the convergence rate.

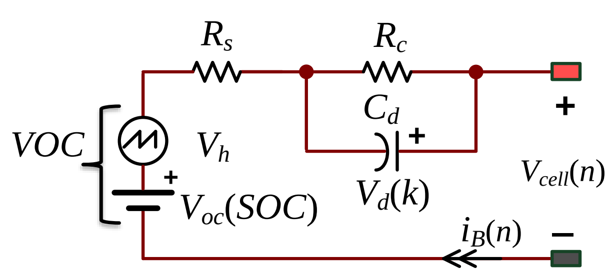


Fig. 7. The first-order RC model with a hysteresis.

TABLE III
Battery Model Parameters

| | | | |
|---|---|---|---|
| $R_s$ | 0.08 | $R_c$ | 0.03 |
| $C_d$ | 3000 | $\rho$ | $2.47 \cdot 10^{-3}$ |
| $V_{hmax}$ | 0.03 | $a_0$ | -0.852 |
| $a_1$ | 63.867 | $a_2$ | 3.692 |
| $a_3$ | 0.559 | $a_4$ | 0.51 |
| $a_5$ | 0.508 | | |

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