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Modified Probabilistic Data Association Algorithms

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***Abstract*—Probabilistic Data Association (PDA) algorithm has shown promising performance in symbol detection and interference cancellation in different communication schemes. This paper proposes new algorithms that build on PDA and introduce modifications in the way the symbol being detected is treated. While PDA models this symbol as a discrete sample from a constellation, PDA with symbol uncertainty (SU-PDA) views it as a sum of a deterministic symbol and random noise, while the Gaussian PDA (G-PDA) models it as a random variable with either a single Gaussian or Gaussian mixture distribution. The proposed algorithms are tested via computer simulations on both simulated and experimentally measured channels. The performance study reveals that the SU-PDA and G-PDA outperform the conventional PDA with the performance gain ranging from few dBs on measured channel with block fading up to and exceeding 10 dB on the simulated channel with fast fading.**

***Index Terms*—probabilistic data association algorithm, large MIMO, machine-to-machine communications, precoding, symbol detection.**

I. Introduction

Machine-to-machine (M2M) communication systems are intended to enable machines to exchange short command and control messages over wireless links. The main design goal is to achieve fast and highly reliable transmission of short messages over wireless channels with small and relatively simple devices employing a small number of antennas (preferably one or two) [1]. One of the major application areas of M2M modems are factories wherein the automated production processes would benefit if machines communicated with each other wirelessly.

We consider a single-input single-output (SISO) communication system with block transmission and detection as a candidate for M2M modems. The type of messages and the required low latency call for short block sizes (e.g. < 100 symbols).

A successful design of M2M modem with pseudo-random phase precoding (PRPP) and Likelihood Ascent Search (LAS) detection has been reported in [2]. This scheme performs extremely well in practice when block size is at least 400 symbols. However, our goal is to reduce the block size without losing much of the performance.

Maximum likelihood detection (MLD) is the optimal symbol detection scheme. However, the complexity of the MLD grows exponentially with the block size. Therefore, a variety of suboptimal detection algorithms with polynomial complexity have been developed. Among the most known are PDA [3], QRD-M [4], LAS [5], Reactive Tabu Search (RTS) [6], and algorithms based on graphical models [7].

The PDA algorithm was originally developed for target tracking and has gained interest in the communications community. As such, the PDA has been applied for multiuser detection in code division multiple access (CDMA) systems [8], [9]. The application of PDA for turbo equalization is reported in [10]. The symbol detection in multiple input multiple output (MIMO) systems using PDA algorithm is presented in [11]. PDA has been combined with the decision feedback equalization (DFE) for detection in MIMO system over a frequency selective channel [12]. A bit-by-bit detection of a higher order quadrature amplitude modulation (QAM) using the PDA algorithm is given in [13]. A new symbol ordering scheme for PDA detection suitable for flat fading channel is proposed in [14].

This paper proposes new algorithms based on PDA which outperform the conventional PDA for short block sizes ($<$ 100 symbols) at lower bit error rates (BERs) of our interest in the M2M communication system. Given the equivalence between the SISO system with block transmission and large MIMO [2], the proposed algorithms are equally applicable for symbol detection in large MIMO.

Computer simulations have been conducted for both the simulated and experimentally measured indoor channels. These tests show that the proposed algorithms outperform the conventional PDA with the gain ranging from few dBs (i.e., one order of magnitude of BER) on the experimentally measured channel with larger blocks, up to and exceeding 10 dB (i.e. more than two orders of magnitude of BER) on a simulated fast fading channel with shorter blocks.

Throughout the paper, boldface uppercase letters denote matrices and boldface lowercase letters denote vectors. An operator $(\cdot)^T$ denotes transpose and $(\cdot)^H$ complex-conjugation, i.e., Hermitian. Unless otherwise specified, all vectors are assumed to be column vectors.

II. System Model

A block diagram of the SISO communication system with block transmission and detection is shown in Fig. 1 and described in this section.

The information to be transmitted is represented with complex symbols. The symbol transmitted at discrete time n is $x_n \in \mathcal{X}$, where $\mathcal{X}$ is a finite dimensional symbol alphabet

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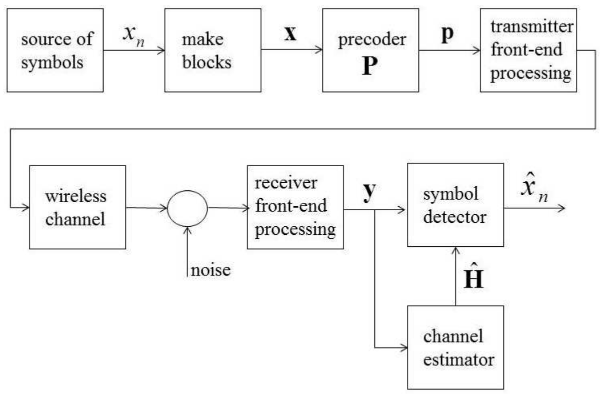

Fig. 1. Block diagram of a SISO communication system with block transmission and detection.

(i.e., constellation). Without loss of generality, we assume that symbol x_n can take any value from $\mathcal{X}$ with equal probability. The consecutive symbols are grouped into blocks of size N so that the transmitted block is $\mathbf{x} \in \mathbb{C}^{N\times 1}$.

The data block $\mathbf{x}$ is processed by a precoder. The precoder essentially spreads the energy of each symbol x_n into N signaling intervals, giving rise to time diversity in the case of a fast fading channel. Formally, the precoder is described by a precoding matrix $\mathbf{P} \in \mathbb{C}^{N\times N}$ and its output is given by

$$\mathbf{p} = \mathbf{P}\mathbf{x}. \tag{1}$$

On the contrary, if the channel experiences very slow or block fading, the precoding does not lead to diversity. Formally, in this case we use the identity precoder, i.e., $\mathbf{P} = \mathbf{I}$.

The precoded block $\mathbf{p}$ is modulated onto a carrier and transmitted over a wireless channel. The signal received on the receiver antenna is filtered, demodulated, converted into the baseband and sampled. The received baseband signal in discrete time is $\mathbf{y} \in \mathbb{C}^{N\times 1}$ and is related with the precoded signal $\mathbf{p}$ through a linear model,

$$\mathbf{y} = \sqrt{\mathrm{SNR}}\,\mathbf{H}\mathbf{p} + \mathbf{v}, \tag{2}$$

where $\mathbf{H} \in \mathbb{C}^{N\times N}$ models distortions caused by the wireless channel. The signal-to-noise ratio is SNR and the additive noise is circularly symmetric zero mean uncorrelated Gaussian process, i.e., $\mathbf{v} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$.

The relation between the received signal $\mathbf{y}$ and transmitted block $\mathbf{x}$ is obtained by substituting (1) into (2) such that

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{v}, \tag{3}$$

where

$$\mathbf{G} = \sqrt{\mathrm{SNR}}\,\mathbf{H}\mathbf{P} \tag{4}$$

is the effective channel matrix. Throughout this paper, we assume the receiver perfectly knows the effective channel matrix $\mathbf{G}$ and focus on symbol detection.

Although derived for a SISO communication system with block detection, expression (3) models the input-output relationship in other communication systems as well. Since the algorithms proposed in this paper do not in particular rely on the additional specifics of the SISO system, they are equally applicable for symbol detection in other communication schemes driven by (3).

III. Probabilistic Data Association Algorithm

The conventional PDA algorithm is outlined in this section. For the sake of easy exposition and without loss of generality, model (3) is after multiplication with $\mathbf{G}^{-1}$ expressed as

$$\mathbf{z} = \mathbf{x} + \mathbf{w}, \tag{5}$$

where $\mathbf{z} = \mathbf{G}^{-1}\mathbf{y}$. Note that the noise $\mathbf{w}$ is circularly symmetric Gaussian, i.e., $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$ with covariance $\mathbf{R} = \left(\mathbf{G}^H\mathbf{G}\right)^{-1}$.

In essence, the PDA algorithm iteratively calculates the posterior probability of each symbol x_n, conditioned on the received signal $\mathbf{z}$,

$$p_n(a) = \mathbb{P}\left[x_n = a \mid \mathbf{z}\right], \quad a \in \mathcal{X}, \quad n = 1, 2, \ldots, N. \tag{6}$$

The algorithm operates as follows. Suppose it is at iteration i and x_n is the desired symbol. Then the received signal $\mathbf{z}$ in (5) is expressed as

$$\mathbf{z} = x_n\mathbf{e}_n + \sum_{i\neq n} x_i\mathbf{e}_i + \mathbf{w}, \tag{7}$$

where $\{\mathbf{e}_i\}_{i=1}^{n}$ is the standard basis (i.e., $\mathbf{e}_i$ contains 1 in entry i and zeros elsewhere). The main idea behind the PDA algorithm is to treat the contribution from all symbols except x_n to the received signal as interference and approximate the sum of the interference and noise with a Gaussian distribution. That is, when detecting x_n, the interference plus noise in (7)

$$\tilde{\mathbf{w}} = \sum_{i\neq n} x_i\mathbf{e}_i + \mathbf{w} \tag{8}$$

is approximately $\mathcal{CN}(\boldsymbol{\mu}_n, \boldsymbol{\Sigma}_n)$[1]. The mean $\boldsymbol{\mu}_n$ and covariance matrix $\boldsymbol{\Sigma}_n$ are matched to the mean and covariance of $\tilde{\mathbf{w}}$, i.e.,

$$\boldsymbol{\mu}_n = \mathbf{E}[\tilde{\mathbf{w}}|\mathbf{z}] = \sum_{i\neq n} \mathbf{E}[x_i|\mathbf{z}]\mathbf{e}_i, \tag{9}$$

and

$$\boldsymbol{\Sigma}_n = \mathrm{cov}(\tilde{\mathbf{w}}, \tilde{\mathbf{w}}|\mathbf{z}) = \sum_{i\neq n} \mathrm{var}(x_i|\mathbf{z})\mathbf{e}_i\mathbf{e}_i^T + \mathbf{R}. \tag{10}$$

In addition, the PDA assumes that the posteriors of all symbols except x_n computed thus far are the true posteriors. Thus, the mean $\boldsymbol{\mu}_n$ and covariance $\boldsymbol{\Sigma}_n$ of the approximating Gaussian distribution are easily obtained by evaluating the mean $\mathbf{E}[x_i|\mathbf{z}]$ and variance $\mathrm{var}(x_i|\mathbf{z})$ of a discrete random variable with known distribution.

Having approximated the statistics of the interference plus noise in (7), the distribution of the received signal $\mathbf{z}$ conditioned on $x_n = a$ is complex Gaussian,

$$p_{\mathbf{z}|x_n}(\mathbf{z} \mid a) = \mathcal{CN}(\mathbf{z}; a\mathbf{e}_n + \boldsymbol{\mu}_n, \boldsymbol{\Sigma}_n), \quad a \in \mathcal{X}. \tag{11}$$

[1]To simplify the exposition, we assume that Gaussian distributions throughout the paper are circularly symmetric. Algorithm development in the real domain as well as the complex domain with non-circular symmetry is analogous.

Therefore, the posterior p_n of a symbol x_n is updated via Bayes' rule

$$p_n(a) = \frac{p_{z|x_n}(\mathbf{z} \mid a)}{\sum_{a' \in \mathcal{X}} p_{z|x_n}(\mathbf{z} \mid a')}, \quad a \in \mathcal{X}, \tag{12}$$

where the summation in the denominator is over alphabet $\mathcal{X}$. Note that in (12) we exploit the assumption that symbols have uniform prior.

The PDA algorithm then continues and updates the posterior of the next symbol following some prespecified/adaptive ordering scheme. After a certain number of iterations or until convergence is established, it outputs the estimates of the posteriors p_n. Hard estimates $\hat{x}_n$ of the corresponding symbols are obtained using the maximum a posteriori probability (MAP) rule,

$$\hat{x}_n = \underset{a \in \mathcal{X}}{\operatorname{argmax}}\, p_n(a). \tag{13}$$

IV. Proposed Algorithms

This section presents new algorithms based on PDA. The common feature of the conventional PDA and the proposed algorithms is that they use the same model (5) (or (3)) and infer posterior distribution p_n of transmitted symbol x_n, conditioned on received signal $\mathbf{z}$ (or $\mathbf{y}$).

A. PDA with Symbol Uncertainty (SU-PDA)

Suppose we are detecting symbol x_n. The main idea behind the SU-PDA algorithm is to model x_n as

$$x_n = \bar{x}_n + \tilde{x}_n, \tag{14}$$

where $\bar{x}_n$ is a deterministic, unknown variable from finite alphabet $\mathcal{X}$ and $\tilde{x}_n$ is a zero mean random variable whose variance is equal to the variance of x_n conditioned on $\mathbf{z}$ and evaluated from the current estimate of p_n,

$$\mathrm{var}(\tilde{x}_n) = \mathrm{var}(x_n|\mathbf{z}). \tag{15}$$

Intuitively, $\tilde{x}_n$ captures the uncertainty in the current knowledge about x_n.

Substituting (14) into (7) yields

$$\mathbf{z} = \bar{x}_n \mathbf{e}_n + \tilde{x}_n \mathbf{e}_n + \sum_{i \neq n} x_i \mathbf{e}_i + \mathbf{w}. \tag{16}$$

The contribution to the received signal $\mathbf{z}$ from the symbols other than x_n is treated as interference, while $\tilde{x}_n \mathbf{e}_n$ is viewed as an additional noise term. Overall, the distribution of the sum of the interference and noise

$$\tilde{\mathbf{w}} = \tilde{x}_n \mathbf{e}_n + \sum_{i \neq n} x_i \mathbf{e}_i + \mathbf{w} \tag{17}$$

is approximated with complex Gaussian distribution with the mean $\boldsymbol{\mu}_n$ and covariance $\boldsymbol{\Sigma}_n$. As in the conventional PDA, the current estimates of posteriors $\{p_i\}_{i=1}^N$ are used to evaluate the mean and covariance of the approximating Gaussian distribution such that

$$\boldsymbol{\mu}_n = \mathbf{E}[\tilde{\mathbf{w}}|\mathbf{z}] = \sum_{i \neq n} \mathbf{E}[x_i|\mathbf{z}] \mathbf{e}_i, \tag{18}$$

and

$$\boldsymbol{\Sigma}_n = \mathrm{cov}(\tilde{\mathbf{w}}, \tilde{\mathbf{w}}|\mathbf{z}) = \sum_{i=1}^{N} \mathrm{var}(x_i|\mathbf{z}) \mathbf{e}_i \mathbf{e}_i^T + \mathbf{R}. \tag{19}$$

Note that the mean vector $\boldsymbol{\mu}_n$ is the same as in the conventional PDA (9). On the other hand, the summation in the expression for covariance matrix (19) includes contributions from all symbols, as opposed to the conventional PDA (10).

Given that interference plus noise $\tilde{\mathbf{w}}$ is Gaussian distributed, the probability distribution of the received signal $\mathbf{z}$, parameterized by $\bar{x}_n$, is

$$p_z(\mathbf{z}; \bar{x}_n = a) = \mathcal{CN}(\mathbf{z}; a\mathbf{e}_n + \boldsymbol{\mu}_n, \boldsymbol{\Sigma}_n), \quad a \in \mathcal{X}. \tag{20}$$

The SU-PDA updates posterior p_n of x_n by normalizing the parameterized distributions with respect to parameter x. That is,

$$p_n(a) = \frac{p_z(\mathbf{z}; \bar{x}_n = a)}{\sum_{a' \in \mathcal{X}} p_z(\mathbf{z}; \bar{x}_n = a')}, \quad a \in \mathcal{X}, \tag{21}$$

where the summation in the denominator is over the finite alphabet $\mathcal{X}$.

A pseudo-code description of how the SU-PDA algorithm evaluates the posteriors of binary phase shift keying (BPSK) modulated symbols is given in Algorithm 1. Note that the conditional mean (18) and variance (19) of symbol x_n taking values from $\mathcal{X} = \{+1, -1\}$ are, respectively, $2p_n^{(i)} - 1$ and $4p_n^{(i)}(1 - p_n^{(i)})$, where $p_n^{(i)}$ is the estimate of $\mathbb{P}[x_n = 1 \mid \mathbf{z}]$ at iteration i.

Algorithm 1 SU-PDA detection of BPSK symbols

Require: received signal $\mathbf{z}$
Ensure: $p_1, p_2, \ldots, p_N$
  Initialize $p_1^{(0)}, \ldots, p_N^{(0)}$
  for $i = 1, 2, \ldots, I$ **do**
    for $n = 1$ to N **do**
      $\boldsymbol{\mu}_n = \sum_{k \neq n} (2p_k^{(i-1)} - 1)\mathbf{e}_k$
      $\boldsymbol{\Sigma}_n = \sum_k 4p_k^{(i-1)}(1 - p_k^{(i-1)})\mathbf{e}_k \mathbf{e}_k^T$
      $p_n^{(i)} = \frac{\mathcal{CN}(\mathbf{z}; \mathbf{e}_n + \boldsymbol{\mu}_n, \boldsymbol{\Sigma}_n)}{\mathcal{CN}(\mathbf{z}; \mathbf{e}_n + \boldsymbol{\mu}_n, \boldsymbol{\Sigma}_n) + \mathcal{CN}(\mathbf{z}; -\mathbf{e}_n + \boldsymbol{\mu}_n, \boldsymbol{\Sigma}_n)}$
    end for
  end for

B. PDA with Gaussian Approximation (G-PDA)

G-PDA starts with model (5) and updates posterior p_n. As in the original PDA, the contribution of other symbols to the received signal $\mathbf{z}$ is viewed as interference. The distribution of the interference plus noise $\tilde{\mathbf{w}}$ is approximated with complex Gaussian distribution with the mean vector $\boldsymbol{\mu}_n$ and covariance matrix $\boldsymbol{\Sigma}_n$, evaluated as in (9) and (10).

In the first step of the G-PDA algorithm, the expected value of the interference is subtracted from the received signal. Since the noise $\mathbf{w}$ has zero mean, the expected value of the interference is $\boldsymbol{\mu}_n$ and the obtained interference-free signal is

$$\tilde{\mathbf{z}} = \mathbf{z} - \boldsymbol{\mu}_n. \tag{22}$$

The interference-free signal $\tilde{\mathbf{z}}$ is using (5) modeled as

$$\tilde{\mathbf{z}} = x_n \mathbf{e}_n + \mathbf{q}, \tag{23}$$

where $\mathbf{q}$ is the equivalent noise, given by

$$\mathbf{q} = \sum_{i \neq n} x_n \mathbf{e}_n - \boldsymbol{\mu}_n + \mathbf{w}. \tag{24}$$

Note that $\mathbf{q} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_n)$, where $\mathbf{\Sigma}_n$ is evaluated using (10).

In the second step of G-PDA, the signal $\tilde{\mathbf{z}}$ in (23) is processed with the minimum mean square error (MMSE) filter in order to estimate symbol x_n. The impulse response of the MMSE filter is given in vector form by

$$\mathbf{w}_{\text{MMSE}} = \left(\mathbf{e}_n \mathbf{e}_n^T + \mathbf{\Sigma}_n\right)^{-1} \mathbf{e}_n. \tag{25}$$

The output from the MMSE filter is scaled and sufficient statistic $\tilde{z}_{\text{o}}$ is given by

$$\tilde{z}_{\text{o}} = \frac{\mathbf{w}_{\text{MMSE}}^H \tilde{\mathbf{z}}}{\mathbf{e}_n^T (\mathbf{e}_n \mathbf{e}_n^T + \mathbf{\Sigma}_n)^{-1} \mathbf{e}_n}. \tag{26}$$

Substituting (25) into (26) and using the matrix inversion lemma for the inverse of the rank one update of a matrix yields

$$\tilde{z}_{\text{o}} = \frac{\mathbf{e}_n^T \mathbf{\Sigma}_n^{-1}}{\mathbf{e}_n^T \mathbf{\Sigma}_n^{-1} \mathbf{e}_n} \tilde{\mathbf{z}}. \tag{27}$$

Finally, substituting (23) into (27), yields a simple model for $\tilde{z}_{\text{o}}$

$$\tilde{z}_{\text{o}} = x_n + \tilde{q}, \tag{28}$$

where $\tilde{q} \sim \mathcal{CN}(0, \sigma^2)$. The variance σ^2 is evaluated as

$$\sigma^2 = \frac{1}{\mathbf{e}_n^T \mathbf{\Sigma}_n^{-1} \mathbf{e}_n}. \tag{29}$$

In the final step, symbol x_n is detected from $\tilde{z}_{\text{o}}$ using model (28). It can be confirmed that if x_n is modeled as a uniform discrete random variable over alphabet $\mathcal{X}$, the conventional PDA algorithm is obtained. Instead, depending on how the distribution of x_n is modeled, we arrive to two versions of G-PDA.

1) Modeling x_n with a single Gaussian: In this version, we model x_n as a single Gaussian random variable whose mean and variance match the mean and variance of x_n. Since x_n is a point from a finite constellation, without loss of generality, we have

$$\tilde{p}_{\mathsf{x}_n}(x) = \mathcal{CN}(x; 0, 1). \tag{30}$$

Hence, given that both x_n and $\tilde{q}$ in (28) are Gaussian distributed scalars, the posterior of x_n conditioned on $\tilde{z}_{\text{o}}$ (and therefore on $\mathbf{z}$ because $\tilde{z}_{\text{o}}$ is a sufficient statistics) is also Gaussian, i.e.,

$$\tilde{p}_{\mathsf{x}_n|\tilde{\mathsf{z}}_{\text{o}}}(x \mid \tilde{z}_{\text{o}}) = \mathcal{CN}(x; \mu'_n, \sigma'^2_n), \tag{31}$$

where the mean and variance are, respectively, given by

$$\mu'_n = \frac{\tilde{z}_{\text{o}}}{1+\sigma^2} \quad \text{and} \quad \sigma'^2_n = \frac{\sigma^2}{1+\sigma^2}. \tag{32}$$

After substituting (27) and (29) into (32), the mean and variance of the a posteriori Gaussian distribution are given by

$$\mu'_n = \frac{\mathbf{e}_n^T \mathbf{\Sigma}_n^{-1} (\mathbf{z} - \boldsymbol{\mu}_n)}{1 + \mathbf{e}_n^T \mathbf{\Sigma}_n^{-1} \mathbf{e}_n} \quad \text{and} \quad \sigma'^2_n = \frac{1}{1 + \mathbf{e}_n^T \mathbf{\Sigma}_n^{-1} \mathbf{e}_n}. \tag{33}$$

Finally, the posterior $p_n(a)$ is updated by integrating Gaussian distribution $\mathcal{CN}(\mu'_n, \sigma'^2_n)$ over region $\mathcal{Z}(a)$ in the complex plane which is defined as the set of points which are closer in the Euclidean sense to the constellation point $x = a$ than to any other constellation point from $\mathcal{X}$. Formally, we have

$$p_n(a) = \int_{\mathcal{Z}(a)} \tilde{p}_{\mathsf{x}_n|\tilde{\mathsf{z}}_{\text{o}}}(x \mid \tilde{z}_{\text{o}}) dx, \tag{34}$$

where

$$\mathcal{Z}(a) = \{z \mid \|z - a\| < \|z - a'\|, a' \in \mathcal{X}\}. \tag{35}$$

For BPSK modulated symbols where $\mathcal{X} = \{+1, -1\}$, $\mathcal{Z}(1) = \{z \in \mathcal{R} \mid z > 0\}$ and thus the posterior $p_n(1)$ is updated with the probability that a random variable distributed according to (31) is positive. Hence,

$$p_n(x_n = 1) = \mathbb{P}\left[\mathcal{N}(\mu'_n, \sigma'^2_n) > 0\right] = \frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{\mu'_n}{\sqrt{2\sigma^{2\prime}_n}}\right), \tag{36}$$

where the error function is defined as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \tag{37}$$

A pseudo-code description of how the G-PDA algorithm with a single Gaussian approximation updates and evaluates the posteriors of BPSK symbols is given in Algorithm 2.

Algorithm 2 G-PDA with single Gaussian approximation

Require: received signal $\mathbf{z}$
Ensure: $p_1, p_2, \ldots, p_N$
  Initialize $p_1^{(0)}, \ldots, p_N^{(0)}$
  for $i = 1, 2, \ldots, I$ **do**
    for $n = 1$ to N **do**
      $\boldsymbol{\mu}_n = \sum_{k \neq n} (2p_k^{(i-1)} - 1)\mathbf{e}_k$
      $\mathbf{\Sigma}_n = \sum_{k \neq n} 4p_k^{(i-1)}(1 - p_k^{(i-1)})\mathbf{e}_k \mathbf{e}_k^T$
      $\mu'_n = \left(1 + \mathbf{e}_n^T \mathbf{\Sigma}_n^{-1} \mathbf{e}_n\right)^{-1} \mathbf{e}_n^T \mathbf{\Sigma}_n^{-1} (\mathbf{z} - \boldsymbol{\mu}_n)$
      $\sigma'^2_n = \left(1 + \mathbf{e}_n^T \mathbf{\Sigma}_n^{-1} \mathbf{e}_n\right)^{-1}$
      $p_n^{(i)} = 0.5 + 0.5\text{erf}\left(\frac{\mu'_n}{\sqrt{2\sigma^{2\prime}_n}}\right)$
    end for
  end for

2) Modeling x_n with a Gaussian Mixture: In this version of the G-PDA, we assume the prior on x_n is a Gaussian mixture whose components have means taking values from the alphabet $\mathcal{X}$, the same variance σ_0^2 and equal weights. Formally,

$$\tilde{p}_{\mathsf{x}_n}(x) = \frac{1}{|\mathcal{X}|} \sum_{a \in \mathcal{X}} \mathcal{CN}(x; a, \sigma_0^2), \tag{38}$$

where σ_0^2 is a parameter that can be tuned. The posterior of x_n is using the Bayes' rule determined to be a Gaussian mixture.

As an example, we consider the BPSK modulation where $\mathcal{X} = \{+1, -1\}$. The distribution of x_n conditioned on $\tilde{z}_o$ (and thus on $\mathbf{z}$) is using the Bayes' rule given by

$$\tilde{p}_{x_n|z}(x|\mathbf{z}) = w_1 \mathcal{CN}(x; \mu_1, \sigma'^2) + w_2 \mathcal{CN}(x; \mu_2, \sigma'^2), \quad (39)$$

where

$$\mu_{1,2} = \frac{\tilde{z}_o \sigma_0^2 \pm \sigma^2}{\sigma^2 + \sigma_0^2} \quad \text{and} \quad \sigma'^2 = \frac{\sigma^2 \sigma_0^2}{\sigma^2 + \sigma_0^2}, \quad (40)$$

while the weights are

$$w_{1,2} \propto \exp\left(-\frac{(\tilde{z}_o \mp 1)^2}{2(\sigma^2 + \sigma_0^2)}\right) \quad \text{where } w_1 + w_2 = 1. \quad (41)$$

The version of the G-PDA with Gaussian mixture which does not update the weights w_1 and w_2 i.e., keeps them equal is referred to as G-PDA with partial update.

The posterior $p_n(a)$, $a \in \mathcal{X}$ is updated by integrating the Gaussian mixture $\tilde{p}_{x_n|z}(x|\mathbf{z})$ as in (34) over region $\mathcal{Z}(a)$, defined in (35). For the BPSK modulation, we obtain

$$p_n(1) = 0.5\left(1 + w_1 \mathrm{erf}\left(\frac{\mu_1}{\sqrt{2\sigma'^2}}\right) + w_2 \mathrm{erf}\left(\frac{\mu_2}{\sqrt{2\sigma'^2}}\right)\right). \quad (42)$$

A pseudo-code description of how the G-PDA algorithm with a Gaussian mixture updates and evaluates the posteriors of BPSK symbols is given in Algorithm 3.

Algorithm 3 G-PDA with a Gaussian mixture

Require: received signal $\mathbf{z}$, variance σ_0^2
Ensure: $p_1, p_2, \ldots, p_N$
  Initialize $p_1^{(0)}, \ldots, p_N^{(0)}$
  for $i = 1, 2, \ldots, I$ **do**
    for $n = 1$ to N **do**
      $\boldsymbol{\mu}_n = \sum_{k \neq n} (2p_k^{(i-1)} - 1)\mathbf{e}_k$
      $\mathbf{\Sigma}_n = \sum_{k \neq n} 4p_k^{(i-1)}(1 - p_k^{(i-1)})\mathbf{e}_k \mathbf{e}_k^T$
      $\tilde{z}_o = \left(\mathbf{e}_n^T \mathbf{\Sigma}_n^{-1} \mathbf{e}_n\right)^{-1} \mathbf{e}_n^T \mathbf{\Sigma}_n^{-1} (\mathbf{z} - \boldsymbol{\mu}_n)$
      $\sigma^2 = \left(\mathbf{e}_n^T \mathbf{\Sigma}_n^{-1} \mathbf{e}_n\right)^{-1}$; $\mu_{1,2} = \frac{\tilde{z}_o \sigma_0^2 \pm \sigma^2}{\sigma^2 + \sigma_0^2}$; $\sigma'^2 = \frac{\sigma^2 \sigma_0^2}{\sigma^2 + \sigma_0^2}$
      $w_{1,2} \propto \exp\left(-\frac{(\tilde{z}_o \mp 1)^2}{2(\sigma^2 + \sigma_0^2)}\right)$, where $w_1 + w_2 = 1$
      $p_n^{(i)} = 0.5\left(1 + w_1 \mathrm{erf}\left(\frac{\mu_1}{\sqrt{2\sigma'^2}}\right) + w_2 \mathrm{erf}\left(\frac{\mu_2}{\sqrt{2\sigma'^2}}\right)\right)$
    end for
  end for

Once the posterior p_n of a symbol x_n is updated, the SU-PDA and G-PDA update the posterior of the next symbol from some ordering scheme and after a certain number of iterations or after it has converged, the algorithm outputs the final estimates of the posteriors and hard estimates of the transmitted symbols obtained using the MAP rule (13).

V. Validation Results

This section presents the performance results of the SU-PDA and G-PDA algorithms. The first part of this section presents simulation results on fast fading channel. The second part presents the results obtained from simulating the proposed algorithms using the channel impulse responses measured in an indoor commercial setting.

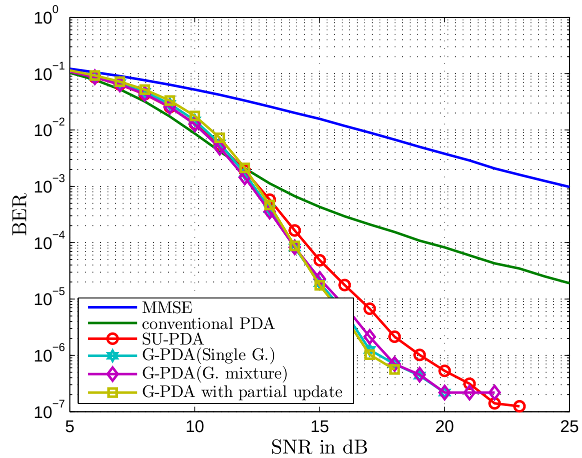

Fig. 2. Block size $N = 16$, QPSK modulation, DFT precoder, five-tap channel with fast fading, ten iterations. G-PDA with partial update is G-PDA with Gaussian mixture and no update on mixture weights.

A. Simulation on Fast Fading Channel

The BER performance of the communication system which uses the proposed algorithms for symbol detection is evaluated using the Monte-Carlo simulations.

Equally likely quadrature phase shift keying (QPSK) symbols are grouped into blocks of size N and precoded with the discrete Fourier transform (DFT) precoder, whose

$$[\mathbf{P}]_{n,m} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}(n-1)(m-1)}, \quad n, m = 1, 2, \ldots, N. \quad (43)$$

The obtained symbols are transmitted over a channel modeled as a linear time-varying filter which has five taps. The coefficients corresponding to a single tap exhibit fast fading such that $f_d t_s = 1$, where f_d is the Doppler and t_s is the symbol duration.

We assume the receiver perfectly tracks the channel and detects the transmitted symbols using the proposed algorithms. For reference, we also consider the MMSE receiver and the conventional PDA algorithm.

The BER performance of the described communication system employing the MMSE, PDA, SU-PDA, G-PDA with single Gaussian approximation, G-PDA with Gaussian mixture and G-PDA with partial update detection is shown in Fig. 2 for $N = 16$ and Fig. 3 for $N = 32$. The BER performance versus SNR for block size $N = 64$ and MMSE, original PDA and SU-PDA receivers is shown in Fig. 4.

As can be seen from the presented performance plots, the conventional PDA suffers from the error floor. On the contrary, the symbol detectors based on the PDA modifications do not exhibit an error floor in the region of considered SNR's and the performance improvement is significant at SNR's corresponding to the BER's of our interest. On the other hand, at relatively lower SNR's the original PDA has slightly better performance and the crossover point appears between 11-14 dB and moves towards higher SNR's as N increases.

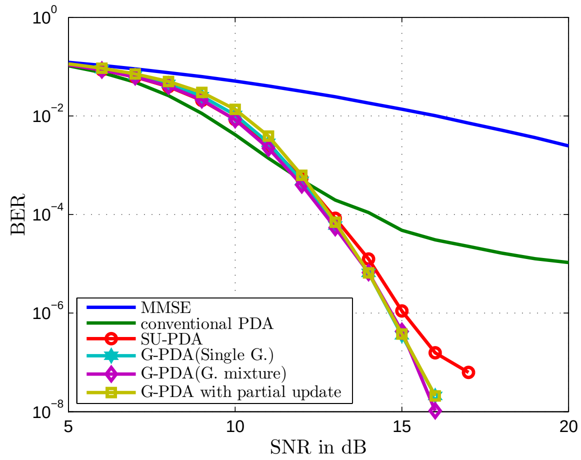

Fig. 3. Block size $N = 32$, QPSK modulation, DFT precoder, five-tap channel with fast fading, ten iterations. G-PDA with partial update is G-PDA with Gaussian mixture and no update on mixture weights.

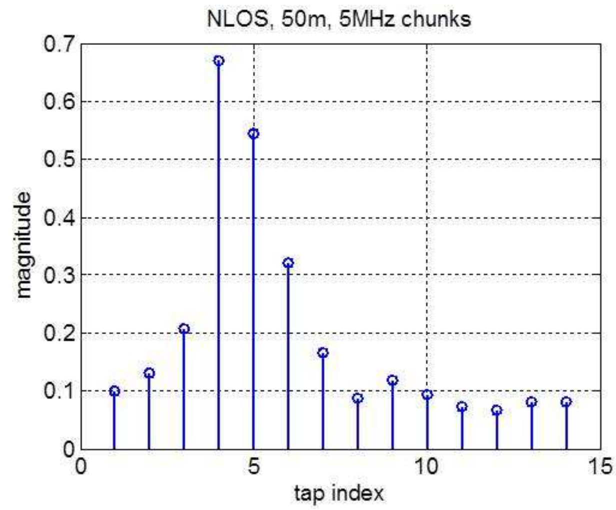

Fig. 5. One realization of a measured channel impulse response.

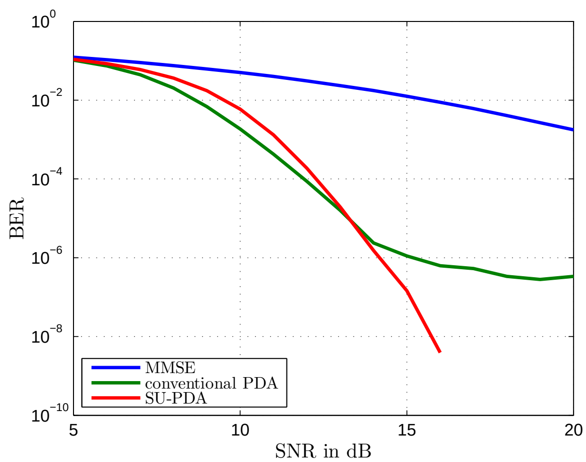

Fig. 4. Block size $N = 64$, QPSK modulation, DFT precoder, five-tap channel with fast fading, ten iterations.

The comparison between different PDA modifications reveals that all versions of the G-PDA outperform the SU-PDA for $N = 16$ and $N = 32$. Among the G-PDA versions, the one which uses the Gaussian mixture approximation of distribution of the symbol being detected has a slightly better performance than the one with a single Gaussian approximation.

B. Simulation on Experimentally Measured Channel

The BER performance of the PDA algorithm and proposed modifications SU-PDA and G-PDA when used to detect the symbols received from the real, experimentally measured channel is presented in this part.

The channel transfer function is experimentally measured in an indoor environment. The realizations of the channel impulse response at 2.4 GHz band and of 5 MHz bandwidth are calculated from the channel transfer functions. One such realization corresponding to the non-line of sight (NLOS) channel with transmitter-receiver range of 50 m is shown in Fig. 5. For simulation results presented in this section we had access to total of $\sim$ 700 measured power delay profile (PDP) realizations, and we obtained additional channel impulse responses by employing random phase rotations to the measured PDP tap coefficients.

The obtained realizations of the channel impulse response are used in computer simulations of the M2M communication system. Equally likely QPSK symbols are generated, grouped into blocks of size N and precoded. The blocks of symbols are transmitted through a channel with the impulse response being one of the experimentally obtained channel impulse responses. A response is normalized to 1 such that the channel gain is 1 and hence the evaluated BER performance corresponds to the received SNR. Since the considered block sizes are relatively short, we assume the channel is time-invariant during the transmission of a single block.

The BER versus received SNR performance of the receivers based on the MMSE, PDA, SU-PDA, G-PDA with a single Gaussian, G-PDA with a Gaussian mixture and G-PDA with partial update (i.e. weights in the mixture are not updated) and $\sigma_0^2 = 1$, is shown in Fig. 6 for block size $N = 16$ and in Fig. 7 for block size $N = 32$.

Two types of precoders are considered in this study: the DFT precoder and identity precoder. Since the channel is assumed invariant during the transmission of a block, precoding does not help in achieving the diversity. In addition, it destroys the diversity offered by the multi-tap (frequency selective) channel. On the other hand, if precoding is skipped (formally, identity precoder is used), the PDA algorithm and its modifications extract the diversity offered by the channel. This can be seen from the performance plots.

The SU-PDA and G-PDA behave worse than the original PDA at relatively small SNR. However, the proposed algorithms start outperforming the PDA at around 12 dB and the performance gain increases as SNR increases. Thus, at

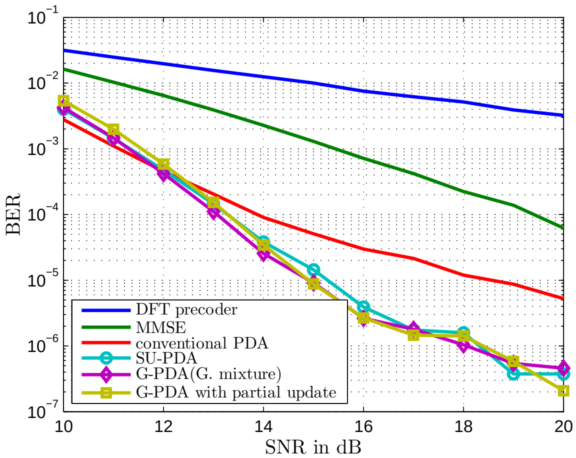

Fig. 6. Block size $N = 16$, QPSK modulation, measured channel, ten iterations. Remark: all algorithms yield the same performance with DFT precoder. All other plots correspond to identity precoder.

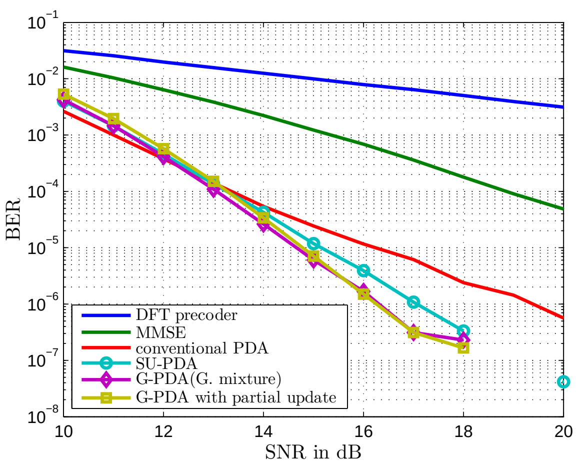

Fig. 7. Block size $N = 32$, QPSK modulation, measured channel, ten iterations. Remark: all algorithms yield the same performance with DFT precoder. All other plots correspond to identity precoder.

SNR of 16dB and above, the SU-PDA and G-PDA attain at least one order of magnitude lower BER compared to the original PDA. More importantly, this happens at SNR's which correspond to BER's of the interest for M2M modem. Among the PDA modifications, the G-PDA with Gaussian mixture approximation behaves slightly better than the SU-PDA.

VI. Conclusions

PDA algorithm has shown promising performance in symbol detection and interference cancellation in a variety of communication techniques. One such application is symbol detection in a SISO system with block transmission and detection. This technique is envisioned to provide fast and highly reliable transmission of short messages over wireless channels. These requirements constitute design goal for the M2M communication system.

This paper proposes new algorithms based on PDA which outperform the conventional PDA algorithm for short blocks ($<$ 100 symbols) and at SNR's corresponding to BER's required for M2M communications. The proposed algorithms differ from the original PDA in the way they treat the symbol being estimated. As such, the PDA-SU models such a symbol as a sum of a discrete deterministic variable and zero mean noise which captures the uncertainty in our knowledge about that symbol. On the other hand, the G-PDA models this symbol as a random variable distributed either as a single Gaussian or Gaussian mixture.

The proposed algorithms have been tested via computer simulations using both simulated and real channel. The simulated channel is multi-tap with fast fading, while for real channel we use the channel impulse responses that were experimentally measured. The tests on both channel types have shown that the proposed algorithms outperform the conventional PDA.

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