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Mouhacine Benosman

***Abstract*— In this brief, we study the problem of rope sway dynamics control for elevator systems. We choose to actuate the system with a force actuator pulling on the compensation sheave. Under these conditions, we formulate this problem as a bilinear control problem and propose several nonlinear controllers based on Lyapunov theory to stabilize the rope sway dynamics, for different elevator operation conditions. We present a stability analysis of the proposed controllers, and illustrate their performance via numerical tests.**

***Index Terms*— Disturbance rejection, elevator system, Lyapunov control, nonlinear time-varying dynamics, rope sway control.**

I. Introduction

THE GROWING demand for high-rise buildings motivates the recent interest in the problem of rope sway control, which is very important to maintain a high safety level of elevator systems. Indeed, even slight external disturbances on the building, e.g., wind gust or earthquake, at such dimensions of structures can lead to large rope sway within the elevator shaft. Considering the length of the ropes and their heavy weight, it is clear that the rope sway can damage the equipments that are installed in the elevator shaft and can also cause damages to the elevator shaft structure itself, not to mention the potential danger caused for the elevator passengers. For these reasons, it is very important to be able to control the rope dynamics within the elevator shaft. Furthermore, due to cost constraints, it is preferable to be able to do so with minimum actuation capabilities. Several papers have been dedicated to the problem of controlling elevator ropes [1]–[5]. Otsuki *et al.* [4] study the rope sway control problem for high-rise building with an actuator mounted on the top of the building, i.e., an actuated capstan, this choice of the actuator placement led to a linear time-varying model, to which an linear quadratic regulator controller was applied to reduce the rope sway. A boundary optimal control based on a partial differential equation (PDE) model of a moving string was introduced in [5]. However, this approach necessitates to be able to actuate the boundary conditions of the string, which is of limited practical value for elevator systems, due to the impracticality of mounting an actuator at the boundary points, e.g., on the top of the elevator car. In [3], a simple model of a cable attached to an actuator at its free end was used to investigate the stiffening effect of the control force on the cable. A numerical energy analysis was used to tune an open-loop sinusoidal force applied to the cable, no feedback controller was proposed. An active stiffness control of the transverse vibrations of elevator ropes was presented in [1]. Kaczmarczyk [1] used a nonlinear modal feedback to drive an actuator pulling on one end of the rope. The control performance was investigated by numerical tests, no stability analysis was reported. Zhu and Chen [2] used a passive damper attached between the car and the rope to realize a boundary control. Numerical analysis of the transverse motion average energy was conducted to find the optimal value of the damper coefficient, which reduced the rope sway, but no analytic analysis of the controller performance was provided; furthermore, as mentioned above, placing a passive or an active controller on the top of elevator cars is of limited industrial feasibility.

We choose in this brief to use an active actuator to pull on one-side of the ropes, similar to [1]. This choice of actuator placement is more feasible than other placement location and is optimal in terms of installation and maintenance costs. We then propose to investigate the problem of elevators' rope sway mitigation as a nonlinear control problem, which leads to a constructive design of the controllers and their stability analysis. The main difference with [1] is that we use Lyapunov theory to derive the nonlinear controllers with rigorous stability analysis and we explicitly consider the external disturbances in the controller design and the stability analysis. We show that with our choice of actuator placement, the model of the elevator rope together with its actuator writes as a bilinear model (in the control theory sense), and we use this bilinear model to develop nonlinear Lyapunov-based feedback controllers to stabilize the rope sway dynamics, for different operating conditions of the elevator system. We present a stability analysis of the closed-loop dynamics and show the performances of these controllers via numerical tests.

This brief is organized as follows. We start this brief with some notations in Section II. In Section III, we recall the model of the system. Next, in Section IV, we present the main results of this brief, namely the nonlinear Lyapunov-based controllers, together with their stability analysis. Section V is dedicated to some numerical results. Finally, we conclude this brief with a brief summary of the results in Section VI.

II. Notations

Throughout this brief, $\mathbb{R}$ and $\mathbb{R}_+$ denote the set of real and nonnegative real numbers, respectively. For $x \in \mathbb{R}^N$, we define $|x| = \sqrt{x^T x}$, we denote by $A_{ij},\ i = 1, \ldots, n,\ j = 1, \ldots, m$ the elements of the matrix A.

III. Elevator Rope Modeling

In this section, we first introduce the infinite dimension model, i.e., PDE, of a moving hoist cable, with

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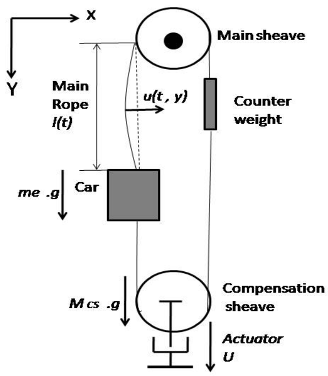


Fig. 1. Schematic representation of an elevator shaft showing different variables used in the model.

nonhomogenous boundary conditions. Second, to be able to reduce the PDE model to an open dynamics engine (ODE) model using a Galerkin reduction method, we introduce a change of variables and rewrite the first PDE model in a new coordinates, where the new PDE model has zero boundary conditions. Let us first enumerate the assumptions under which our model is valid: 1) the elevator ropes are modeled within the framework of string theory; 2) the elevator car is modeled as a point mass; 3) the vibration in the second lateral direction is not included; and 4) the suspension of the car against its guide rails is assumed to be rigid. Under the previous assumption, following [6], the general PDE model of an elevator rope, shown in Fig. 1, is given by

$$\rho\left(\frac{\partial^2}{\partial t^2}+v^2(t)\frac{\partial^2}{\partial y^2}+2v(t)\frac{\partial}{\partial y\partial t}+a\frac{\partial}{\partial y}\right)u(y,t)$$
$$-\frac{\partial}{\partial y}T(y,t)\frac{\partial u(y,t)}{\partial y}+c_p\left(\frac{\partial}{\partial t}+v(t)\frac{\partial}{\partial y}\right)u(y,t)=0 \quad (1)$$

where $u(y,t)$ is the lateral displacement of the rope. ρ is the mass of the rope per unit length. T is the tension in the rope, which varies depending on which rope in the elevator system we are modeling, i.e., main rope, compensation rope, and so on. c_p is the damping coefficient of the rope per unit length. $v=\partial l(t)/\partial t$ is the elevator rope velocity, where $l:\ \mathbb{R}\to\mathbb{R}$ is a function (at least C^2) modeling the time-varying rope length. $a=\partial^2 l(t)/\partial t^2$ is the elevator rope acceleration.

The PDE (1) is associated with the following two boundary conditions:

$$u(0,t)=f_1(t)\quad u(l(t),t)=f_2(t) \quad (2)$$

where $f_1(t)$ is the time-varying disturbance acting on the rope at the level of the machine room, due to external disturbances, e.g., wind gust. $f_2(t)$ is the time-varying disturbance acting at the level of the car, due to external disturbances. In this brief, we assume that the two boundary disturbances acting on the rope are related via the relation

$$f_2(t)=f_1(t)\sin\left(\frac{\pi(H-l)}{2H}\right),\quad H\in\mathbb{R} \quad (3)$$

where H is the height of the building. This expression is an approximation of the propagation of the boundary disturbance f_1 along the building structure, based on the length l, it leads to $f_2=f_1$ for a length zero (which is expected), and a decreasing force along the building until it vanishes at $l=H$, $f_2=0$ (which makes sense, since the effect of any disturbance f_1, for example, wind gusts, is expected to vanish at the bottom of the building). However, other equations can be used to relate f_1 and f_2 along the building structure, without affecting the results of this brief. As we mentioned earlier, the tension of the rope $T(y)$ depends on the type of the rope that we are dealing with. In the sequel, we concentrate on the main rope of the elevator, the remaining ropes are modeled using the same steps by simply changing the rope tension expression.

For the case of the main rope, the tension is given by

$$T(y,t)=(m_e+\rho(l(t)-y))(g-a(t))+0.5M_{\text{cs}}g+U(t) \quad (4)$$

where g is the standard gravity constant, m_e and M_{cs} are the mass of the car and the compensating sheave, respectively, and $U(t)$ is the control tension due to the actuator attached to the compensation sheave (the same actuator placement has been considered in [1]). Next, we reduce the PDE model (1) to a more tractable model for control, using a projection Galerkin method or assumed mode approach, see [7].

To be able to apply the assumed mode approach, let us first apply the following one-to-one change of coordinates to (1):[1]

$$u(y,t)=w(y,t)+\frac{l(t)-y}{l(t)}f_1(t)+\frac{y}{l(t)}f_2(t). \quad (5)$$

One can easily observe that this change of coordinates implies trivial boundary conditions.

After some algebraic and integral manipulations, the PDE model (1) writes in the new coordinates as

$$\rho\frac{\partial^2 w}{\partial t^2}+2v(t)\rho\frac{\partial^2 w}{\partial y\partial t}+\left(\rho v^2-T(y,t)\right)\frac{\partial^2 w}{\partial y^2}+G(t)\frac{\partial w}{\partial y}+c_p\frac{\partial w}{\partial t}$$
$$=y\left(-\rho s_1(t)-c_p s_2(t)\right)-\rho f_1^{(2)}+s_4(t) \quad (6)$$

where $G(t)=\rho a(t)-\partial T/\partial y+c_p v(t)$, and the s_i variables are defined as

$$\begin{aligned}
s_1(t)&=\frac{l l^{(2)}-2\dot{l}^2}{l^3}f_1(t)+2\frac{\dot{l}}{l^2}\dot{f}_1+\frac{(l^3 f_2^{(2)}-f_2 l^2 l^{(2)}+2l\dot{l}^2 f_2-2l^2\dot{l}\dot{f}_2)}{l^4}-\frac{f_1^{(2)}}{l}\\
s_2(t)&=\frac{\dot{l}}{l^2}f_1-\frac{\dot{f}_1}{l}+\frac{\dot{f}_2}{l}-f_2\frac{\dot{l}}{l^2}\\
s_3(t)&=\frac{f_2-f_1}{l}\\
s_4(t)&=-2v(t)\rho s_2(t)-G(t)s_3(t)-c_p\dot{f}_1(t)
\end{aligned} \quad (7)$$

associated with the two-point boundary conditions

$$w(0,t)=0,\quad w(l(t),t)=0. \quad (8)$$

Now, instead of dealing with the PDE (1) with nonzero boundary conditions, we can use the equivalent model, given by (6) associated with trivial boundary conditions (8).

[1]This change of coordinates is needed to write to original PDE model as an equivalent PDE with homogenous boundary conditions. This change of coordinates is well known in boundary value problems [8], but to the best of our knowledge, it has not been proposed in previous work on elevator ropes modeling.

Following the assumed-mode technique, the solution of (6) and (8) writes as:

$$w(y,t)=\sum_{j=1}^{j=N} q_j(t)\phi_j(y,t),\ N\in\mathbb{N} \tag{9}$$

where N is the number of bases (modes), included in the discretization, $\phi_j,\ j=1,\ldots,N$ are the discretization bases, and $q_j,\ j=1,\ldots,N$ are the discretization coordinates. To simplify the analytic manipulation of the equations, the base functions are chosen to satisfy the following normalization constraints: $\int_0^{l(t)}\phi_j^2(y,t)dy=1$, $\int_0^{l(t)}\phi_i(y,t)\phi_j(y,t)dy=0,\ \forall i\neq j$. To further simplify the base functions, we define the normalized variable, (see [2], [6]) $\xi(t)=y(t)/l(t)$ and the normalized base functions $\phi_j(y,t)=\psi_j(\xi)/\sqrt{l(t)},\ j=1,\ldots,N$. In these new coordinates, the previous normalization constraints write as: $\int_0^1\psi_j^2(\xi)d\xi=1,\ \int_0^1\psi_i(\xi)\psi_j(\xi)d\xi=0,\ \forall i\neq j$. After discretization (see [6]) of the PDE-based model (6), (8), we can write the reduced ODE-model based on N-modes as

$$M\ddot{q}+C\dot{q}+(K+\beta U)q=F(t),\ q\in\mathbb{R}^N,\ F\in\mathbb{R}^N \tag{10}$$

where

$$\begin{aligned}
M_{ij}&=\rho\delta_{ij}\\
C_{ij}&=\rho l^{-1}\dot{l}\left(2\int_0^1(1-\xi)\psi_i(\xi)\psi_j'(\xi)d\xi-\delta_{ij}\right)+c_p\delta_{ij}\\
K_{ij}&=\frac{1}{4}\rho l^{-2}\dot{l}^2\delta_{ij}-\rho l^{-2}\dot{l}^2\int_0^1(1-\xi)^2\psi_i'(\xi)\psi_j'(\xi)d\xi\\
&\quad+\rho l^{-1}(g-a(t))\int_0^1(1-\xi)\psi_i'(\xi)\psi_j'(\xi)d\xi+m_e l^{-2}\\
&\quad\times(g-a(t))\int_0^1\psi_i'(\xi)\psi_j'(\xi)d\xi+\rho(l^{-2}\dot{l}^2-l^{-1}\ddot{l})\\
&\quad\times\left(0.5\delta_{ij}-\int_0^1(1-\xi)\psi_i(\xi)\psi_j'(\xi)d\xi\right)\\
&\quad-c_p\dot{l}l^{-1}\left(\int_0^1\psi_i(\xi)\psi_j'(\xi)\xi d\xi+0.5\delta_{ij}\right)+0.5M_{cs}gl^{-2}\\
&\quad\times\int_0^1\psi_i'(\xi)\psi_j'(\xi)d\xi\\
\beta_{ii}&=l^{-2}\int_0^1\psi_i'^2d\xi=l^{-2}\tilde{\beta}_{ii}\\
\beta_{ij}&=\tilde{\beta}_{ij}=0\quad\forall\ i\neq j\\
F_i(t)&=-l\sqrt{l}\left(\rho s_1(t)+c_p s_2(t)\right)\int_0^1\psi_i(\xi)\xi d\xi\\
&\quad+\sqrt{l}\left(s_4(t)-\rho f_1^{(2)}(t)\right)\int_0^1\psi_i(\xi)d\xi\\
\delta_{ij}&=\begin{cases}0, & i\neq j\\ 1, & i=j\end{cases}
\end{aligned} \tag{11}$$

where $s_i,\ i=1,2,3,4$ are given in (7).

If we use the classical definition of the state vector $x=(q,\dot{q})^T$, then it is easy to see that the obtained ODE model is a bilinear model in the state x and the control vector U.

IV. Main Result: Lyapunov-Based Controllers

In this section, we present several Lyapunov-based feedback controllers, each one tailored for a specific practical situation, and designed to stabilize the rope sway dynamics.

The first controller deals with the case where the building, hosting a static elevator, e.g., night operation of commercial buildings, sustains a brief (impulse-like) external disturbance. For example, an earthquake impulse with a sufficient force to make the top of the building oscillate, or a strong wind gust that happens over a short period, exciting the building structure and implying residual vibrations of the building even after the wind gust interruption. In these cases, the elevator ropes will vibrate, starting from nonzero initial conditions, due to the impulse-like external disturbances (i.e., happening over a short time interval), and this case corresponds to the model (10), (11) with nonzero initial conditions and zero external disturbances.

Theorem 1: Consider the rope dynamics (10), (11), with nonzero initial conditions, with no external disturbances, i.e., $f_1(t)=f_2(t)=0,\forall t$, and with constant length l, then the feedback control

$$U_{\text{nom}-1}(x)=\text{Max}\left(0,u_{\max}\frac{\dot{q}^T\tilde{\beta}q}{\sqrt{1+(\dot{q}^T\tilde{\beta}q)^2}}\right) \tag{12}$$

where $x=(q^T,\dot{q}^T)^T$ renders the closed-loop equilibrium point $(0,0)$ globally asymptotically stable, with $|U_{\text{nom}-1}|\leq u_{\max}$; furthermore, $|U_{\text{nom}-1}|$ decreases with the decrease of $\dot{q}^T\tilde{\beta}q$.

Proof: We define the control Lyapunov function as

$$V(z)=\frac{1}{2}\dot{q}^T(t)M\dot{q}(t)+\frac{1}{2}q^T(t)Kq(t) \tag{13}$$

where $x=(q^T,\dot{q}^T)^T$.

First, we compute the derivative of the Lyapunov function along the dynamics (10), without disturbances, i.e., $F(t)=0,\ \forall t$

$$\begin{aligned}\dot{V}(z)&=\dot{q}^T(-C\dot{q}-Kq-\beta Uq)+q^TK\dot{q}\\&=-\dot{q}^TC\dot{q}-\dot{q}^T\beta qU.\end{aligned} \tag{14}$$

To ensure the negative definiteness of $\dot{V}(x)$, we define the first controller (12). Using the continuity of (12) at $\dot{q}^T\tilde{\beta}q=0$ and LaSalle theorem, see [9], we can conclude that the states of the closed-loop dynamics converge to the set $S=\{z=(q^T,\dot{q}^T)^T\in\mathbb{R}^{2N},\ \text{s.t.}\ \dot{q}=0\}$. Next, we analyze the closed-loop dynamics. Since the boundedness of V implies boundedness of $\dot{q},\ q$ and by (10), boundedness of $\ddot{q}$. Boundedness of $\dot{q},\ \ddot{q}$ implies the uniform continuity of $q,\ \dot{q}$, which again by (10), implies the uniform continuity of $\ddot{q}$. Next, since $\dot{q}\to 0$, using Barbalat's Lemma, see [9], we conclude that $\ddot{q}\to 0$, and by invertibility of the stiffness matrix $K+\beta U$, we conclude that $q\to 0$. Finally, the fact that V is a radially unbounded function ensures that the equilibrium point $(q,\dot{q})=(0,0)$ is globally asymptotically stable. Furthermore, the fact that $|U_{\text{nom}-1}|\leq u_{\max}$, and the decrease of $|U_{\text{nom}-1}|$ with the decrease of $\dot{q}^T\tilde{\beta}q$ is deduced from (12). ■

Remark 1: By examining the Lyapunov derivative (14), we can see that instead of the C^0 controller (12), we could use a smooth controller of the form

$$U_{\text{nom}-1}(x) = u_{\max}\frac{\dot{q}^T\tilde{\beta}q}{\sqrt{1+(\dot{q}^T\tilde{\beta}q)^2}}.$$

However, the advantage of the switching controller (12) is the fact that it necessitates less control energy, since when the condition $\dot{q}^T\tilde{\beta}q \le 0$ is satisfied, it does not apply any extra control and only uses the system's natural damping.

Next, we present a controller which deals with the case of static elevator in a building under sustained external disturbances, e.g., sustained wind forces. In practical applications, we seldom have access to direct measurements of the disturbance signal $F(t)$; to overcome this problem, we use the so-called Lyapunov reconstruction technique, see [10], to augment the nominal controller $U_{\text{nom}-1}$ with an additional feedback term, which is based only on an upper bound of the disturbance signal $F(t)$ (i.e., does not require the exact measurements of $F(t)$) and ensures the stabilization of the sway to a small amplitude, which can be tuned by the choice of the feedback gains.

First, let us state the following assumption.

Assumption 1: The time-varying disturbance functions f_1, f_2 are such that, the function $F(t)$ is bounded, i.e., $\exists F_{\max},$ s.t. $|F(t)| \le F_{\max},\ \forall t$.

Theorem 2: Consider the rope dynamics (10), (11), under nonzero external disturbances, i.e., $f_1(t) \neq 0,\ f_2(t) \neq 0$, and with constant length l, then under Assumption 1, the feedback control

$$U(x) = U_{\text{nom}-1}(x) + k(\dot{q}^T\tilde{\beta}q)(F_{\max}+\epsilon)|\dot{q}|,\ k>0,\ \epsilon>0 \tag{15}$$

where $x = (q^T, \dot{q}^T)^T$, ensures that the solutions of (10), (11), and (15) converge to the invariant set $\tilde{S} = \{(q^T,\dot{q}^T)^T \in \mathbb{R}^{2N},\ \text{s.t. } kl^{-2}(\dot{q}^T\tilde{\beta}q)^2 \le 1\}$.

Proof: Using the same Lyapunov function (13), and writing its derivative along (10)

$$\dot{V}(x) = -\dot{q}^TCq - \dot{q}^T\beta qU(x) + \dot{q}^TF(t) \tag{16}$$

if we denote $U(x) = U_{\text{nom}-1}(x) + v(x)$, where $v(x) = k(\dot{q}^T\tilde{\beta}q)(F_{\max}+\epsilon)|\dot{q}|,\ k>0,\ \epsilon>0$, we obtain

$$\dot{V}(x) = -\dot{q}^TC\dot{q} - \dot{q}^T\beta qU_{\text{nom}-1}(x) - \dot{q}^T\beta qv(x) + \dot{q}^TF(t) \tag{17}$$

and by the definition of $U_{\text{nom}-1}(x)$, we know that

$$-\dot{q}^TC\dot{q} - \dot{q}^T\beta qU_{\text{nom}-1}(x) < 0$$

thus, using Assumption 1, we can write

$$\begin{aligned}\dot{V}(x) &\le -\dot{q}^T\beta qv(x) + \dot{q}^TF(t)\\ &\le -kl^{-2}(\dot{q}^T\tilde{\beta}q)^2(F_{\max}+\epsilon)|\dot{q}| + |\dot{q}||F(t)|\\ &\le -kl^{-2}(\dot{q}^T\tilde{\beta}q)^2(F_{\max}+\epsilon)|\dot{q}| + |\dot{q}|F_{\max}\\ &\le -kl^{-2}(\dot{q}^T\tilde{\beta}q)^2|\dot{q}|\epsilon + |\dot{q}|F_{\max}(1-kl^{-2}(\dot{q}^T\tilde{\beta}q)^2)\\ &\le +|\dot{q}|F_{\max}(1-kl^{-2}(\dot{q}^T\tilde{\beta}q)^2)\end{aligned}$$

which proves the decrease of $V(x)$ until reaching the invariant set

$$\tilde{S} = \{(q^T,\dot{q}^T)^T \in \mathbb{R}^{2N},\ \text{s.t. } kl^{-2}(\dot{q}^T\tilde{\beta}q)^2 \le 1\}.$$

■

Remark 2: It can be deduced from the proof of Theorem 2, using similar reasoning as in the proof of Theorem 1, that controller (15) implies $q \to 0$ in the case of zero external disturbances, i.e., the case treated in Theorem 1. However, the controller (15) has an extra term, i.e., the Lyapunov reconstruction term $v(x)$, needed to compensate for the effect of $F(t)$. This term implies extra control effort, which is not necessary in the case of zero external disturbances. Thus, to avoid using unnecessary control effort in real application, one can switch between the two controllers, based on the detection of a sustained external force (case of Theorem 2), or an impulse-like disturbance force (case of Theorem 1).

In the previous theorems, we have consider the case of static elevators, e.g., night operation of commercial buildings. We analyze now the case of moving elevators, i.e., with time-varying rope length $l(t)$. This case encompass the situations where the elevator is in motion and an external disturbance starts acting on the building. In such situations, the controller goal is to minimize the effect of this disturbance on the rope sway amplitude, to avoid damaging the elevator shaft and ensures the passengers security while the elevator is moving. Similar to the case of static elevators, we study two scenarios: 1) impulse-like and 2) sustained external disturbances.

To deal with the analysis of this case, we need to add the following assumption.

Assumption 2: The time-varying length function $l : \mathbb{R}_+ \to \mathbb{R}_+$ is $\mathbb{C}^2$, and satisfies: $l(t) \in [l_{\min},\ l_{\max}]$, $\dot{l}(t) \in [0,\ \dot{l}_{\max}]$, $\ddot{l}(t) \in [0,\ \ddot{l}_{\max}]$, $\forall t \in \mathbb{R}_+$, where $l_{\min},\ l_{\max},\ \dot{l}_{\max},\ \ddot{l}_{\max}$ are given constants.

We can now state the following theorem.

Theorem 3: Consider the rope dynamics (10), (11), with nonzero initial conditions, with no external disturbances, i.e., $f_1(t) = f_2(t) = 0, \forall t$, and with time-varying length l satisfying Assumption 2, then the feedback control

$$U_{\text{nom}-2}(x) = u_{\max}\frac{\dot{q}^T\tilde{\beta}q|q|^2}{\sqrt{1+(\dot{q}^T\tilde{\beta}q)^2}} \tag{18}$$

where $x = (q^T, \dot{q}^T)^T$ implies that $q(t) \to 0,\ for\ t \to 0$; furthermore, $|U_{\text{nom}-2}|$ decreases with the decrease of $|q|^2$.

Proof: We define the time-varying control Lyapunov function as

$$V(x,t) = \frac{1}{2}\dot{q}^T(t)M\dot{q}(t) + \frac{1}{2}q^T(t)K(t)q(t) \tag{19}$$

where $x = (q^T, \dot{q}^T)^T$.

First, we compute the derivative of the Lyapunov function along the dynamics (10), without disturbances, i.e., $F(t) = 0,\ \forall t$

$$\begin{aligned}\dot{V}(x,t) &= \dot{q}^T(-C\dot{q} - Kq - \beta Uq) + q^TK\dot{q} + q^T\dot{K}(t)q\\ &= -\dot{q}^TC\dot{q} - \dot{q}^T\beta qU + q^T\dot{K}(t)q.\end{aligned} \tag{20}$$

Next, based on Assumption 2, we can write

$$\exists c > 0,\ \text{s.t. } \dot{K}(t) \leq c I_{n\times n} \quad \forall t$$

which leads to

$$\dot{V}(x,t) \leq -\dot{q}^T \beta q U + \frac{1}{2} c |q|^2 \tag{21}$$

using U defined in (18), we have

$$\begin{aligned}\dot{V}(x,t) &\leq -u_{\max} l^{-2} \frac{(\dot{q}^T\tilde{\beta}q)^2|q|^2}{\sqrt{1+(\dot{q}^T\tilde{\beta}q)^2}} + \frac{1}{2}c|q|^2 \\ &\leq \left(\frac{1}{2}c - u_{\max} l^{-2}\frac{(\dot{q}^T\tilde{\beta}q)^2}{\sqrt{1+(\dot{q}^T\tilde{\beta}q)^2}}\right)|q|^2 \end{aligned}\tag{22}$$

this shows that $\dot{V}(x,t)$ decreases along (10), as long as $(q,\dot{q})$ satisfies $1/2c - u_{\max} l^{-2}(\dot{q}^T\tilde{\beta}q)^2/\sqrt{1+(\dot{q}^T\tilde{\beta}q)^2} < 0$, and when $(q,\dot{q})$ enters the set $\{(q^T,\dot{q}^T)^T \in \mathbb{R}^{2N},\ \text{s.t. } 1/2c - u_{\max} l^{-2}(\dot{q}^T\tilde{\beta}q)^2/\sqrt{1+(\dot{q}^T\tilde{\beta}q)^2} \geq 0\}$, it stays in it, which makes $V(x,t)$ bounded.

Next, from (22), we can write

$$\dot{V}(x,t) \leq \tfrac{1}{2} q^T c q \tag{23}$$

thus

$$\begin{aligned} &+\infty > V(x(0),0) - V(x(t),t) \geq -\frac{1}{2}\int_0^t q^T c q dt \\ &\Rightarrow \int_0^t q^T \text{cqdt is bounded.} \end{aligned}\tag{24}$$

Since the boundedness of V implies boundedness of $\dot{q}$, we conclude about the uniform continuity of q and finally using Barbalat's Lemma, see [9], we conclude that $\lim_{t\to+\infty} q(t) = 0$. The fact that $|U_{\text{nom}-2}|$ decreases with the decrease of $|q|^2$ is concluded from the upper bound

$$|U_{\text{nom}-2}(t)| \leq u_{\max}|q|^2.$$

■

Let us examine now the case of nonzero disturbances, i.e., $F(t) \neq 0$ over a nonzero time interval.

Theorem 4: Consider the rope dynamics (10), (11), under nonzero external disturbances, i.e., $f_1(t) \neq 0,\ f_2(t) \neq 0$ satisfying Assumption 1, and with time-varying length l satisfying Assumption 2, then the feedback control

$$\begin{aligned} U(x) = &\frac{u_{\max}\dot{q}^T\tilde{\beta}q}{\sqrt{1+(\dot{q}^T\tilde{\beta}q)^2}} + k_1(\dot{q}^T\tilde{\beta}q)(F_{\max}+\epsilon)|\dot{q}| \\ &+ k_2(\dot{q}^T\tilde{\beta}q)|q|^2, \quad k_1>0, \quad k_2>0,\ \epsilon>0 \end{aligned}\tag{25}$$

where $x = (q^T,\dot{q}^T)^T$ ensures that the state vector x converges to the ω-limit set $S_1 = \{(q^T,\dot{q}^T)^T \in \mathbb{R}^{2N},$ such that $c/2 - k_2 l^{-2}(\dot{q}^T\tilde{\beta}q)^2 \to 0\}$ or the invariant set $\{(q^T,\dot{q}^T)^T \in \mathbb{R}^{2N},$ s.t. $l^{-2}(\dot{q}^T\tilde{\beta}q)^2 \leq 1/k_1\}$ if $(c/2k_2) > (1/k_1)$, and converges to the ω-limit set $S_2 = \{(q^T,\dot{q}^T)^T \in \mathbb{R}^{2N},$ s.t. $1 - k_1 l^{-2}(\dot{q}^T\tilde{\beta}q)^2 \to 0\}$ or the invariant set $\{(q^T,\dot{q}^T)^T \in \mathbb{R}^{2N},$ s.t. $l^{-2}(\dot{q}^T\tilde{\beta}q)^2 \leq (c/2k_2)\}$ if $(c/2k_2) \leq (1/k_1)$, where c is such that $\dot{K}(t) < c I_{n\times n},\ \forall t$.

Proof: Let us consider again the time-varying Lyapunov function (19). Its derivative along the dynamics (10), with nonzero disturbance $F(t)$, writes as

$$\begin{aligned}\dot{V}(x,t) &= \dot{q}^T(-C\dot{q} - Kq - \beta U q) + q^T K\dot{q} + q^T\dot{K}(t)q + \dot{q}^T F(t) \\ &= -\dot{q}^T C\dot{q} - \dot{q}^T\beta q U + q^T\dot{K}(t)q + \dot{q}^T F(t)\end{aligned}\tag{26}$$

under Assumption (2), we can write

$$\dot{V}(x,t) \leq -\dot{q}^T\beta q U + \dot{q}^T F(t) + \frac{1}{2}cq^2$$

which under Assumption 1 gives

$$\leq -\dot{q}^T\beta q U + |\dot{q}|F_{\max} + \frac{1}{2}cq^2.$$

Substituting U by the controller (25) leads to

$$\begin{aligned} &\leq -k_1 l^{-2}(\dot{q}^T\tilde{\beta}q|)^2\epsilon|\dot{q}| + |\dot{q}|F_{\max}(1 - k_1 l^{-2}(\dot{q}^T\tilde{\beta}q)^2) \\ &\quad + |q^2|(\frac{c}{2} - k_2 l^{-2}(\dot{q}^T\tilde{\beta}q)^2) \\ &\leq |\dot{q}|F_{\max}(1 - k_1 l^{-2}(\dot{q}^T\tilde{\beta}q)^2) + |q^2|\left(\frac{c}{2} - k_2 l^{-2}(\dot{q}^T\tilde{\beta}q)^2\right). \end{aligned}$$

Case 1: $(c/2k_2) > (1/k_1)$. In this case, as long as $(\dot{q}^T\tilde{\beta}q)^2 > (c/2k_2) > (1/k_1)$, then $\dot{V} < 0$, which makes x decreasing until it enters the invariant set $\{(q^T,\dot{q}^T)^T \in \mathbb{R}^{2N}$, such that $l^{-2}(\dot{q}^T\tilde{\beta}q)^2 \leq (c/2k_2)\}$, which makes $V(x,t)$ bounded. Here, we have to distinguish two cases as follows.

1) The trajectories keep decreasing until they reach the invariant set

$$\{(q^T,\dot{q}^T)^T \in \mathbb{R}^{2N},\ \text{s.t. } l^{-2}(\dot{q}^T\tilde{\beta}q)^2 \leq (1/k_1)\}.$$

2) The trajectories are stuck in the domain, where $(\dot{q}^T\tilde{\beta}q)^2 \leq (c/2k_2 l^{-2})$ and $(\dot{q}^T\tilde{\beta}q)^2 > (1/k_1 l^{-2})$. Since in this set we have $|\dot{q}|F_{\max}(1 - k_1 l^{-2}(\dot{q}^T\tilde{\beta}q)^2) \leq 0$, we can write

$$\dot{V}(x,t) \leq |q^2|(\frac{c}{2} - k_2 l^{-2}(\dot{q}^T\tilde{\beta}q)^2)$$

which together with the boundedness of V gives

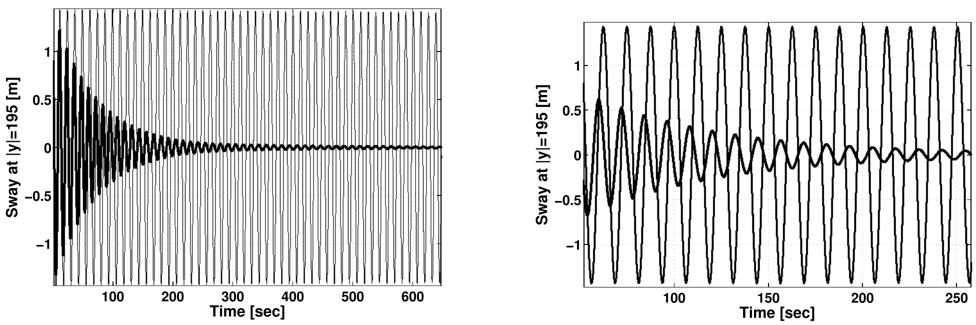
$$\begin{aligned} &+\infty > V(x(0),0) - V(x(t),t) \\ &\geq -\int_0^t q^2(t)(\frac{c}{2} - k_2 l^{-2}(\dot{q}(t)^T\tilde{\beta}q(t))^2)\text{dt} \\ &\Rightarrow \int_0^t q^2(t)(\frac{c}{2} - k_2 l^{-2}(\dot{q}(t)^T\tilde{\beta}q(t))^2)\text{dt} \ \text{ is bounded.} \end{aligned}$$

Now, due to the boundedness of V, we conclude about the boundedness of $q,\ \dot{q}$; furthermore, using Assumption 2 and the system equations (10), we conclude about the boundedness of $\ddot{q}$. Boundedness of $\dot{q}$ and $\ddot{q}$ implies that $q^2(t)((c/2) - k_2 l^{-2}(\dot{q}(t)^T\tilde{\beta}q(t))^2)$ is uniform continuous. Finally, using Barbalat's Lemma, we conclude that $\lim_{t\to\infty} q^2(t)((c/2) - k_2 l^{-2}(\dot{q}(t)^T\tilde{\beta}q(t))^2) = 0$. Next, by examining the system equations (10), we can conclude that $\lim_{t\to\infty} q^2(t) = 0$ cannot be a solution of (10), since there is no assumption on $F(t)$ converging to zero when $t \to 0$. Thus, we finally conclude that in this second case, the solution $q,\dot{q}$ satisfies $\lim_{t\to\infty}(c/2) - k_2 l^{-2}(\dot{q}(t)^T\tilde{\beta}q(t))^2 = 0$.

Case 2: $(c/2k_2) \leq (1/k_1)$: Following the same reasoning as in case 1, we can conclude that the solution $q,\dot{q}$ either converges to the invariant set $\{(q^T,\dot{q}^T)^T \in$

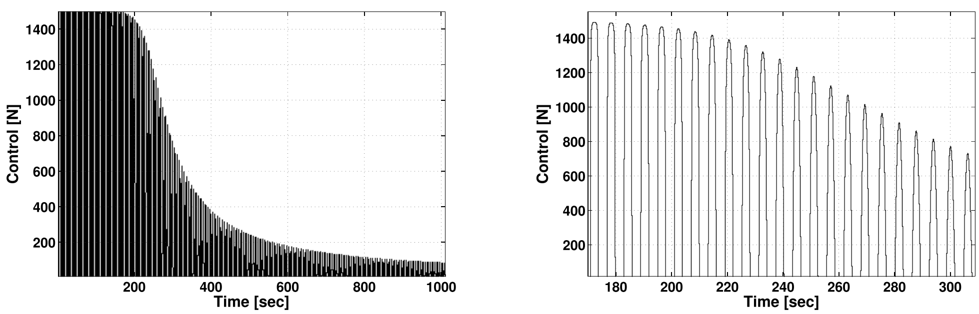
TABLE I
NUMERICAL VALUES OF THE MECHANICAL PARAMETERS

Parameter's symbols	Parameter's definitions	Parameter's values
n	Number of ropes	$8\ [-]$
m_e	Mass of the car	$3500\ [kg]$
ρ	Main rope linear mass density	$2.11\ [kg/m]$
l	Rope maximum length	$390\ [m]$
H	Building height	$402.8\ [m]$
c_p	Damping coefficient	$0.0315\ [N.sec/m]$



(a) (b)

Fig. 2. Rope sway at $y = 195$ m in the case of nonzero initial conditions and zero external disturbance. (a) Rope sway at $y = 195$ m: no control (thin line) and with control (12) (bold line). (b) Zoom of the rope sway at $y = 195$ m: no control (thin line) and with control (12) (bold line).



(a) (b)

Fig. 3. Output of controller (12) in the case of nonzero initial conditions and zero external disturbance. (a) Output of controller (12). (b) Output of controller (12) zoomed-in view.

$\mathbb{R}^{2N}$, s.t. $l^{-2}(\dot{q}^T\tilde{\beta}q)^2 \leq (c/2k_2)\}$ or satisfies $\lim_{t\to\infty} 1 - k_1 l^{-2}(\dot{q}^T\tilde{\beta}q)^2 = 0$. ∎

Remark 3: Similarly to Remark 2, we can point out here that the controller (25) can deal with the case treated by the controller (18); however, it does necessitate more control power. We can make the same observation regarding the controllers (18), (25) proposed for the case of time-varying length versus the controllers (15) and (18) for the case of constant length. It is easy to see from the proofs of Theorems 3 and 4 that the controllers (18) and (25) stabilize the rope sway in the case of constant rope length, as well. They require, however, an extra control term [due to the time varying matrix $K(t)$], which is not needed in the case of constant rope length. Thus, depending on the practical application, i.e., stationary versus moving elevators and actuator power availability, one can choose one controller versus the other, or consider switching between the different controllers.

Remark 4: The controllers (12), (15), (18), and (25) are state feedbacks based on q, $\dot{q}$, these states can be easily computed from the sway measurements at N given positions $y(1), \ldots, y(N)$, via (9). The sway $w(y,t)$ can be measured by laser displacement sensors placed at the positions $y(i)$, $i = 1, 2, \ldots, N$, along the rope, see [4]; subsequently, q can be computed by simple algebraic inversion of (9), and $\dot{q}$ can be obtained by direct numerical differentiation of q.

V. NUMERICAL EXAMPLE

In this section, we present some numerical results obtained on the system presented in [1]. The case of an elevator system with the mechanical parameters summarized in Table I has been considered for the tests presented hereafter. We underline that in the following we write the controllers based on the model (10), (11) with one mode, but we test them a model

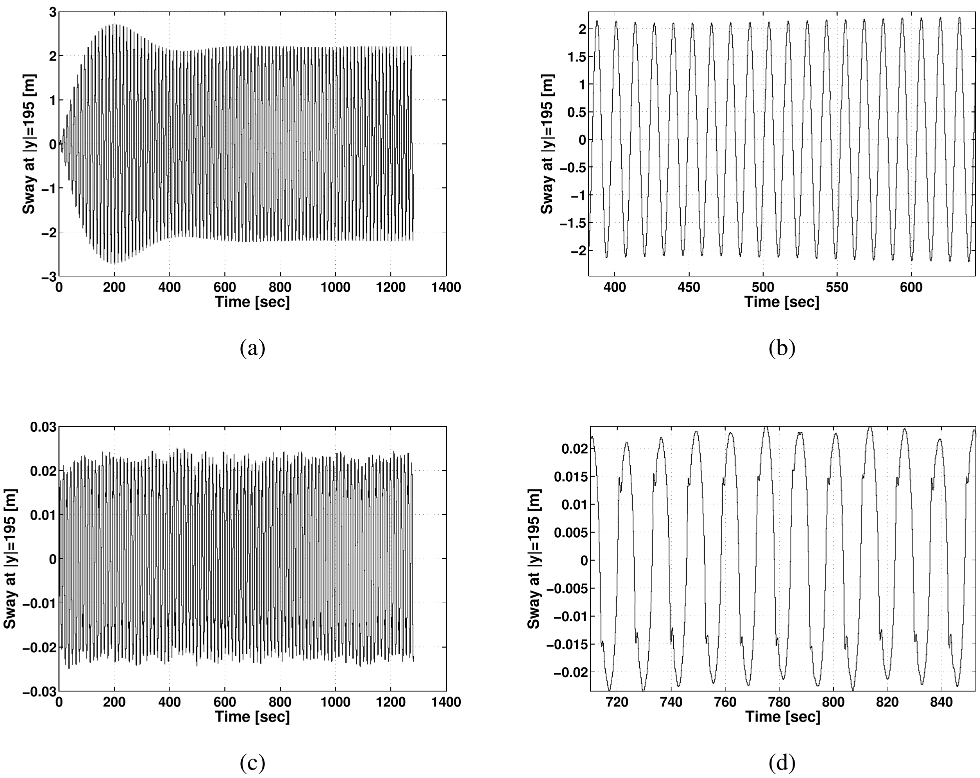


Fig. 4. Rope sway at $y = 195$ m in the case of nonzero external disturbance. (a) Rope sway at $y = 195$ m without control. (b) Zoomed-in-view of rope sway at $y = 195$ m without control. (c) Rope sway at $y = 195$ m with control (15). (d) Zoomed-in view of the rope sway at $y = 195$ m with control (15).

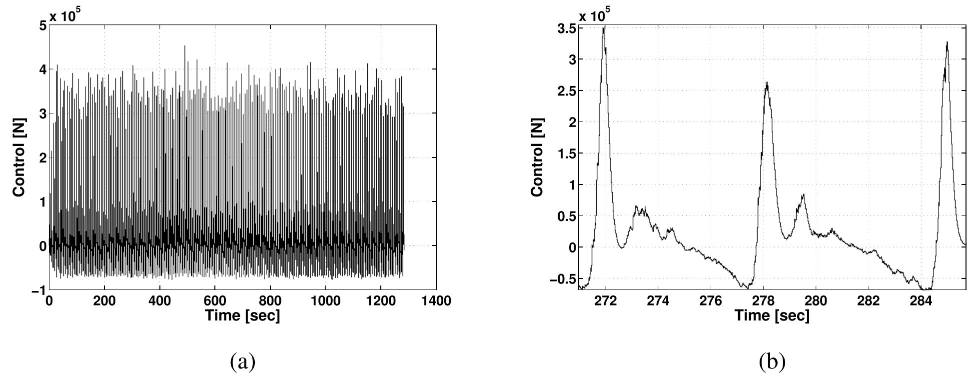


Fig. 5. Output of controller (15) in the case of nonzero external disturbance. (a) Output of controller (15). (b) Output of controller (15) zoomed-in view.

with three modes. The fact is that one mode is enough since when comparing the solution of the PDE (6) with the discrete model (10), the higher modes shown to be negligible, and a discrete model with one mode showed a very good match with the PDE model, but to make the simulation tests more realistic, we chose to test the controllers on a three-mode model. Furthermore, to make the simulation tests more challenging, we added a random white noise to the states fed back to the controller (equivalent to about ± 1 cm of error on the rope sway measurement from which the states are computed, see Remark 4), and we filtered the control signal with a first-order filter with a cut frequency of 10 hz and a delay term of five sampling times, to simulate actuator dynamics and delays to signal transmission and computation time.

First, to validate Theorem 1, we present the results obtained by applying the controller (12), to the model (10), (11), with

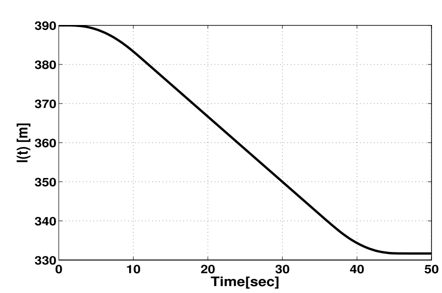
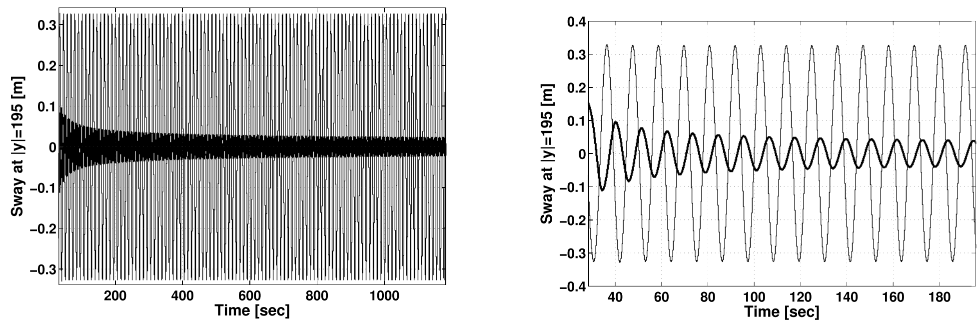


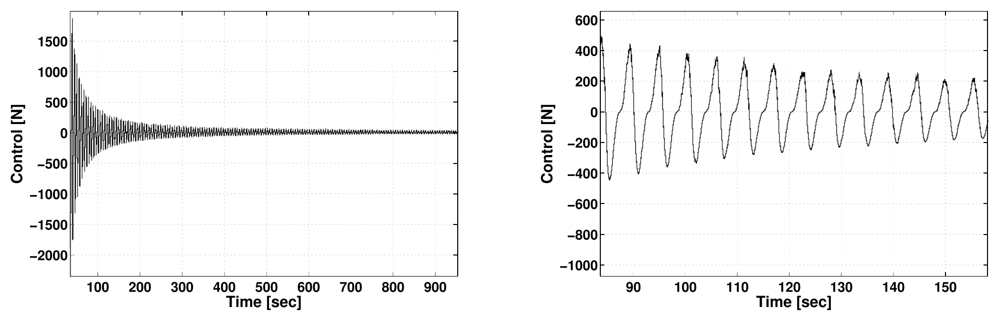
Fig. 6. Time-varying rope length.

nonzero initial conditions $q(0) = 20$, $\dot{q}(0) = 0$, and zero external disturbances, i.e., $f_1(t) = f_2(t) = 0$, $\forall t$. In these first tests, to show the effect of the controller (12) alone, without the help of the system's natural damping, we fix the damping coefficient to zero, i.e., $c_p = 0$. Fig. 2(a) and (b)



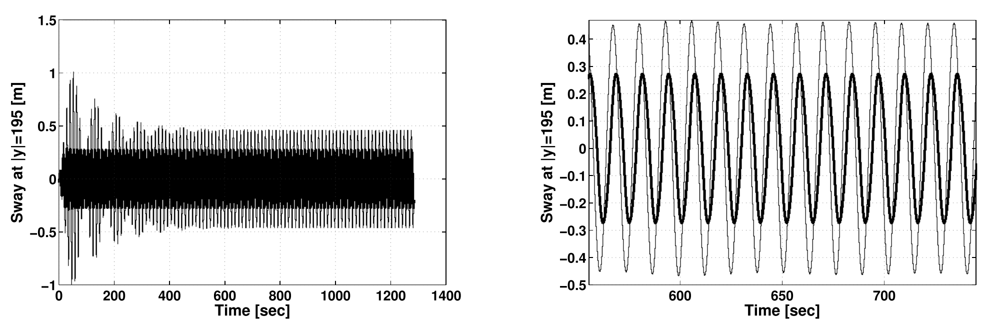
(a) (b)

Fig. 7. Rope sway at $y = 195$ m in the case of nonzero initial conditions, without external disturbance and time-varying rope length. (a) Rope sway at $y = 195$ m: no control (thin line) and with control (18) (bold line). (b) Zoomed-in-view of rope sway at $y = 195$ m: no control (thin line) and with control (18) (bold line).



(a) (b)

Fig. 8. Output of controller (18) in the case of nonzero initial conditions, without external disturbance and time-varying rope length. (a) Output of controller (18). (b) Output of controller (18) zoomed-in view.



(a) (b)

Fig. 9. Rope sway at $y = 195$ m with external disturbance and time-varying rope length: no control (thin line) and with control (bold line). (a) Rope sway at $y = 195$ m: no control (thin line) and with control (25) (bold line). (b) Zoomed-in-view of rope sway at $y = 195$ m: no control (thin line) and with control (25) (bold line).

(thin line) shows the rope sway obtained at half rope length $y = 195$ m without control. It reaches a maximum value of about 1.45 m. We show on Fig. 2(a) and (b) (bold line) the rope sway obtained at the same rope length but this time with the controller (12), with $u_{\max} = 1500$ N. We see the expected effect of the controller on the sway, which is reduced by half in about 60 s and vanishes asymptotically. The corresponding control force is shown in Fig. 3(a) and (b). We see that, as expected from the theoretical analysis of Theorem 1, the control force remains bounded by $u_{\max}$ and decreases with the decrease of the sway. The zoom of the control signal reported in Fig. 3(b) shows that the control is C^0 continuous.

Let us consider now the controller (15) introduced in Theorem 2. We consider the model (10), (11) with nonzero disturbance signals: $f_1(t) = 0.2\sin(2\pi.0.08t)$, and f_2 being deduced form f_1 via (3). We have purposely selected the disturbance frequency to be equal to the first resonance frequency of the rope, to simulate the worst case scenario. We apply (15), with the parameters $u_{\max} = 1500$ N, $F_{\max} = 1.6$, $\epsilon = 0.1$, and $k = 10^6$. The effect of the control on the rope sway amplitude is shown in Fig. 4. The rope sway is effectively reduced. The control force is shown in Fig. 5, which shows some noise (because of the feedback noise) and a high amplitude, due to the selected high gain value for k. We underline here that, in

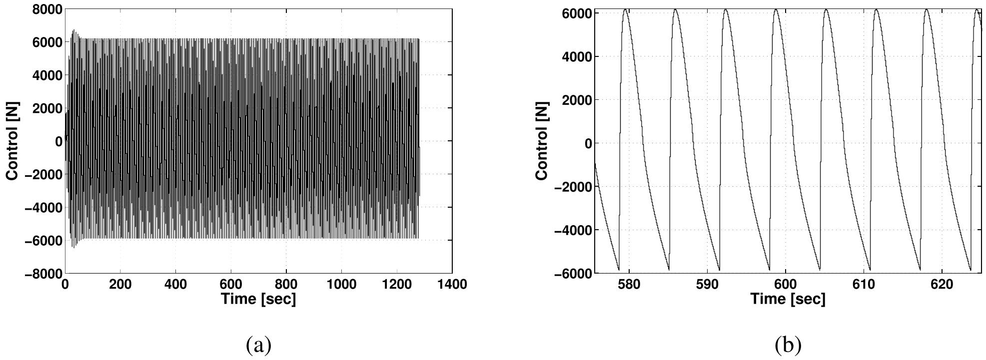


Fig. 10. Output of controller (25) in the case of nonzero external disturbance and time-varying rope length. (a) Output of controller (25). (b) Output of controller (25) zoomed-in view.

this type of applications, due to the large masses involved, the actuators used to pull on the car are shakers with very large force load, which can easily reproduce such desired control force (e.g., electrohydraulic force shakers can generate a force of $37e4\ N$ [11]).

To end this section, we finally report the results obtained in the case of a moving car, i.e., time-varying rope length. We first start with the validation of Theorem 3. The controller (18) has been implemented with $u_{\max} = 1500\ N$, starting the simulation with nonzero initial condition $q(0) = 4,\ \dot{q}(0) = 1$, and with zero external disturbances $f_1(t) = f_2(t) = 0$. We also fixed the damping coefficient c_p to zero, to see the damping effect of the controller alone. Following [2], the tested time-varying rope length is shown in Fig. 6. The sway signal is shown in Fig. 7, where both the controlled and the uncontrolled sway signals are reported. The corresponding control signal is shown in Fig. 8. These numerical results are in concordance with the asymptotic convergence results of Theorem 3.

Eventually, we report the numerical results corresponding to Theorem 4. We tested controller (25), with the gains: $u_{\max} = 150,\ F_{\max} = 1,\ k_1 = 3000,\ k_2 = 15, and\ \epsilon = 0.1$. Fig. 9 shows the sway without control versus the sway with control at half-rope length. The effect of the controller (25) is clear, i.e., the maximum sway in transient phase is reduced from 0.8 to about 0.3 m, and the steady-state sway is reduced by half. The corresponding continuous control signal is shown in Fig. 10.

VI. Conclusion

In this brief, we have studied the problem of active control of elevator rope sway dynamics occurring due to external force disturbances acting on the elevator system. We have selected one actuation configuration, namely a force actuator placed at the bottom of the elevator shaft pulling on the compensation sheave. For the selected actuation configuration, we have proposed several nonlinear controllers based on Lyapunov theory. The proposed controllers deal with several elevator system operating conditions. We have presented the stability analysis of these controllers and shown their efficiency using numerical tests. The numerical results reported here show a very good performance of the proposed controllers when applied to a force actuator pulling on the ropes via the compensation sheave. However, other actuation methods might be feasible; therefore, one future research direction is to compare on the same test case the performance and feasibility of different controllers designed for different actuation configurations.

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