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Cognitive MIMO Relaying with Multiple Primary Transceivers

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***Abstract*—We examine the impact of clusters of primary transceivers in cognitive multiple-input multiple-output (MIMO) relay networks with underlay spectrum sharing. In such a network, we propose antenna selection as an interference-aware design to satisfy the power constraints in the primary and secondary networks. To demonstrate this, we consider transmit antenna selection with maximal ratio combining (TAS/MRC) in the primary and secondary networks. With this in mind, we derive new closed-form asymptotic expressions for the outage probability and the symbol error rate (SER) over independent Nakagami-m fading channels. Our results lead to several new fundamental insights. In particular, we highlight that TAS/MRC achieves a full diversity gain when the maximum transmit power in the secondary network is proportional to the peak interference temperature in the primary network.**

I. Introduction

Cognitive spectrum sharing in cooperative networks is a revolutionary paradigm to support the proliferation of wireless devices and combat spectrum scarcity [1]. From a power perspective, cognitive spectrum sharing with network cooperation addresses fundamental constraints on the transmit power at the secondary users (SUs), while keeping the interference temperature at the primary users (PUs) to a minimum [2]. Against this background, how to manage the transmit power relative to the interference temperature in cognitive spectrum sharing with multiple antennas at the primary and secondary users remains an open question.

A common approach to cognitive spectrum sharing is the underlay model in which the SUs are permitted to transmit over the same spectrum as the PUs [3]. In underlay spectrum sharing, the transmit power at the SUs must be managed under a peak interference temperature to guarantee reliable communication between the PUs. Under this paradigm, majority of works in the literature have considered single antennas at the primary and secondary users. Among them, the exact outage probability of cognitive spectrum sharing was derived in [4] for decode-and-forward (DF) relaying in Nakagami-m fading. Considering amplify-and-forward (AF) relaying in Rayleigh fading, the exact outage probability was examined in [5]. In [6], AF relaying was found to achieve a full diversity gain in Nakagami-m fading when the transmit power at the SU transmitters (SU-Tx) is managed according to the peak interference temperature of a single PU receiver (PU-Rx). More recently in [7], the impact of multiple PU-Rx on the outage probability was addressed. To do so, the transmit power at the SU transmitters (SU-Tx) was adapted to the maximum interference constraint arising from the multiple PU-Rx. In [8], [9], the impact of interference from PU transmitters (PU-Tx) on the SU receivers (SU-Rx) was considered. In [10], several relay selection and power allocation constraints were proposed.

In this paper, different from [4]–[10], we investigate cognitive spectrum sharing from the viewpoint of multiple-input multiple-output (MIMO) in the primary and secondary networks. In the primary network, we consider a cluster of L PU-Tx transmitting to a cluster of L PU-Rx with N_{Q} antennas at each of the PUs. In the secondary network, we assume N_{S}, N_{R}, and N_{D} antennas at the source, relay, and destination, respectively. To harness the diversity benefits of MIMO, we consider transmit antenna selection with receive maximal-ratio combining (TAS/MRC) in both the primary and secondary networks. In TAS/MRC, a single antenna at the transmitter is selected for transmission, and all the antennas at the receiver are combined using MRC. In this setting, we seek to address the impact of three cognitive design parameters, namely, 1) maximum transmit power at the SU-Tx, $\mathcal{P}_{\text{T}}$, 2) peak interference temperature at the PU-Rx, $\mathcal{Q}$, and 3) interference power at the SU-Rx due to the PU-Tx, $\mathcal{P}_{\text{I}}$. In doing so, we derive new closed-form expressions for the outage probability with Nakagami-m fading in the primary and secondary networks. Based on this, we derive the symbol error rate (SER) under the higher order constellations of M-ary quadrature amplitude modulation (M-QAM) and M-ary phase-shift keying (M-PSK). Our results are given in terms of concise asymptotic expressions which accurately characterize the network performance when $\mathcal{P}_{\text{T}}$ grows large.

II. Network and Channel Description

We consider a cognitive MIMO relay network as illustrated in Fig. 1. The primary network comprises L PU-Tx transmitting to L PU-Rx, each equipped with N_{Q} antennas.

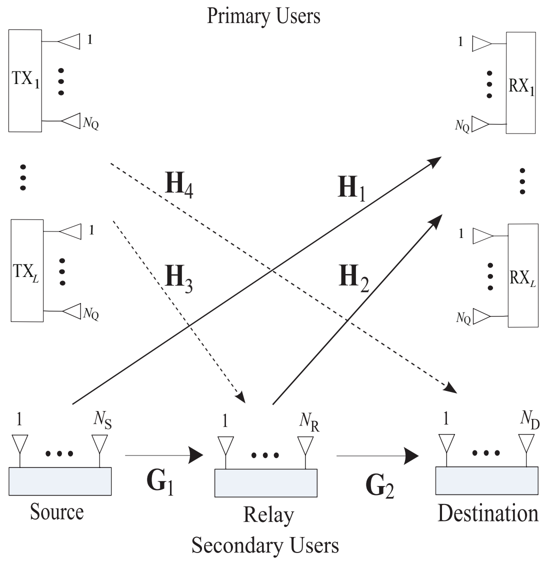

Fig. 1. Cognitive spectrum sharing with MIMO in the primary and secondary networks.

The secondary network consists of a secondary source (S), a secondary relay (R), and a secondary destination (D), equipped with N_{S}, N_{R}, and N_{D} antennas, respectively.

In the secondary network, $\mathbf{G}_1$ denotes the $N_{\mathrm{S}} \times N_{\mathrm{R}}$ channel for the $\mathsf{S} \rightarrow \mathsf{R}$ link, and $\mathbf{G}_2$ denotes the $N_{\mathrm{R}} \times N_{\mathrm{D}}$ channel for the $\mathsf{R} \rightarrow \mathsf{D}$ link. The interference channels from the SU-Tx to the PU-Rx are represented by: 1) $\mathbf{H}_{1\ell}$ which denotes the $N_{\mathrm{S}} \times N_{\mathrm{Q}}$ channel from S to the ℓ-th PU-Rx, and 2) $\mathbf{H}_{2\ell}$ which denotes the $N_{\mathrm{R}} \times N_{\mathrm{Q}}$ channel from R to the ℓ-th PU-Rx. The interference channels from the PU-Tx to the SU-Rx are represented by: 1) $\mathbf{H}_{3\ell}$ which denotes the $N_{\mathrm{Q}} \times N_{\mathrm{R}}$ channel from the ℓ-th PU-Tx to R, and 2) $\mathbf{H}_{4\ell}$ which denotes the $N_{\mathrm{Q}} \times N_{\mathrm{D}}$ channel from the ℓ-th PU-Tx to D. The channel coefficients in $\mathbf{G}_1$, $\mathbf{G}_2$, $\mathbf{H}_{1\ell}$, $\mathbf{H}_{2\ell}$, $\mathbf{H}_{3\ell}$, and $\mathbf{H}_{4\ell}$ follow a Nakagami-m distribution with independent fading parameters m_{g_1}, m_{g_2}, m_{h_1}, m_{h_2}, m_{h_3}, m_{h_4}, and channel powers Ω_{g_1}, Ω_{g_2}, Ω_{h_1}, Ω_{h_2}, Ω_{h_3}, Ω_{h_4}, respectively. In the following, $||\cdot||$ is the Euclidean norm, $|\cdot|$ is the absolute value, and $\mathbb{E}[\cdot]$ is the expectation.

We now detail the interference-limited scenario with TAS/MRC in the primary and secondary networks[1]. In the $\mathsf{S} \rightarrow \mathsf{R}$ link, a single antenna at S that maximizes the signal-to-interference ratio (SIR) at R is selected to transmit, whereas all the antennas at R are combined using MRC. In the $\mathsf{R} \rightarrow \mathsf{D}$ link, the signal received at R is decoded and forwarded to D. In doing so, a single antenna at R that maximizes the SIR at D is selected to transmit, whereas all the antennas at D are combined using MRC. In the primary network, a single transmit antenna is selected at each PU-Tx and all the antennas at each PU-Rx are combined using MRC.

According to underlay spectrum sharing, the interference at the PU-Rx inflicted by S and R should not exceed a given maximum tolerable level, $\mathcal{Q}$. Further to satisfying $\mathcal{Q}$, S and R are power-limited terminals such that the maximum allowable transmit power is $\mathcal{P}_{\mathrm{T}}$. As such, the transmit powers at S and R are constrained as [11]

$$\mathcal{P}_{\mathrm{S}} = \min\left(\mathcal{P}_{\mathrm{T}}, \frac{\mathcal{Q}}{||\mathbf{h}_{1\ell^* i^*}||^2}\right), \tag{1}$$

and

$$\mathcal{P}_{\mathrm{R}} = \min\left(\mathcal{P}_{\mathrm{T}}, \frac{\mathcal{Q}}{||\mathbf{h}_{2\ell^* j^*}||^2}\right), \tag{2}$$

respectively. In (1), due to MRC at the PU-Rx, $||\mathbf{h}_{1\ell^* i^*}|| = \max_\ell\{||[h_{1\ell i^* 1}, \ldots, h_{1\ell i^* N_{\mathrm{Q}}}]||\}$ is the largest channel vector from S to the L PU-Rx where $h_{1\ell iq}$ are channel coefficients in $\mathbf{H}_{1\ell}$ with $\ell \in \{1, \ldots, L\}$, $i \in \{1, \ldots N_{\mathrm{S}}\}$, $q \in \{1, \ldots, N_{\mathrm{Q}}\}$. In (2), $||\mathbf{h}_{2\ell^* j^*}|| = \max_\ell\{||[h_{2\ell j^* 1}, \ldots, h_{2\ell j^* N_{\mathrm{Q}}}]||\}$ is the largest channel vector from R to the L PU-Rx where $h_{2\ell jq}$ are channel coefficients in $\mathbf{H}_{2\ell}$ with $\ell \in \{1, \ldots, L\}$, $i \in \{j, \ldots N_{\mathrm{R}}\}$, $q \in \{1, \ldots, N_{\mathrm{Q}}\}$.

Given the co-existence of transmissions in the primary and secondary networks, the received signals at S and R are impacted by interference from the PU-Tx denoted by $\mathcal{P}_{\mathrm{I}}$. As such, the instantaneous received SIR in the $\mathsf{S} \rightarrow \mathsf{R}$ link can be written as

$$\tilde{\gamma}_1 = \frac{\mathcal{P}_{\mathrm{S}}||\mathbf{g}_{1i^*}||^2}{\sum_{\ell=1}^{L} \mathcal{P}_{\mathrm{I}}||\mathbf{h}_{3\ell q^*}||^2}. \tag{3}$$

In (3), we define $||\mathbf{g}_{1i^*}|| = \max_i\{||[g_{1i1}, \ldots, g_{1iN_{\mathrm{R}}}]||\}$ as the largest channel vector from S to R where g_{1ij} are channel coefficients in $\mathbf{G}_1$ with $i \in \{1, \ldots N_{\mathrm{S}}\}$, $j \in \{1, \ldots, N_{\mathrm{R}}\}$. We also define $||\mathbf{h}_{3\ell q^*}|| = ||[h_{3\ell q^* 1}, \ldots, h_{3\ell q^* N_{\mathrm{R}}}]||$ as the channel vector from a single transmit antenna at the ℓ-th PU-Tx to R, where $h_{3\ell qj}$ are channel coefficients in $\mathbf{H}_{3\ell}$ with $q \in \{1, \ldots, N_{\mathrm{Q}}\}$ and $j \in \{1, \ldots N_{\mathrm{R}}\}$.

In the $\mathsf{R} \rightarrow \mathsf{D}$ link, the instantaneous received SIR is given by

$$\tilde{\gamma}_2 = \frac{\mathcal{P}_{\mathrm{R}}||\mathbf{g}_{2j^*}||^2}{\sum_{\ell=1}^{L} \mathcal{P}_{\mathrm{I}}||\mathbf{h}_{4\ell q^*}||^2}. \tag{4}$$

In (4), we define $||\mathbf{g}_{2j^*}|| = \max_j\{||[g_{2j1}, \ldots, g_{2jN_{\mathrm{D}}}]||\}$ as the largest channel vector from R to D where g_{2jk}, are channel coefficients in $\mathbf{G}_2$ with $j \in \{1, \ldots N_{\mathrm{R}}\}$, $k \in \{1, \ldots, N_{\mathrm{D}}\}$. We also define $||\mathbf{h}_{4\ell q^*}|| = ||[h_{4\ell q^* 1}, \ldots, h_{4\ell q^* N_{\mathrm{D}}}]||$ as the channel vector from a single transmit antenna at the ℓ-th PU-Tx to D, where $h_{4\ell qk}$ are channel coefficients in $\mathbf{H}_{4\ell}$ with $q \in \{1, \ldots, N_{\mathrm{Q}}\}$ and $k \in \{1, \ldots N_{\mathrm{D}}\}$.

Finally, the instantaneous end-to-end SIR with TAS/MRC and DF relaying is defined as [12]

$$\gamma_{\mathsf{D}} = \min(\tilde{\gamma}_1, \tilde{\gamma}_2). \tag{5}$$

III. Outage Probability

In this section, we seek to address the impacts of the maximum transmit power at the SU-Tx $\mathcal{P}_{\mathrm{T}}$, the peak interference temperature at the PU-Rx $\mathcal{Q}$, and the interference power at the SU-Rx $\mathcal{P}_{\mathrm{I}}$, on the outage probability of cognitive MIMO relaying. The cognitive network is considered to be in outage

[1]We focus on the interference-limited scenario where the interference power is dominant relative to the noise, and therefore the effects of noise is neglected.

when γ_{D} falls below a minimum threshold γ_{th}. As such, the outage probability is

$$P_{\text{out}} = \Pr\{\gamma_{\mathsf{D}} \le \gamma_{\text{th}}\} = F_{\gamma_{\mathsf{D}}}(\gamma_{\text{th}}), \tag{6}$$

where $F_{\gamma_{\mathsf{D}}}(\gamma_{\text{th}})$ is the cumulative distribution function (cdf) of γ_{D}. Based on (5), the cdf of γ_{D} is evaluated as

$$F_{\gamma_{\mathsf{D}}}(\gamma) = F_{\tilde{\gamma}_1}(\gamma) + F_{\tilde{\gamma}_2}(\gamma) - F_{\tilde{\gamma}_1}(\gamma)F_{\tilde{\gamma}_2}(\gamma), \tag{7}$$

where $F_{\tilde{\gamma}_1}(\gamma)$ is the cdf of $\tilde{\gamma}_1$ and $F_{\tilde{\gamma}_2}(\gamma)$ is the cdf of $\tilde{\gamma}_2$.

We now present the asymptotic cdf of $\tilde{\gamma}_1$ in the following theorem.

Theorem 1: The asymptotic cdf of $\tilde{\gamma}_1$ as $\mathcal{P}_{\text{T}} \to \infty$ is given by (8). Note that (8) contains easy-to-evaluate finite summations of the gamma function $\Gamma(\cdot)$ [13, eq. (8.310.1)] and the upper incomplete gamma function $\Gamma(\cdot,\cdot)$ [13, eq. (8.350.2)].

Proof: See Appendix A. ■

The asymptotic cdf of $\tilde{\gamma}_2$ follows directly from (8) by exchanging the parameters $\epsilon_1 \to \epsilon_2$, $m_{h_1} \to m_{h_2}$, $m_{h_3} \to m_{h_4}$, $m_{g_1} \to m_{g_2}$, $\Omega_{h_1} \to \Omega_{h_2}$, $\Omega_{h_3} \to \Omega_{h_4}$, $\Omega_{g_1} \to \Omega_{g_2}$, $N_{\text{S}} \to N_{\text{R}}$, and $N_{\text{R}} \to N_{\text{D}}$.

Applying the asymptotic cdf's of $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$, the asymptotic outage probability for the end-to-end SIR is presented as

$$P_{\text{out}} \overset{\mathcal{P}_{\text{T}}\to\infty}{\approx} \begin{cases} \Theta_1\left(\frac{\gamma\mathcal{P}_{\text{I}}}{\mathcal{P}_{\text{T}}}\right)^{m_{g_1}N_{\text{S}}N_{\text{R}}} + \Delta_1\left(\frac{\gamma\mathcal{P}_{\text{I}}}{\mathcal{Q}}\right)^{m_{g_1}N_{\text{S}}N_{\text{R}}}, \\ \qquad m_{g_1}N_{\text{S}}N_{\text{R}} < m_{g_2}N_{\text{R}}N_{\text{D}}, \\ \Theta_2\left(\frac{\gamma\mathcal{P}_{\text{I}}}{\mathcal{P}_{\text{T}}}\right)^{m_{g_2}N_{\text{R}}N_{\text{D}}} + \Delta_2\left(\frac{\gamma\mathcal{P}_{\text{I}}}{\mathcal{Q}}\right)^{m_{g_2}N_{\text{R}}N_{\text{D}}}, \\ \qquad m_{g_1}N_{\text{S}}N_{\text{R}} > m_{g_2}N_{\text{R}}N_{\text{D}}, \\ (\Theta_1+\Theta_2)\left(\frac{\gamma\mathcal{P}_{\text{I}}}{\mathcal{P}_{\text{T}}}\right)^{m_{g_1}N_{\text{S}}N_{\text{R}}} \\ \qquad + (\Delta_1+\Delta_2)\left(\frac{\gamma\mathcal{P}_{\text{I}}}{\mathcal{Q}}\right)^{m_{g_1}N_{\text{S}}N_{\text{R}}}, \\ \qquad m_{g_1}N_{\text{S}}N_{\text{R}} = m_{g_2}N_{\text{R}}N_{\text{D}}, \end{cases} \tag{9}$$

where

$$\Theta_1 = \frac{\epsilon_1\left(\frac{m_{g_1}\Omega_{h_3}}{\Omega_{g_1}m_{h_3}}\right)^{m_{g_1}N_{\text{S}}N_{\text{R}}}\Gamma(m_{g_1}N_{\text{S}}N_{\text{R}} + m_{h_3}N_{\text{R}}L)}{(\Gamma(m_{g_1}N_{\text{R}}+1))^{N_{\text{S}}}\Gamma(m_{h_3}N_{\text{R}}L)}, \tag{10}$$

$$\Theta_2 = \frac{\epsilon_2\left(\frac{m_{g_2}\Omega_{h_4}}{\Omega_{g_2}m_{h_4}}\right)^{m_{g_2}N_{\text{R}}N_{\text{D}}}\Gamma(m_{g_2}N_{\text{R}}N_{\text{D}} + m_{h_4}N_{\text{D}}L)}{(\Gamma(m_{g_2}N_{\text{D}}+1))^{N_{\text{R}}}\Gamma(m_{h_4}N_{\text{D}}L)}, \tag{11}$$

$$\begin{aligned}\Delta_1 &= \frac{L\left(\frac{m_{g_1}\Omega_{h_1}\Omega_{h_3}}{m_{h_1}\Omega_{g_1}m_{h_3}}\right)^{m_{g_1}N_{\text{S}}N_{\text{R}}}\Gamma(m_{g_1}N_{\text{S}}N_{\text{R}} + m_{h_3}N_{\text{R}}L)}{\Gamma(m_{h_1}N_{\text{Q}})(\Gamma(m_{g_1}N_{\text{R}}+1))^{N_{\text{S}}}\Gamma(m_{h_3}N_{\text{R}}L)} \\ &\times \sum_{l=0}^{L-1}\binom{L-1}{l}(-1)^l \prod_{r=1}^{m_{h_1}N_{\text{Q}}-1}\left[\sum_{a_r=0}^{a_{r-1}}\left(\frac{1}{r!}\right)^{a_r-a_{r+1}}\right] \\ &\times (l+1)^{-m_{g_1}N_{\text{S}}N_{\text{R}} - m_{h_1}N_{\text{Q}} - \sum_{r=1}^{m_{h_1}N_{\text{Q}}-1}a_r} \\ &\times \Gamma\left(m_{g_1}N_{\text{S}}N_{\text{R}} + m_{h_1}N_{\text{Q}} + \sum_{r=1}^{m_{h_1}N_{\text{Q}}-1}a_r, \frac{\mathcal{Q}(l+1)m_{h_1}}{\mathcal{P}_{\text{T}}\Omega_{h_1}}\right),\end{aligned} \tag{12}$$

and

$$\begin{aligned}\Delta_2 &= \frac{L\left(\frac{m_{g_2}\Omega_{h_2}\Omega_{h_4}}{m_{h_2}\Omega_{g_2}m_{h_4}}\right)^{m_{g_2}N_{\text{R}}N_{\text{D}}}\Gamma(m_{g_2}N_{\text{R}}N_{\text{D}} + m_{h_4}N_{\text{D}}L)}{\Gamma(m_{h_2}N_{\text{Q}})(\Gamma(m_{g_2}N_{\text{D}}+1))^{N_{\text{R}}}\Gamma(m_{h_4}N_{\text{D}}L)} \\ &\times \sum_{l=0}^{L-1}\binom{L-1}{l}(-1)^l \prod_{r=1}^{m_{h_2}N_{\text{Q}}-1}\left[\sum_{a_r=0}^{a_{r-1}}\left(\frac{1}{r!}\right)^{a_r-a_{r+1}}\right] \\ &\times (l+1)^{-m_{g_2}N_{\text{R}}N_{\text{D}} - m_{h_2}N_{\text{Q}} - \sum_{r=1}^{m_{h_2}N_{\text{Q}}-1}a_r} \\ &\times \Gamma\left(m_{g_2}N_{\text{R}}N_{\text{D}} + m_{h_2}N_{\text{Q}} + \sum_{r=1}^{m_{h_2}N_{\text{Q}}-1}a_r, \frac{\mathcal{Q}(l+1)m_{h_2}}{\mathcal{P}_{\text{T}}\Omega_{h_2}}\right).\end{aligned} \tag{13}$$

Our result in (9) clearly highlights that the outage probability of TAS/MRC increases with the interference power at the SU-Rx, $\mathcal{P}_{\text{I}}$. We also see that the outage probability decreases with increasing $\mathcal{P}_{\text{T}}$ and $\mathcal{Q}$.

IV. Symbol Error Rate

In this section, we derive new asymptotic expressions for the SER of TAS/MRC under the higher order constellations of M-QAM and M-PSK. In our closed-form expressions, we define the ratio of the peak interference temperature to the maximum transmit power as $\mu = \mathcal{Q}/\mathcal{P}_{\text{T}}$.

The asymptotic SER with M-QAM is evaluated using [14]

$$\begin{aligned}P_e^{\infty} &= \frac{6\left(1-\frac{1}{\sqrt{\mathcal{M}}}\right)}{\pi(\mathcal{M}-1)}\int_0^{\frac{\pi}{2}}\int_0^{\infty}\frac{e^{-\gamma\frac{a_{\text{MQAM}}}{\sin^2\theta}}}{\sin^2\theta}F_{\gamma_{\mathsf{D}}}^{\infty}(\gamma)d\gamma d\theta \\ &- \frac{6\left(1-\frac{1}{\sqrt{\mathcal{M}}}\right)^2}{\pi(\mathcal{M}-1)}\int_0^{\frac{\pi}{4}}\int_0^{\infty}\frac{e^{-\gamma\frac{a_{\text{MQAM}}}{\sin^2\theta}}}{\sin^2\theta}F_{\gamma_{\mathsf{D}}}^{\infty}(\gamma)d\gamma d\theta,\end{aligned} \tag{14}$$

where $a_{\text{MQAM}} = 3/2(\mathcal{M}-1)$, and $\mathcal{M}$ represents the constellation size.

We substitute (9) into (14) and solve the nested integrals which results in

$$\begin{aligned}P_e &\overset{\mathcal{P}_{\text{T}}\to\infty}{\approx} \Xi_{\text{TAS/MRC}} \times \frac{3\left(1-\frac{1}{\sqrt{\mathcal{M}}}\right)\mathcal{P}_{\text{I}}^q}{\mathcal{P}_{\text{T}}^q\pi(\mathcal{M}-1)a_{\text{MQAM}}^{q+1}}\left(\sqrt{\pi}\,\Gamma\left(\frac{1}{2}+q\right)\right. \\ &\left. - \left(1-\frac{1}{\sqrt{\mathcal{M}}}\right)\Gamma(q+1)B_{\frac{1}{2}}\left(\frac{1}{2}+q,\frac{1}{2}\right)\right),\end{aligned} \tag{15}$$

where $q = \min(m_{g_1}N_{\text{S}}N_{\text{R}}, m_{g_2}N_{\text{R}}N_{\text{D}})$,

$$\Xi_{\text{TAS/MRC}} = \begin{cases} \Theta_1 + \frac{\Delta_1}{\mu^{m_{g_1}N_{\text{S}}N_{\text{R}}}},\ m_{g_1}N_{\text{S}}N_{\text{R}} < m_{g_2}N_{\text{R}}N_{\text{D}}, \\ \Theta_2 + \frac{\Delta_2}{\mu^{m_{g_2}N_{\text{R}}N_{\text{D}}}},\ m_{g_1}N_{\text{S}}N_{\text{R}} > m_{g_2}N_{\text{R}}N_{\text{D}}, \\ \Theta_1 + \Theta_2 + \frac{(\Delta_1+\Delta_2)}{\mu^{m_{g_1}N_{\text{S}}N_{\text{R}}}}, \\ \qquad m_{g_1}N_{\text{S}}N_{\text{R}} = m_{g_2}N_{\text{R}}N_{\text{D}}. \end{cases} \tag{16}$$

and $\text{B}_z(a,b) = \int_0^z t^{a-1}(1-t)^{b-1}dt$ is the incomplete beta function.

The asymptotic SER with M-PSK can be derived as [14]

$$P_e = \frac{a_{\text{MPSK}}}{\pi}\int_0^{\tau}\int_0^{\infty}\frac{e^{-\gamma\frac{a_{\text{MPSK}}}{\sin^2\theta}}}{\sin^2\theta}F_{\gamma_{\mathsf{D}}}^{\infty}(\gamma)d\gamma d\theta, \tag{17}$$

$$F_{\tilde{\gamma}_1}(\gamma) \overset{\mathcal{P}_{\mathrm{T}}\to\infty}{\approx} \frac{\epsilon_1 \Gamma(m_{g_1}N_{\mathrm{S}}N_{\mathrm{R}}+m_{h_3}N_{\mathrm{R}}L)}{(\Gamma(m_{g_1}N_{\mathrm{R}}+1))^{N_{\mathrm{S}}}\Gamma(m_{h_3}N_{\mathrm{R}}L)} \left(\frac{\gamma m_{g_1}\mathcal{P}_{\mathrm{I}}\Omega_{h_3}}{\mathcal{P}_{\mathrm{T}}\Omega_{g_1}m_{h_3}}\right)^{m_{g_1}N_{\mathrm{S}}N_{\mathrm{R}}}$$
$$+\sum_{l=0}^{L-1}\frac{\binom{L-1}{l}(-1)^l L\Gamma(m_{g_1}N_{\mathrm{S}}N_{\mathrm{R}}+m_{h_3}N_{\mathrm{R}}L)}{\Gamma(m_{h_1}N_{\mathrm{Q}})(\Gamma(m_{g_1}N_{\mathrm{R}}+1))^{N_{\mathrm{S}}}\Gamma(m_{h_3}N_{\mathrm{R}}L)}\left(\frac{\gamma m_{g_1}\Omega_{h_1}\mathcal{P}_{\mathrm{I}}\Omega_{h_3}}{\mathcal{Q}m_{h_1}\Omega_{g_1}m_{h_3}}\right)^{m_{g_1}N_{\mathrm{S}}N_{\mathrm{R}}}\prod_{r=1}^{m_{h_1}N_{\mathrm{Q}}-1}\left[\sum_{a_r=0}^{a_{r-1}}\left(\frac{1}{r!}\right)^{a_r-a_{r+1}}\right]$$
$$\times (l+1)^{-m_{g_1}N_{\mathrm{S}}N_{\mathrm{R}}-m_{h_1}N_{\mathrm{Q}}-\sum_{r=1}^{m_{h_1}N_{\mathrm{Q}}-1}a_r}\Gamma\left(m_{g_1}N_{\mathrm{S}}N_{\mathrm{R}}+m_{h_1}N_{\mathrm{Q}}+\sum_{r=1}^{m_{h_1}N_{\mathrm{Q}}-1}a_r, \frac{\mathcal{Q}(l+1)m_{h_1}}{\mathcal{P}_{\mathrm{T}}\Omega_{h_1}}\right). \quad (8)$$

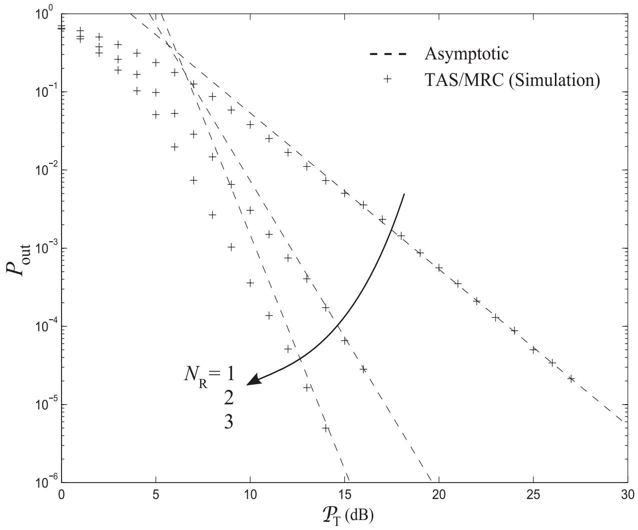

Fig. 2. Outage probability with $N_{\mathrm{S}}=N_{\mathrm{D}}=1$, $L=N_{\mathrm{Q}}=2$, $\mathcal{Q}=2\mathcal{P}_{\mathrm{T}}$, and $\mathcal{P}_{\mathrm{I}}=5$ dB.

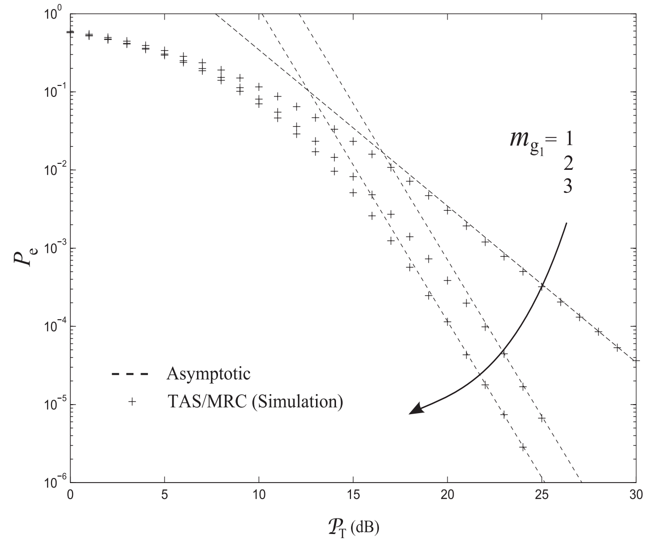

Fig. 3. SER of 16-QAM with $N_{\mathrm{S}}=N_{\mathrm{D}}=1$, $N_{\mathrm{R}}=2$, $L=N_{\mathrm{Q}}=2$, $\mathcal{Q}=2\mathcal{P}_{\mathrm{T}}$, and $\mathcal{P}_{\mathrm{I}}=3$ dB.

where $\tau=\pi(\mathcal{M}-1)/\mathcal{M}$, and $a_{\mathrm{MPSK}}=\sin^2(\pi/\mathcal{M})$.

Substituting (9) into (17) and solving the nested integrals results in

$$P_e \overset{\mathcal{P}_{\mathrm{T}}\to\infty}{\approx} \Xi_{\text{TAS/MRC}} \times \frac{\mathcal{P}_{\mathrm{I}}^q\Gamma(q+1)}{\mathcal{P}_{\mathrm{T}}^q\pi a_{\mathrm{MPSK}}^q}\left(\frac{\sqrt{\pi}\Gamma(\frac{1}{2}+q)}{2\Gamma(1+q)}+ \times\sqrt{1-a_{\mathrm{MPSK}}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}-q;\frac{3}{2};1-a_{\mathrm{MPSK}}\right)\right). \quad (18)$$

Our new closed-form expression for the SER of TAS/MRC is given in terms of the Gauss hypergeometric function, ${}_2F_1(a,b;c;z)$ [13, eq. (9.100)], which is straightforward to evaluate with numerical software such as Matlab.

Our asymptotic solutions can be re-expressed as

$$P_e \overset{\mathcal{P}_{\mathrm{T}}\to\infty}{\approx} (\mathcal{G}_A\mathcal{P}_{\mathrm{T}})^{-\mathcal{G}_D}+o\left(\mathcal{P}_{\mathrm{T}}^{-\mathcal{G}_D}\right), \quad (19)$$

where $\mathcal{G}_D$ is the diversity gain, which determines the asymptotic slope of the SER curve, and $\mathcal{G}_A$ is the array gain, which represents the shift of the SER curve with respect to a reference curve of $\mathcal{P}_{\mathrm{T}}^{-\mathcal{G}_D}$. Based on (15) and (18), we see that the diversity gain of TAS/MRC is $\mathcal{G}_D=\min(m_{g_1}N_{\mathrm{S}}N_{\mathrm{R}}, m_{g_2}N_{\mathrm{R}}N_{\mathrm{D}})$.

V. Numerical Examples

Numerical examples are provided to highlight the impact of TAS/MRC on the outage probability and the SER of cognitive MIMO networks. As shown in Fig. 1, we consider a cluster of PU-Tx nodes communicating with a cluster of PU-Rx nodes in the primary network. In the secondary network, we consider a simple collinear topology where S, R, and D are placed along the same straight line, with R located halfway between S and D. In the following examples, we assume a two dimensional network topology where the three nodes in the secondary network are placed along the x-axis, with S located at $(0,0)$, R located at $(\frac{1}{2},0)$, D located at $(1,0)$, and the channel mean power of the link from S to D is normalized to unity. We further assume an exponential decaying path loss where the channel mean power is proportional to $d^{-\nu}$ with d denoting the distance between the transceivers and $\nu=3$ denoting the path loss coefficient. As such, the channel mean powers for the links in the secondary network are $\Omega_{g_1}=\Omega_{g_2}=8$. The channel mean powers for the links from the primary network to the secondary network are defined by the the locations of the PU-Tx and PU-Rx clusters, for example, $\Omega_{h_3}=(d_{T_x}-1/2)^2+d_{T_y}^2)^{-3/2}$ and $\Omega_{h_1}=(d_{R_x}^2+d_{R_y}^2)^{-3/2}$

where (d_{T_x}, d_{T_y}) and (d_{R_x}, d_{R_y}) are the coordinates of the PU-Tx and PU-Rx clusters, respectively.

Fig. 2 plots the outage probability versus the maximum transmit power in the secondary network, $\mathcal{P}_{\rm T}$. We set the outage threshold as $\gamma_{\rm th} = 3$ dB and the coordinates of the PU-Tx and PU-Rx clusters are (0, 1) and (1, 1), respectively. The dashed lines represent the asymptotic outage probability derived in (9) and '+' denotes the simulation points. We see in the figure that there is a good agreement between our analytical results and the simulations. We further see that the outage probability of TAS/MRC decreases with increasing number of antennas at the relay, $N_{\rm R}$.

Fig. 3 plots the SER with 16-QAM versus $\mathcal{P}_{\rm T}$. The dashed lines represent the asymptotic SER derived in (15). We see that the asymptotic SER provides an accurate approximation of the simulation points at medium to high $\mathcal{P}_{\rm T}$. Furthermore, we observe that the SER decreases as the fading parameter in the secondary network m_{g_1} increases. We note that TAS/MRC achieves a full diversity gain of $\mathcal{G}_D = \min(m_{g_1} N_{\rm S} N_{\rm R}, m_{g_2} N_{\rm R} N_{\rm D})$, which serves to verify our analytical results.

VI. Conclusion

We proposed TAS/MRC with DF relaying in underlay spectrum sharing with cognitive MIMO relaying. For such networks, we derived new closed-form expressions for the asymptotic outage probability and symbol error rate with L primary transceivers equipped with $N_{\rm Q}$ antennas, and a secondary source, relay, and destination equipped with $N_{\rm S}$, $N_{\rm R}$, and $N_{\rm D}$ antennas, respectively. Our results are valid for Nakagami-m fading with distinct fading parameters in the primary and secondary networks. We show that TAS/MRC achieves a full diversity gain of $\mathcal{G}_D = \min(m_{g_1} N_{\rm S} N_{\rm R}, m_{g_2} N_{\rm R} N_{\rm D})$ for both M-PSK and M-QAM modulations.

Appendix A
Proof of Theorem 1

The cdf of $\tilde{\gamma}_1$ conditioned on Z_1 is derived as

$$\begin{aligned} F_{\tilde{\gamma}_1|Z_1}(\gamma) &= \Pr\left\{\min\left(\mathcal{P}_{\rm T}, \frac{\mathcal{Q}}{||\mathbf{h}_{1\ell^* i^*}||^2}\right)\frac{||\mathbf{g}_{1i^*}||^2}{Z_1} \leq \gamma\right\} \\ &= \underbrace{\Pr\left\{||\mathbf{g}_{1i^*}||^2 \leq \frac{\gamma Z_1}{\mathcal{P}_{\rm T}} \ \& \ ||\mathbf{h}_{1\ell^* i^*}||^2 \leq \frac{\mathcal{Q}}{\mathcal{P}_{\rm T}}\right\}}_{\mathcal{I}_1(Z_1)} \\ &+ \underbrace{\Pr\left\{||\mathbf{g}_{1i^*}||^2 \leq \frac{\gamma ||\mathbf{h}_{1\ell^* i^*}||^2 Z_1}{\mathcal{Q}} \ \& \ ||\mathbf{h}_{1\ell^* i^*}||^2 \geq \frac{\mathcal{Q}}{\mathcal{P}_{\rm T}}\right\}}_{\mathcal{I}_2(Z_1)} \end{aligned} \tag{20}$$

where we re-express $\tilde{\gamma}_1$ as

$$\tilde{\gamma}_1 = \min\left(\mathcal{P}_{\rm T}, \frac{\mathcal{Q}}{Y_1}\right)\frac{X_1}{Z_1}, \tag{21}$$

with $Y_1 = ||\mathbf{h}_{1\ell^* i^*}||^2$, $X_1 = ||\mathbf{g}_{1i^*}||^2$, and $Z_1 = \sum_{\ell=1}^{L} \mathcal{P}_{\rm I} ||\mathbf{h}_{3\ell q^*}||^2$.

The first term $\mathcal{I}_1(Z_1)$ can be evaluated as

$$\begin{aligned} \mathcal{I}_1(Z_1) &= \Pr\left(||\mathbf{g}_{1i^*}||^2 \leq \frac{\gamma Z_1}{\mathcal{P}_{\rm T}}\right)\Pr\left(||\mathbf{h}_{1\ell^* i^*}||^2 \leq \frac{\mathcal{Q}}{\mathcal{P}_{\rm T}}\right) \\ &= F_{||\mathbf{g}_{1i^*}||^2}\left(\frac{\gamma Z_1}{\mathcal{P}_{\rm T}}\right) F_{||\mathbf{h}_{1\ell^* i^*}||^2}\left(\frac{\mathcal{Q}}{\mathcal{P}_{\rm T}}\right) \end{aligned} \tag{22}$$

where $F_{||\mathbf{g}_{1i^*}||^2}(\cdot)$ is the cdf of $||\mathbf{g}_{1i^*}||^2$ and $F_{||\mathbf{h}_{1\ell^* i^*}||^2}(\cdot)$ is the cdf of $||\mathbf{h}_{1\ell^* i^*}||^2$. The cdf of $||\mathbf{g}_{1i^*}||^2$ is given by

$$F_{||\mathbf{g}_{1i^*}||^2}(x) = \left(1 - e^{-x\frac{m_{g_1}}{\Omega_{g_1}}} \sum_{p=0}^{m_{g_1} N_{\rm R}-1} \frac{\left(x\frac{m_{g_1}}{\Omega_{g_1}}\right)^p}{p!}\right)^{N_{\rm S}}, \tag{23}$$

where the channel gains in $||\mathbf{g}_{1i^*}||$ follow a Gamma distribution with Nakagami-m fading parameter m_{g_1} and channel power Ω_{g_1}. The cdf of $||\mathbf{h}_{1\ell^* i^*}|^2$ can be written as

$$F_{||\mathbf{h}_{1\ell^* i^*}||^2}(x) = \left(1 - e^{-x\frac{m_{h_1}}{\Omega_{h_1}}} \sum_{r=0}^{m_{h_1} N_{\rm Q}-1} \frac{\left(x\frac{m_{h_1}}{\Omega_{h_1}}\right)^r}{r!}\right)^{L}, \tag{24}$$

where the channel gains in $||\mathbf{h}_{1\ell^* i^*}||^2$ follow a Gamma distribution with Nakagami-m fading parameter m_{h_1} and channel power Ω_{h_1}. At high $\mathcal{P}_{\rm T}$, (22) can be approximated as

$$\mathcal{I}_1(Z_1) \overset{\mathcal{P}_{\rm T}\to\infty}{\approx} \frac{\epsilon_1 \left(\frac{\gamma Z_1 m_{g_1}}{\mathcal{P}_{\rm T}\Omega_{g_1}}\right)^{m_{g_1} N_{\rm S} N_{\rm R}}}{(\Gamma(m_{g_1} N_{\rm R}+1))^{N_{\rm S}}} \tag{25}$$

where the asymptotic cdf of $||\mathbf{g}_{1i^*}||^2$ is

$$F_{||\mathbf{g}_{1i^*}||^2}(x) \overset{x\to 0^+}{\approx} \frac{\left(\frac{x m_{g_1}}{\Omega_{g_1}}\right)^{m_{g_1} N_{\rm S} N_{\rm R}}}{(\Gamma(m_{g_1} N_{\rm R}+1))^{N_{\rm S}}}, \tag{26}$$

and we define

$$\begin{aligned} \epsilon_1 &= F_{||\mathbf{h}_{1\ell^* i^*}||^2}\left(\frac{\mathcal{Q}}{\mathcal{P}_{\rm T}}\right) \\ &= \left(1 - e^{-\frac{\mathcal{Q} m_{h_1}}{\mathcal{P}_{\rm T}\Omega_{h_1}}} \sum_{r=0}^{m_{h_1} N_{\rm Q}-1} \frac{\left(\frac{\mathcal{Q} m_{h_1}}{\mathcal{P}_{\rm T}\Omega_{h_1}}\right)^r}{r!}\right)^{L}. \end{aligned} \tag{27}$$

Integrating (25) with respect to the pdf of Z_1 given by

$$f_{Z_1}(x) = \left(\frac{m_{h_3}}{\mathcal{P}_{\rm I}\Omega_{h_3}}\right)^{m_{h_3} N_{\rm R} L} \frac{x^{m_{h_3} N_{\rm R} L-1} e^{-x\frac{m_{h_3}}{\mathcal{P}_{\rm I}\Omega_{h_3}}}}{\Gamma(m_{h_3} N_{\rm R} L)}. \tag{28}$$

yields

$$\mathbb{E}_{Z_1}\{\mathcal{I}_1(Z_1)\} \overset{\mathcal{P}_{\rm T}\to\infty}{\approx} \frac{\epsilon_1 \left(\frac{\gamma m_{g_1}\mathcal{P}_{\rm I}\Omega_{h_3}}{\mathcal{P}_{\rm T}\Omega_{g_1} m_{h_3}}\right)^{m_{g_1} N_{\rm S} N_{\rm R}} \Gamma(m_{g_1} N_{\rm S} N_{\rm R} + m_{h_3} N_{\rm R} L)}{(\Gamma(m_{g_1} N_{\rm R}+1))^{N_{\rm S}} \Gamma(m_{h_3} N_{\rm R} L)} \tag{29}$$

where we solve the integral according to [13, 3.326.2]

$$\int_0^\infty x^m \exp(-\beta x) dx = \frac{\Gamma(m+1)}{\beta^{m+1}} \tag{30}$$

The second term $\mathcal{I}_2(Z_1)$ can be evaluated as

$$\mathcal{I}_2(Z_1) = \int_{\frac{\mathcal{Q}}{\mathcal{P}_\mathrm{T}}}^{\infty} f_{||\mathbf{h}_{1\ell^* i^*}||^2}(y) \int_0^{\frac{y\gamma Z_1}{\mathcal{Q}}} f_{||\mathbf{g}_{1i^*}||^2}(x) dx dy$$
$$= \int_{\frac{\mathcal{Q}}{\mathcal{P}_\mathrm{T}}}^{\infty} f_{||\mathbf{h}_{1\ell^* i^*}||^2}(y) F_{||\mathbf{g}_{1i^*}||^2}\left(\frac{y\gamma Z_1}{\mathcal{Q}}\right) dy \quad (31)$$

where $f_{||\mathbf{h}_{1\ell^* i^*}||^2}(\cdot)$ is the probability density function (pdf) of $||\mathbf{h}_{1\ell^* i^*}||^2$ given by

$$f_{||\mathbf{h}_{1\ell^* i^*}||^2}(x) = L\left(\frac{m_{h_1}}{\Omega_{h_1}}\right)^{m_{h_1}N_\mathrm{Q}} \frac{x^{m_{h_1}N_\mathrm{Q}-1} e^{-x\frac{m_{h_1}}{\Omega_{h_1}}}}{\Gamma(m_{h_1}N_\mathrm{Q})}$$
$$\times \left(1 - e^{-x\frac{m_{h_1}}{\Omega_{h_1}}} \sum_{r=0}^{m_{h_1}N_\mathrm{Q}-1} \frac{\left(x\frac{m_{h_1}}{\Omega_{h_1}}\right)^r}{r!}\right)^{L-1}, \quad (32)$$

where the channel coefficients in $||\mathbf{h}_{1\ell^* i^*}||^2$ follow a Gamma distribution with fading severity parameter m_{h_1} and channel power Ω_{h_1}. We perform a change of variables $y = \frac{\mathcal{Q}}{\mathcal{P}_\mathrm{T}}t$ in (31) which results in

$$\mathcal{I}_2(Z_1) = \int_1^{\infty} f_{||\mathbf{h}_{1\ell^* i^*}||^2}\left(\frac{\mathcal{Q}}{\mathcal{P}_\mathrm{T}}t\right) F_{||\mathbf{g}_{1i^*}||^2}\left(\frac{\gamma Z_1}{\mathcal{P}_\mathrm{T}}t\right) \frac{\mathcal{Q}}{\mathcal{P}_\mathrm{T}} dt \quad (33)$$

where $dy = \frac{\mathcal{Q}}{\mathcal{P}_\mathrm{T}}dt$. Applying the Taylor series expansion as $\mathcal{P}_\mathrm{T} \to \infty$, we approximate the cdf of $||\mathbf{g}_{1i^*}||^2$ as

$$F_{||\mathbf{g}_{1i^*}||^2}\left(\frac{\gamma Z_1}{\mathcal{P}_\mathrm{T}}t\right) \overset{\mathcal{P}_\mathrm{T}\to\infty}{\approx} \frac{\left(\frac{\gamma Z_1 m_{g_1}}{\mathcal{P}_\mathrm{T}\Omega_{g_1}}t\right)^{m_{g_1}N_\mathrm{S}N_\mathrm{R}}}{(\Gamma(m_{g_1}N_\mathrm{R}+1))^{N_\mathrm{S}}} \quad (34)$$

Substituting (32) and (34) into (33) results in

$$\mathcal{I}_2(Z_1) \overset{\mathcal{P}_\mathrm{T}\to\infty}{\approx} \sum_{l=0}^{L-1} \frac{\binom{L-1}{l}(-1)^l L \left(\frac{\gamma Z_1 m_{g_1}\Omega_{h_1}}{\mathcal{Q} m_{h_1}\Omega_{g_1}}\right)^{m_{g_1}N_\mathrm{S}N_\mathrm{R}}}{\Gamma(m_{h_1}N_\mathrm{Q})(\Gamma(m_{g_1}N_\mathrm{R}+1))^{N_\mathrm{S}}}$$
$$\times \prod_{r=1}^{m_{h_1}N_\mathrm{Q}-1} \left[\sum_{a_r=0}^{a_{r-1}} \left(\frac{1}{r!}\right)^{a_r - a_{r+1}}\right]$$
$$\times (l+1)^{-m_{g_1}N_\mathrm{S}N_\mathrm{R} - m_{h_1}N_\mathrm{Q} - \sum_{r=1}^{m_{h_1}N_\mathrm{Q}-1} a_r}$$
$$\times \Gamma\left(m_{g_1}N_\mathrm{S}N_\mathrm{R} + m_{h_1}N_\mathrm{Q} + \sum_{r=1}^{m_{h_1}N_\mathrm{Q}-1} a_r, \frac{\mathcal{Q}(l+1)m_{h_1}}{\mathcal{P}_\mathrm{T}\Omega_{h_1}}\right) \quad (35)$$

where we solve the resulting integral using [13, 3.351/2]

$$\int_1^{\infty} = x^n e^{-\mu x} dx = \mu^{-n-1}\Gamma(n+1, \mu). \quad (36)$$

with $\Gamma(\cdot, \cdot)$ denoting the upper incomplete gamma function [13, eq. (8.350.2)]. We expand the power sum according to

$$\left[\sum_{n=0}^{N-1} \frac{x^n}{n!}\right]^l = \prod_{n=1}^{N-1} \left[\sum_{a_n=0}^{a_{n-1}} \binom{a_{n-1}}{a_n} \left(\frac{1}{n!}\right)^{a_n - a_{n+1}}\right] x^{\sum_{n=1}^{N-1} a_n}, \quad (37)$$

where $N > 1$, $a_0 = l$, and $a_N = 0$. Integrating (35) with respect to (28) results in

$$\mathbb{E}_{Z_1}\{\mathcal{I}_2(Z_1)\} \overset{\mathcal{P}_\mathrm{T}\to\infty}{\approx} \sum_{l=0}^{L-1} \frac{\binom{L-1}{l}(-1)^l L}{\Gamma(m_{h_1}N_\mathrm{Q})(\Gamma(m_{g_1}N_\mathrm{R}+1))^{N_\mathrm{S}}}$$
$$\times \left(\frac{\gamma m_{g_1}\Omega_{h_1}\mathcal{P}_\mathrm{I}\Omega_{h_3}}{\mathcal{Q} m_{h_1}\Omega_{g_1} m_{h_3}}\right)^{m_{g_1}N_\mathrm{S}N_\mathrm{R}}$$
$$\times \frac{\Gamma(m_{g_1}N_\mathrm{S}N_\mathrm{R} + m_{h_3}N_\mathrm{R}L)}{\Gamma(m_{h_3}N_\mathrm{R}L)} \prod_{r=1}^{m_{h_1}N_\mathrm{Q}-1} \left[\sum_{a_r=0}^{a_{r-1}} \left(\frac{1}{r!}\right)^{a_r - a_{r+1}}\right]$$
$$\times (l+1)^{-m_{g_1}N_\mathrm{S}N_\mathrm{R} - m_{h_1}N_\mathrm{Q} - \sum_{r=1}^{m_{h_1}N_\mathrm{Q}-1} a_r}$$
$$\times \Gamma\left(m_{g_1}N_\mathrm{S}N_\mathrm{R} + m_{h_1}N_\mathrm{Q} + \sum_{r=1}^{m_{h_1}N_\mathrm{Q}-1} a_r, \frac{\mathcal{Q}(l+1)m_{h_1}}{\mathcal{P}_\mathrm{T}\Omega_{h_1}}\right) \quad (38)$$

Finally, the asymptotic cdf of $\tilde{\gamma}_1$ is the sum of (29) and (38) as presented in (8) which completes the proof.

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