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A Fast and Robust Load Flow Method for Distribution Systems with Distributed Generations

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***Abstract*—This paper proposes a fast and robust load flow method for balanced power distribution systems with distributed generation sources. The method formulates the power flow equations in PQ decoupled form with polar coordinates. Second-order terms are included in the active power mismatch iteration, and resistances are fully modeled without any simplifications. The impacts of zero-impedance branches are explicitly modeled through reconfiguring of the adjacent branches with impedances. Typical distribution generation models and distribution load models are included. A hybrid direct and indirect solution technique is used to achieve efficiency and robustness of the algorithm. Active power correction is solved by means of a sparse LU decomposition algorithm with partial pivoting, and the reactive power correction is solved by means of restarted Generalized Minimal Residual algorithm with incomplete LU pre-conditioner. The numerical examples on a sample distribution system with widespread Photovoltaic installations are given to demonstrate the effectiveness of the proposed method.**

***Keywords*—Power distribution systems; distribution generations; load flow; zero-impedance branches; direct method; indirect method**

I. Introduction

Load flow calculation is one of the most common computational procedures used in distribution system analysis. Planning, operation and control of distribution systems require such calculations in order to analyze the steady-state performance of the systems under various operating conditions and equipment configurations. With the increasing penetration of various distribution generations and implementation of advanced control techniques, the analysis of distribution systems plays even more critical role than before, and the complexity of analysis has significantly increased as well.

Various methods for solving the power flow problem are known [1] ~ [17]. Those methods differ in either the form of the equation describing the system, or the numerical techniques used. The bus admittance matrix based methods are widely used. Typical methods include the Gauss-Seidel method [1], the Newton-Raphson method [2], and the Fast Decoupled method [6]. Those methods formulate power flow problems as linear systems, and solve the problem by either direct or iterative techniques. The method proposed in this paper also belongs to this category.

Based on the characteristics of distribution systems with distribution generation, this paper proposes a new efficient and robust load flow method for balanced distribution systems. The features of the new method are summarized as follows:

- Accurate modeling of the zero impedance branches by merging the zero-impedance branches with adjacent impedance branches to avoid convergence problems resulting from modeling those as small impedance branches.
- Formulating the decoupled PQ equations in polar form. The resistance impacts are modeled in both active and reactive power equations, and the necessary trigonometric operations have been avoided by using an appropriate polynomial approximation. It includes second terms of phase angle in active power correction equations to reduce the required iterations for phase angle updating.
- Using direct and iterative solution techniques to handle active and reactive power corrections respectively. This hybrid solution technique fully takes advantage of the different characteristics of active and reactive power updating to speed up the load flow solution.
- Seamless integration of various types of distribution generation sources and distribution load models with the solution process.

II. Proposed Method

A. Modeling of zero-impedance branches

Many branches in a power distribution system have very low impedance, such as voltage regulators, switches, ideal transformers, ideal phase shifters, elbows, and jumpers.

In practice, these low impedances are ignored and set to zero in conventional models. The consequence is that some

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entries in the resultant bus admittance matrix are infinite, and thus the admittance matrix based approaches are inapplicable. In order to use bus admittance matrix based approaches, conventional methods have arbitrarily assigned small non-zero impedances to those branches. However, assigning such small impedances makes the analysis ill-conditioned, and power flows are difficult to converge. This paper uses a different approach to handle the zero-impedance branches in power flow analysis.

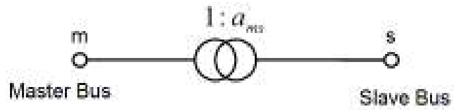

Fig.1. Generalized zero-impedance branch model.

Fig. 1 shows a generalized model for representing zero-impedance branches in a distribution system. A branch has a master bus *m* and a slave bus *s*. The buses are connected by an ideal transformer. The transformer has a ratio $1\mathbf{:}\,a_{ms}$, where a_{ms} is a complex number.

The complex transformer ratio becomes 1, when the branch is a switch or small conductor, a real number when it is an ideal voltage regulator or transformer, and a complex number with magnitude 1.0 when it is an ideal phase shifter.

The current flowing to the slave bus through the branch is equal to the current flowing from the master bus divided by the conjugate of the complex ratio. The voltage at the slave bus is equal to the voltage at the master bus multiplied by the complex ratio.

When constructing the bus admittance matrix, only non-slave buses are considered. Zero-impedance branches are not used. The impacts of zero impedance branches are represented through the associated master buses, and the branches adjacent to the slave buses as shown in Fig. 2.

Fig. 2 shows an example construction of an equivalent distribution system model with non-zero impedances. The construction transforms a model of distribution system with zero impedance branches to the equivalent distribution system model with non-zero impedances.

In Fig. 2, a zero-impedance branch is connected to three branches (broken lines) by the slave bus and to two branches (double lines) by the master bus. Taking one adjacent branch between slave bus *s* and bus *k* as example, the branch admittance matrix is

$$\begin{array}{c} \\ s \\ k \end{array} \begin{array}{c} \quad s \qquad k \\ \begin{bmatrix} Y_{ss} & Y_{sk} \\ Y_{ks} & Y_{kk} \end{bmatrix} \end{array}$$

where, Y_{ss} and Y_{kk} are the self admittances of the branch at the slave bus *s* and the bus *k*, and Y_{sk} and Y_{ks} are the mutual admittances of the branch between the bus *s* to bus *k*, and bus *k* to bus s, respectively. The master bus *m* provides an injected complex current I_m, an injected complex power S_m, and a shunt compensator with admittance Y_m^{sh}. The slave bus *s* provides an injected complex current I_s, an injected complex power S_s and a shunt compensator with admittance Y_s^{sh}.

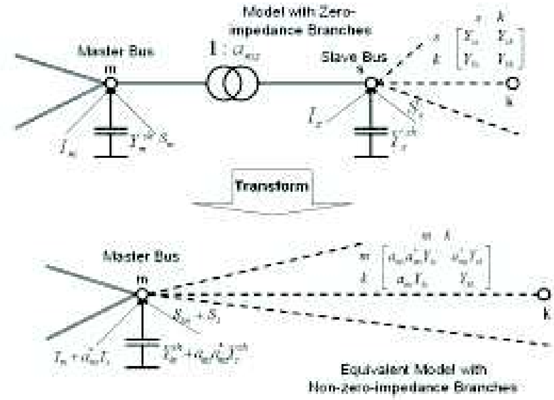

Fig.2. Equivalent model for distribution system with zero impedance branches.

In the equivalent model, the zero-impedance branch and the slave bus *s* are removed. There are no changes for the branches connected to the master bus *m*. The branches connected to the slave bus *s* are reconnected to bus *m*, and the branch admittance matrices are modified accordingly.

The branch between buses *s* and *k* in the system is replaced with a new branch between bus *m* and bus *k* in the equivalent system and the branch admittance matrix is

$$\begin{array}{c} \\ m \\ k \end{array} \begin{array}{c} \qquad m \qquad\qquad k \\ \begin{bmatrix} a_{ms}a_{ms}^{*}Y_{ss} & a_{ms}^{*}Y_{sk} \\ a_{ms}Y_{ks} & Y_{kk} \end{bmatrix} \end{array}$$

where, a_{ms}^{*} is the conjugate of zero-impedance branch ratio. The self admittance at bus *m* is determined from the product of self admittance at bus *s* in the model and the square of the zero-impedance branch ratio. The mutual admittance for bus *m* to *k* is the product of the conjugate of the zero-impedance branch ratio and mutual admittance for bus *s* to *k* in the original system. The mutual admittance for bus *k* to bus *m* is the product of the zero-impedance branch ratio and mutual admittance for bus *k* to bus *s* in the original model.

The current at the slave bus *s* is multiplied by the conjugate of the zero-impedance branch ratio to add to the master bus *m*, and the equivalent current at bus *m* is:

$$I_m + \sum_{s\in M} a_{ms}^{*} I_s$$

where M is the set of buses that have connected with bus m through zero-impedance branches.

The powers at bus *s* are directly added to bus *m*, and the resulting equivalent complex power at bus *m* is:

$$S_m + \sum_{s\in M} S_s$$

The shunt compensation admittance at bus s is multiplied with the square of the zero-impedance branch ratio to add to bus *m*, and the equivalent shunt compensation admittance at bus m is:

$$Y_m^{sh} + \sum_{s\in M} a_{ms}a_{ms}^* Y_s^{sh}$$

B. Modeling of distribution generation sources and loads

The generation source for the power distribution system is usually a power transmission system, and corresponding equivalent source models are expressed as a swing bus, or a PV bus in the power flow analysis.

In addition to the equivalent sources, the power distribution system can also have distributed power generators. Depending on the types of energy sources and energy converters, the distribution power generators are specified by a constant power factor model, constant voltage model, or variable reactive power model [18].

The buses connected to the constant power factor generators or the variable reactive power generators are treated as PQ buses. For the constant power factor generator, the specified values are the active power output and power factor. The reactive power output is determined from the active power and the power factor. For the variable reactive power generator, the active power output is specified, and the reactive power output is determined by applying a predetermined polynomial function to the active power output.

The buses connected to constant voltage generators are treated as PV buses, and the specified values are the outputs of the active powers and the magnitudes of bus voltages. These buses are also selected as master buses when the equivalent system model is constructed.

The distribution load models include a constant impedance load, a constant power load, and a constant current load. The constant impedance load is directly treated as connected bus shunt impedance, which is embedded into the bus admittance matrix.

The constant power load is modeled as bus injected power. The constant current load is converted to equivalent bus injected powers to be modeled. The equivalent injected powers are based on estimated bus voltages. The powers are recalculated when the current bus voltages become available during the iterations of the solution.

By converting to PV or PQ buses or shunt compensations, the distribution source and load models are seamlessly integrated into the solution process.

C. Decoupled Power Flow Equations with Full Impedances and Second Order Terms

The power flow equations for all non-slave buses are

$$P_i = V_i \sum_j V_j [G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)] \quad (1)$$

$$Q_i = V_i \sum_j V_j [G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)] \quad (2)$$

where, P_i and Q_i are the net injected active power and reactive power at bus i, V_i and θ_i are the voltage magnitude and phase angle at bus i, and G_{ij} and B_{ij} are the real and imaginary part of the bus admittance matrix element associated with bus i and bus j.

Similarly to the Fast Decoupled method, active power is expressed as a function of bus phase angles, and reactive power is a function of bus voltage magnitudes. By applying the Taylor expansion to Eqns. (1) and (2), and retaining up to second order terms, one obtains

$$\frac{\Delta P}{V} = J_\theta \Delta\theta + \frac{1}{2}\Delta\theta^T H_\theta \Delta\theta \quad (3)$$

$$\frac{\Delta Q}{V} = J_V \Delta V \quad (4)$$

where ΔP and ΔQ are vectors of bus active and reactive power changes, $\Delta\theta$ and ΔV are the vectors of bus phase angle and voltage magnitude changes, J_θ and H_θ are the Jacobian and Hessian matrices of bus active powers with respect to bus phase angles, and J_V is the Jacobian matrix of bus reactive powers with respect to bus voltage magnitudes.

The element of the Jacobian J_θ that is associated with the active power at bus i and phase angle at bus j is $\frac{1}{V_i}\frac{\partial P_i}{\partial \theta_j}$.

The element of the Hessian H_θ that is associated with the active power at bus i and the phase angles at bus j and bus k is $\frac{1}{V_i}\frac{\partial^2 P_i}{\partial\theta_j \partial\theta_k}$.

Similarly, the element of Jacobian J_V that is associated with the reactive power at bus i and the voltage magnitude at bus j is $\frac{1}{V_i}\frac{\partial Q_i}{\partial V_j}$.

During the formulation of the Jacobian and Hessian matrices, the trigonometric functions are replaced with the Taylor series up to 2-orders respectively to simplify the formulation and speed up the calculation:

$$\sin(x) \approx x \quad (5)$$

$$\cos(x) \approx 1 - x^2/2 \quad (6)$$

D. Hybrid Direct and Indirect Procedures

The power flow equations are usually solved either by means of a direct solution technique [19] or an iterative solution technique [20]. Considering the characteristic difference between active power and reactive power problems, the proposed method uses a hybrid procedure to solve the power flow equations described in Eqns. (3) and (4), in which the direct solution technique is used to solve the active power mismatch equations, and the iterative procedure is used to solve the reactive power mismatch equations.

For the active power mismatch problem, the following equation is used:

$$J_\theta(V^{(0)}, \theta^{(0)})\Delta\theta = \frac{\Delta P^{(k)}}{V^{(k)}} - \frac{1}{2}\Delta\theta^{(k)T} H_\theta(V^{(0)}, \theta^{(0)})\Delta\theta^{(k)} \quad (7)$$

The Jacobian matrix $J_\theta(V^{(0)},\theta^{(0)})$ and the Hessian matrix $H_\theta(V^{(0)},\theta^{(0)})$ are determined by using the initial bus voltage magnitude $V^{(0)}$ and phase angle $\theta^{(0)}$, which remain constant during the iterations. The first item in the right hand side is the bus active power mismatch divided by the corresponding bus voltage magnitude that was determined by means of the bus voltage magnitude and phase angle obtained during the previous iteration *k*. The second item is the additional mismatch added by the second order of phase angle changes, also determined by means of the phase angles obtained at previous iteration. This linear equation is solved by means of a sparse LU decomposition with partial pivoting. The bus phase angle vector θ is updated when the phase angle correction vector $\Delta\theta$ is determined.

For the reactive power mismatch problem, the following equation is used

$$J_V(V^{(0)},\theta^{(0)})\Delta V = \frac{\Delta Q^{(k)}}{V^{(k)}} \quad (8)$$

The Jacobian matrix $J_V(V^{(0)},\theta^{(0)})$ is determined from initial bus voltage magnitudes and phase angles, which remain constant during the iterations. The right hand side is the bus reactive power mismatch divided by the corresponding bus voltage magnitude that was determined from the bus voltage magnitude and phase angle obtained during the previous iteration k. This linear equation is solved by means of the Restarted Generalized Minimal Residual method with incomplete LU pre-conditioner. The diagonal elements of the Jacobian matrix are taken to be the preconditioned matrix. The bus voltage magnitude vector V is updated when the voltage magnitude correction vector ΔV is determined.

The ideal values are used to set the initial values for bus voltage magnitudes and phase angles. It is assumed that the impedances of all branches are zero.

The initial voltage magnitude of a bus is set as the result of multiplying the swing bus voltage magnitude by all voltage increasing ratios resulting from the transformers along the shortest path from the swing bus to the study bus.

The bus initial phase angle is set as the swing bus phase angle plus all phase angle changes resulting from the phase shifters along the shortest path from the swing bus to the bus.

III. Numerical Examples

The developed algorithm has been tested on several sample systems, and satisfactory results have been obtained. The testing results on a sample 6.6-kV distribution system and computation performance compared with other existing methods are provided here.

As shown in the Fig. 3, the test system has 6 feeders and 122 nodes. The first feeder of the system is used for power generation only, and each node along the feeder has a photovoltaic unit installed at it. The other 5 feeders are used for both power generation and for serving power demand from customers, with each node along the feeder also having a photovoltaic unit and a service load transformer installed. The load demand at each node contains 30% constant power load, 30% constant current load, and 30% impedance load. The system has 12 zero-impedance branches, including 6 closed switches, and 6 voltage regulators.

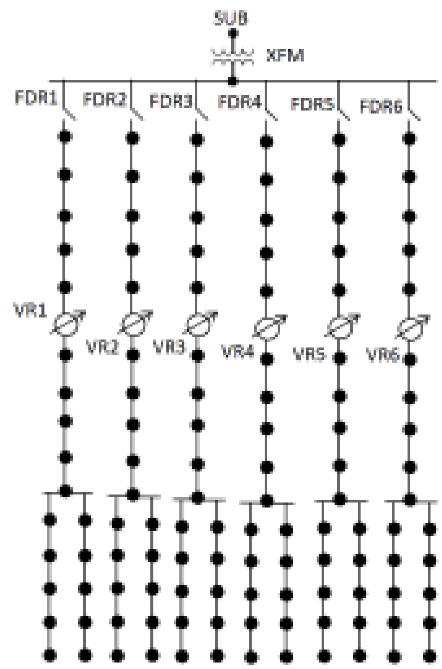

Fig. 3. The sample 6.6 kV distribution system.

Four different cases are simulated as shown in Table 1. Case I and II simulate normal power supply scenarios. In those cases, the main grid satisfies the major portion of total system load demand, and the remaining portion is satisfied by local photovoltaic units. Case III and IV simulate back-feeding scenarios. Besides satisfying the total load demands of local customers, the system still has power surplus that can be fed back to the main grid. The photovoltaic units only generate active power in case I and III, and both active power and reactive power in case II and IV.

Table 1: Test Scenarios

Scenarios	Case	Load Demands		PV Generations	
		MVA	Power Factor	MVA	Power Factor
Normal Power Supply	I	11.486	0.95	3	1.0
	II	11.486	0.95	3	0.95
Back-feed to Main Grid	III	11.486	0.95	30	1.0
	IV	11.486	0.95	30	0.95

Five different algorithms have been implemented to calculate the load flows of the sample micro-grid system, including the method proposed in this paper, the Gauss-Seidel method, the Newton-Raphson method, and the BX and XB versions of the fast decoupled method. The computational performance is shown in Table 2-5. The allowed maximum power mismatch is set to 10^{-5} per unit, and the maximum iteration number is set to 2000.

Table 2: Computational Performance for Case I

Algorithm	Computation Time(Seconds)	Iterations	Convergence
Proposed Method	0.029	12	Converged
Gauss-Seidel Method	5.668	1387	Converged
Newton-Raphson Method	0.191	7	Converged
Fast Decoupled Method, XB Version	0.741	2000	Not Converged
Fast Decoupled Method, BX Version	0.812	2000	Not Converged

Table 3: Computational Performance for Case II

Algorithm	Computation Time(Seconds)	Iterations	Convergence
Proposed Method	0.032	13	Converged
Gauss-Seidel Method	5.191	1393	Converged
Newton-Raphson Method	0.152	7	Converged
Fast Decoupled Method, XB Version	0.631	2000	Not Converged
Fast Decoupled Method, BX Version	0.633	2000	Not Converged

Table 4: Computational Performance for Case III

Algorithm	Computation Time (Seconds)	Iterations	Convergence
Proposed Method	0.028	15	Converged
Gauss-Seidel Method	7.887	1917	Converged
Newton-Raphson Method	0.189	5	Converged
Fast Decoupled Method, XB Version	0.818	2000	Not Converged
Fast Decoupled Method, BX Version	0.792	2000	Not Converged

Table 5: Computational Performance for Case IV

Algorithm	Computation Time (Seconds)	Iterations	Convergence
Proposed Method	0.038	19	Converged
Gauss-Seidel Method	6.529	1693	Converged
Newton-Raphson Method	0.149	5	Converged
Fast Decoupled Method, XB Version	0.740	2000	Not Converged
Fast Decoupled Method, BX Version	0.711	2000	Not Converged

Taking Case I as an example, it took 29 ms and 12 iterations for the proposed algorithm to find the final solution with the required precision. In comparison, it took 5668 ms and 1387 iterations for the Gauss-Seidel algorithm, and 191 ms and 7 iterations for the Newton-Raphson algorithm to find the solution with the same precision. The two fast decoupled algorithms, either the BX version or the XB version, failed to converge to a solution within the given maximum number of iterations. Similar results can be found in the other three tables for the other three cases.

From those test results, we can see that the proposed algorithm is much more efficient than the Gauss-Seidel and Newton-Raphson algorithms, and has much better convergence than the Fast Decoupled ones.

IV. Conclusion

The paper has proposed a fast and robust method for load flow analysis of balanced distribution systems with distributed generations. It models zero-impedance branches accurately, and avoids solution divergence that is usually caused by zero or small impedance branches in conventional methods. The method formulates the power flow equations in PQ decoupled form with constant Jacobian and Hessian matrices. It uses a hybrid procedure to solve the power flow equations, in which the direct method is used to solve the active power equations, and the indirect method is used to solve the reactive power equations.

The test results have proven experimentally that the proposed method is much faster than both the Gauss-Seidel and Newton-Raphson algorithms, and has better convergence than the Fast Decoupled algorithms.

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