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***Abstract*—In this paper, we study the impact of uncertain channel state information (CSI) on the performance of demodulate-and-forward relaying protocols with higher order modulation formats such as pulse-amplitude modulation (PAM) and rectangular quadrature-amplitude modulation (QAM). Assuming a single source and a single destination node assisted by N relay nodes, we study the average bit error probability (BEP) performance of M-ary PAM and rectangular QAM constellations with Gray code mapping and imperfect CSI at the relay nodes as well as the destination. The main contributions of this paper are the derivation of closed-form expressions for a) the cumulative distribution functions of the demodulator test statistics, b) the transition probability of error at a given relay, and c) the average BEP for independent and not necessarily identically distributed Rayleigh fading channels with imperfect receiver CSI.**

***Index Terms*—Cooperative communication, demodulate-and-forward relaying, fading channel estimation, imperfect channel knowledge.**

I. Introduction

Cooperative relaying has recently received significant attention, both from academia and industry, as a new approach to enhance the performance of wireless cellular, ad-hoc and sensor networks. Cooperative relaying, sometimes termed as distributed spatial diversity [1], enables shared usage of single-antenna transceivers to realize virtual antenna arrays [2] to increase coverage [3], enhance data rates [4] and improve reliability [5] of energy-constrained mobile devices.

Two prominent relay signal processing approaches that have been studied extensively are amplify-and-forward (AF or non-regenerative) and decode-and-forward (DF or regenerative) relaying protocols. With AF signal processing, the relay forwards a scaled version of the received signal from the source to the destination. The scale factor (or the amplification gain) at the relay is chosen as a function of either the instantaneous [6] or the average channel power [7]. Performance of coherent AF relaying is studied in [8] with perfect receiver channel state information (CSI), whereas the impact of imperfect CSI on AF relaying is investigated in [9] and [10].

A relay implementing DF signal processing decodes the source transmission and re-encodes it before forwarding it to the destination [11]. To enable error detection at the relay, in practice, cyclic redundant check (CRC) symbols are appended to the source transmissions [12]. Demodulate-and-forward signal processing is an alternative to DF signal processing to reduce receiver power consumption due to channel decoding at the relay as well as to minimize the overall delay at the destination. With the knowledge of the relay decision errors, optimum demodulate-and-forward receivers are presented in [13] with perfect and no receiver CSI, whereas a cooperative maximal ratio combiner (MRC) receiver is presented in [14] to extract near maximum-likelihood performance with perfect receiver CSI. Unlike [13], [14], the present work does not assume knowledge of the relay probability of error at the destination node. Average bit error probability (BEP) performance of DF protocol with PSK, PAM and QAM signal sets is studied in [15] over independent and not necessarily identically distributed (i.n.i.d.) Rayleigh fading channels with perfect receiver CSI. Symbol error probability analysis for multi-hop transmission with M-PSK and M-QAM signaling and regenerative relaying are reported in [16] and [17], respectively, over i.n.i.d. Nakagami fading channels with perfect receiver CSI.

In this contribution, we investigate the impact of imperfect CSI on the performance of demodulate-and-forward cooperative relaying protocols with higher order signal constellations. In particular, we focus on analytical quantification of the degradation in the average BEP performance at the destination with multiple relay nodes on i.n.i.d. Rayleigh fading channels when the source node employs M-ary rectangular QAM constellations with Gray mapping. We introduce a model for imperfect receiver CSI that is general enough to encompass various scenarios such as pilot-symbol assisted modulation, outdated channel knowledge due to fading decorrelation, and linear minimum mean-square error channel estimation. We present exact yet simple closed-form expressions for the average BEP via the distribution of demodulator test statistics with imperfect CSI. Importantly, our analysis takes into account demodulation errors at the relay due to finite average received SNR, fading and imperfect receiver CSI. Our numerical and simulation results show that relay demodulation errors lead to a serious degradation in average error performance, whereas imperfect receiver CSI causes an irreducible error floor in the average BEP.

The rest of this paper is organized as follows. In Section II, we introduce our signal and channel models. Closed-form expressions for the distribution of the demodulator test statistics and transition probability of error at the relay are presented in Section III, and used in Section IV to provide a closed-form expression for the average error performance of multi-relay demodulate-and-forward protocol with M-ary rectangular QAM modulation and imperfect receiver CSI. Numerical and simulation results are presented in Section V whereas Section VI concludes this work.

II. System Model

We consider the system model illustrated in Fig. 1, where a single source, S, communicates with a single destination, D, with the assistance of N relay nodes. The channels across the nodes are assumed to be random, independent, frequency-flat, and constant over the signaling duration. We employ low-pass equivalent complex-valued representation for the transmit and receive signals, channel gains and background additive noise. Specifically, the channel gain on the $S \rightarrow D$ link is denoted by G_0, which is assumed to be a zero-mean, circularly symmetric, complex Gaussian random variable (RV) with variance $E[|G_0|^2] = \Omega_0$, where $E[.]$ denotes the expectation operator. Similarly, for the jth relay (R_j), we denote the gain on the $S \rightarrow R_j$ link by G_1^j, and that on the $R_j \rightarrow D$ link by G_2^j, with

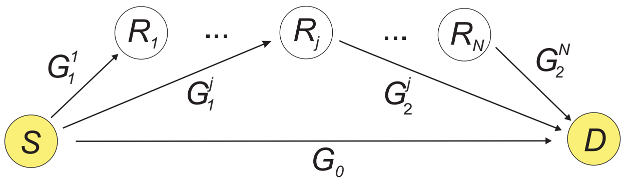

Fig. 1. Sketch of the multi-relay demodulate-and-forward system model under consideration.

variances $E[|G_1^j|^2] = \Omega_1^j$ and $E[|G_2^j|^2] = \Omega_2^j$, respectively. We note that the average path loss across various links in the relay network can be modeled by an approach choice of values for Ω_0 and $\{\Omega_1^j, \Omega_2^j\}_{j=1}^N$. The noise RVs, denoted as η_0 on the $S \to D$ link and η_1^j and η_2^j for each $S \to R_j$ and $R_j \to D$ link, respectively, are assumed to be independent, and are also independent of the transmitted symbols and channel gains. Further, η_0, $\{\eta_j^1, \eta_j^2\}_{j=1}^N$ are modeled as zero-mean additive circularly symmetric complex Gaussian RVs each with a variance of σ_N^2. We denote by $\mathcal{S}$ the signal constellation employed by the source. In this paper, we restrict $\mathcal{S}$ to be a rectangular QAM constellation of size $M_1 \times M_2$, consisting of the product of two PAM constellations, of sizes M_1 and M_2, on the in-phase and quadrature-phase branches, respectively.

In this paper, we assume a two-phase coherent relay signal processing protocol, according to which, the source broadcasts its message during the first phase, while the relays and destination are in listening mode. During the second phase, the source remains silent and the relays perform independent demodulation of the source signal. They then forward their respective versions of the demodulated signal to the destination on orthogonal channels. Finally, the destination node performs coherent combining of the signals received during the first and second phases. Clearly, knowledge of the fading channel realizations at each relay and destination nodes is crucial for the operation of this two-phase protocol. Quite often, pilot symbols are embedded at the source in order for the relays and destination to estimate the channel gains from the source. Similarly, each relay node can send its own pilot symbols to enable the destination to estimate the corresponding channel gain from the relay. The CSI at the receivers may not always be perfect for a variety of reasons such as the presence of noise on the received pilots and the time-variation of the fading channel due to node mobility.

Notwithstanding the specifics of the CSI acquisition process, we model the channel estimates on the $S \to D$, $S \to R_j$ and $R_j \to D$ links as zero-mean complex-Gaussian RVs that are independent of each other. This model captures quite accurately such practical scenarios as $i)$ a pilot-symbol assisted modulation (PSAM) technique [18] is employed to estimate the channel gains of interest, $ii)$ the channel estimates are obtained by linear filtering of the received pilot symbols, or $iii)$ the channel estimates are delayed (i.e., stale or outdated, due to Doppler) versions [19] of the true channels at the respective receiver nodes. From a notational viewpoint, we denote the channel estimate on the $S \to D$ link by H_0 with variance $E[|H_0|^2] = \Lambda_0$. For relay R_j, the estimates on the $S \to R_j$ and $R_j \to D$ links are denoted as H_1^j and H_2^j, respectively, with respective variances $E[|H_1^j|^2] = \Lambda_1^j$ and $E[|H_2^j|^2] = \Lambda_2^j$. The normalized correlation coefficient between the true and the estimated channel gains on the $S \to D$, $S \to R_j$ and $R_j \to D$ links are denoted by $\rho_0 = E[G_0 H_0^*]/\sqrt{\Omega_0 \Lambda_0}$, $\rho_1(j) = E[G_1^j H_1^{j*}]/\sqrt{\Omega_1^j \Lambda_1^j}$ and $\rho_2(j) = E[G_2^j H_2^{j*}]/\sqrt{\Omega_2^j \Lambda_2^j}$, respectively.

The received signal at D in the first time slot is given by

$$Y_0 = G_0 S_0 + \eta_0, \tag{1}$$

where $S_0 \in \mathcal{S}$, whereas at the jth relay it is given by

$$Y_1^j = G_1^j S_0 + \eta_1^j, \quad j = 1, \ldots, N. \tag{2}$$

We denote by S_j the demodulated signal at R_j based on Y_1^j and the channel estimate H_1^j. That is,

$$S_j = \underset{X \in \mathcal{S}}{\operatorname{argmin}} \{|X - Y_1^j / H_1^j|^2\}. \tag{3}$$

In the second time-slot, relay R_j transmits the demodulated symbol S_j which is received at D as

$$Y_2^j = S_j G_2^j + \eta_2^j \quad j = 1, \ldots, N. \tag{4}$$

The destination coherently combines Y_0 in (1) and $\{Y_2^j\}_{j=1}^N$ in (4) using the rule of MRC, hence the demodulator test statistic (DTS) Z at node D, as given by

$$Z = Z_I + \Im Z_Q = \frac{Y_0 H_0^* + \sum_{j=1}^N Y_2^j H_2^{j*}}{|H_0|^2 + \sum_{j=1}^N |H_2^j|^2}, \tag{5}$$

where $\Im = \sqrt{-1}$. We note that the DTS with ideal demodulate-and-forward relaying is obtained by setting $H_2^j = G_2^j$ and $H_0 = G_0$ (i.e., perfect CSI at D) in (5) along with $S_j = S_0$ (i.e., error-free demodulation by relay R_j) in (4).

III. Distribution of Demodulator Test Statistic

We first notice that both the real part, Z_I, and imaginary part, Z_Q, of the DTS (5), are sufficient test statistics to demodulate the bits of the in-phase M_1-ary PAM, and quadrature-phase M_2-ary PAM constellations, respectively. Thus, for the remainder of this section, we focus on deriving expressions for the CDF of Z_I and Z_Q. For the sake of notational convenience, let us define $Y_2^0 = Y_0$, $G_2^0 = G_0$, $H_2^0 = H_0$, $\eta_2^0 = \eta_0$, $\Omega_2^0 = \Omega_0$, $\Lambda_2^0 = \Lambda_0$ and $\rho_2(0) = \rho_0$. Then, consider the following RVs for $k = 0, \ldots, N$:

$$\Psi_k(x; S_k) = Y_2^k H_2^{k*} + Y_2^{k*} H_2^k - 2x H_2^k H_2^{k*}, \tag{6}$$

and

$$\Xi_k(x; S_k) = \Im Y_2^{k*} H_2^k - \Im Y_2^k H_2^{k*} - 2x H_2^k H_2^{k*}. \tag{7}$$

Note that the dependency of $\Psi_k(\cdot;\cdot)$ and $\Xi_k(\cdot;\cdot)$ in (6) and (7) on S_k is due to the fact that Y_2^k depends on S_k via (4). Clearly, conditioned on S_k, $\Psi_k(x; S_k)$ and $\Xi_k(x; S_k)$ are real-valued quadratic forms in the complex Gaussian RVs Y_2^k and H_2^k.

Using results from [20], the characteristic function (CHF) of Ψ_k conditioned on S_j, $\Phi_k(\Im\nu; S_k) := E[e^{\Im \nu \Psi_k} | S_k]$, can be expressed as

$$\Phi_k(\Im\nu; S_k) = -\frac{\nu_1(k)\nu_2(k)}{(\nu + \Im\nu_1(k))(\nu - \Im\nu_2(k))}, \tag{8}$$

where

$$\nu_1(k) = \sqrt{\omega^2(k) + \frac{1}{\sigma_Y^2(k)\sigma_H^2(k) - |\sigma_{YH}(k)|^2}} - \omega(k), \tag{9}$$

$$\nu_2(k) = \sqrt{\omega^2(k) + \frac{1}{\sigma_Y^2(k)\sigma_H^2(k) - |\sigma_{YH}(k)|^2}} + \omega(k), \tag{10}$$

$$\omega(k) = \frac{-x\sigma_H^2(k) + \text{Real}\{\sigma_{YH}(k)\}}{\sigma_Y^2(k)\sigma_H^2(k) - |\sigma_{YH}(k)|^2}, \tag{11}$$

$$\sigma_Y^2(k) \triangleq E\left[|Y_2^k|^2 \,|S_k\right] = |S_k|^2 \Omega_2^k + \sigma_N^2, \tag{12}$$

$$\sigma_H^2(k) \triangleq E\left[|H_2^k|^2\right] = \Lambda_2^k, \tag{13}$$

$$\sigma_{YH}(k) \triangleq E\left[Y_2^k H_2^{k*} \,|S_k\right] = S_k \rho_2(k) \sqrt{\Omega_2^k \Lambda_2^k}. \tag{14}$$

Likewise, the CHF of Ξ_k conditioned on S_k, $\Upsilon_k(\Im\mu; S_k) := E[e^{\Im\mu\Xi_k}|S_k]$, can be obtained as

$$\Upsilon_k(\Im\mu; S_k) = -\frac{\mu_1(k)\mu_2(k)}{(\mu + \Im\mu_1(k))(\mu - \Im\mu_2(k))}, \quad (15)$$

where

$$\mu_1(k) = \sqrt{\tau^2(k) + \frac{1}{\sigma_Y^2(k)\sigma_H^2(k) - |\sigma_{YH}(k)|^2}} - \tau(k), \quad (16)$$

$$\mu_2(k) = \sqrt{\tau^2(k) + \frac{1}{\sigma_Y^2(k)\sigma_H^2(k) - |\sigma_{YH}(k)|^2}} + \tau(k), \quad (17)$$

$$\text{and } \tau(k) = -\frac{x\sigma_H^2(k) + \text{Imag}\{\sigma_{YH}(k)\}}{\sigma_Y^2(k)\sigma_H^2(k) - |\sigma_{YH}(k)|^2}. \quad (18)$$

Conditioned on the modulation symbols S_l, $l = 0, 1, \ldots, N$, the CDF of Z_I, $F_{Z_I}\left(x; \{S_l\}_{l=0}^N\right) := \text{Prob}\left(Z_I \le x \,\middle|\{S_l\}_{l=0}^N\right)$, can be expressed as

$$\begin{aligned} F_{Z_I}\left(x; \{S_l\}_{l=0}^N\right) &= \text{Prob}\left(\sum_{k=0}^N \Psi_k(x, S_j) \le 0 \,\middle|\{S_l\}_{l=0}^N\right) \\ &= -\frac{1}{2\pi\Im}\int_{-\infty+\Im\epsilon}^{\infty+\Im\epsilon} \frac{d\nu}{\nu}\prod_{k=0}^N \Phi_k(\Im\nu; S_k) \\ &= \frac{(-1)^N}{2\pi\Im}\int_{-\infty+\Im\epsilon}^{\infty+\Im\epsilon} \frac{d\nu}{\nu}\prod_{k=0}^N \frac{\nu_1(k)\nu_2(k)}{(\nu + \Im\nu_1(k))(\nu - \Im\nu_2(k))}, \end{aligned} \quad (19)$$

where $\epsilon > 0$ in order to ensure convergence of the integral. Since the $\nu_1(k)$s and $\nu_2(k)$s are distinct for i.n.i.d. $S \to D$ and $\{R_k \to D\}_{k=1}^N$ links, a closed-form solution for the conditional CDF can be obtained by inverting (19), thus leading to[1]

$$F_{Z_I}\left(x; \{S_l\}_{l=0}^N\right) = \sum_{l=0}^N \frac{1}{\nu_2(l)}\prod_{k=0}^N \frac{\nu_1(k)\nu_2(k)}{\nu_1(k) + \nu_2(l)}\prod_{m=0, m\ne l}^N \frac{1}{\nu_2(m) - \nu_2(l)}. \quad (20)$$

Along the similar steps, the conditional CDF of the imaginary part Z_Q of Z, $F_{Z_Q}\left(x; \{S_l\}_{l=0}^N\right) := \text{Prob}\left(Z_Q \le x \,\middle|\{S_l\}_{l=0}^N\right)$, can be reduced to

$$\begin{aligned} F_{Z_Q}\left(x; \{S_l\}_{l=0}^N\right) &= \text{Prob}\left(\sum_{k=0}^N \Xi_k(x, S_j) \le 0 \,\middle|\{S_l\}_{l=0}^N\right) \\ &= -\frac{1}{2\pi\Im}\int_{-\infty+\Im\epsilon}^{\infty+\Im\epsilon} \frac{d\mu}{\mu}\prod_{k=0}^N \Upsilon_k(\Im\mu; S_k) \\ &= \frac{(-1)^N}{2\pi\Im}\int_{-\infty+\Im\epsilon}^{\infty+\Im\epsilon} \frac{d\mu}{\mu}\prod_{k=0}^N \frac{\mu_1(k)\mu_2(k)}{(\mu + \Im\mu_1(k))(\mu - \Im\mu_2(k))} \\ &= \sum_{l=0}^N \frac{1}{\mu_2(l)}\prod_{k=0}^N \frac{\mu_1(k)\mu_2(k)}{\mu_1(k) + \mu_2(l)}\prod_{m=0, m\ne l}^N \frac{1}{\mu_2(m) - \mu_2(l)}. \end{aligned} \quad (21)$$

Upon averaging over the relay constellation symbols, $\{S_1, \ldots, S_N\}$, the CDFs of Z_I and Z_Q can respectively be expressed as

$$F_{Z_I}(x; S_0) \triangleq \sum_{S_1}\cdots\sum_{S_N}\left\{\prod_{k=1}^N \text{Prob}(S_k \mid S_0)\right\} \times F_{Z_I}\left(x; \{S_l\}_{l=0}^N\right) \quad (22)$$

[1]For non-distinct values of $\nu_1(k)$s and $\nu_2(k)$s, the CHF can still be inverted following the approach proposed in [21].

and

$$F_{Z_Q}(x; S_0) \triangleq \sum_{S_1}\cdots\sum_{S_N}\left\{\prod_{k=1}^N \text{Prob}(S_k \mid S_0)\right\} \times F_{Z_Q}\left(x; \{S_l\}_{l=0}^N\right), \quad (23)$$

where $\text{Prob}(S_k \mid S_0)$ is the probability that the kth relay demodulates the source constellation symbol S_0 as S_k. A closed-form expression for $\mathbb{P}_k(i|j) := \text{Prob}(S_k = i \mid S_0 = j)$ is derived in the journal version of this paper [22] and is provided here without proof as (24), given at the top of next page, where

$$\alpha_k + \Im\beta_k = \frac{E[G_1^k H_1^{k*}]}{\Lambda_1^k} = \rho_1(k)\sqrt{\frac{\Omega_1^k}{\Lambda_1^k}}, \quad (25)$$

$$\Theta_{j,k} = \frac{2\Lambda_1^k}{\sigma_N^2 + \left(\Omega_1^k - \Lambda_1^k(\alpha_k^2 + \beta_k^2)\right)|s_j|^2}, \quad (26)$$

$$\begin{aligned} &\mathcal{H}(a, b) = \mathcal{G}(a, b)\mathbf{1}_{\{a\ge 0, b\ge 0\}} + \\ &\{2\mathcal{G}(a, 0) - \mathcal{G}(a, |b|)\}\,\mathbf{1}_{\{a\ge 0, b<0\}} \\ &+ \{1 - 2\mathcal{G}(|a|, 0) - 2\mathcal{G}(0, |b|) + \mathcal{G}(|a|, |b|)\}\,\mathbf{1}_{\{a<0, b<0\}} \\ &+ \{2\mathcal{G}(0, b) - \mathcal{G}(|a|, b)\}\,\mathbf{1}_{\{a<0, b\ge 0\}}, \end{aligned} \quad (27)$$

and

$$\begin{aligned} \mathcal{G}(|a|, |b|) = \; & \frac{1}{4} - \frac{1}{2\pi}\sqrt{\frac{b^2}{2 + b^2}}\tan^{-1}\left(\sqrt{\frac{2 + b^2}{a^2}}\right) \\ & - \frac{1}{2\pi}\sqrt{\frac{a^2}{2 + a^2}}\tan^{-1}\left(\sqrt{\frac{2 + a^2}{b^2}}\right). \end{aligned} \quad (28)$$

In (27), $\mathbf{1}_{\{\mathsf{A}\}}$ is the indicator function that evaluates to one (zero) when the condition A is true (false).

IV. Average Bit Error Probability Analysis

Let us denote the $M_1 \times M_2$-QAM constellation points as $s_m = s_x + \Im s_y$, $m = 0, 1, \ldots, M - 1$, where $M = M_1M_2$, $x = 0, 1, \ldots, M_1 - 1$, $y = 0, 1, \ldots, M_2 - 1$, $s_x = (2x - M_1 + 1)d$, $s_y = (2y - M_2 + 1)d$, and $2d$ is the minimum distance between two constellation points. We define $k_1 = \log_2(M_1)$, $k_2 = \log_2(M_2)$, and the sets $\mathcal{X} = \{0, 1, \ldots, M_1 - 1\}$ and $\mathcal{Y} = \{0, 1, \ldots, M_2 - 1\}$. The Gray code mapping is denoted by the vectors $(a_{k_1-1}, a_{k_1-2}, \ldots, a_0)$, and $(b_{k_2-1}, b_{k_2-2}, \ldots, b_0)$ for the in-phase signal s_x and the quadrature-phase signal s_y, respectively. Additionally, for $i = 0, \ldots, k_1 - 1$, we define the sets $X_1(i) = \{x : (x \bmod 2^{i+2}) = 2^i + l, l = 0, \ldots, 2^i - 1\} \cup \{x : (x \bmod 2^{i+2}) = 2^{i+1} + l, l = 0, \ldots, 2^i - 1\}$ and $X_0(i) = \{x : (x \bmod 2^{i+2}) = l, l = 0, \ldots, 2^i - 1\} \cup \{x : (x \bmod 2^{i+2}) = 3 \times 2^i + l, l = 0, \ldots, 2^i - 1\}$. For $j = 0, \ldots, k_2 - 1$, we define the sets $Y_1(j) = \{y : (y \bmod 2^{j+2}) = 2^j + l, l = 0, \ldots, 2^j - 1\} \cup \{y : (y \bmod 2^{j+2}) = 2^{j+1} + l, l = 0, \ldots, 2^j - 1\}$ and $Y_0(j) = \{y : (y \bmod 2^{j+2}) = l, l = 0, \ldots, 2^j - 1\} \cup \{y : (y \bmod 2^{j+2}) = 3 \times 2^j + l, l = 0, \ldots, 2^j - 1\}$.

With the help of above sets, tabulation of which can be found in [23, Table 1], the decision rule for each bit a_i, $i = 0, \ldots, k_1 - 1$, is given by the following disjoint union of intervals on the x-axis [24]:

$$\widehat{a}_i = \begin{cases} 1 & \text{if } Z_I \in \cup_{x\in X_1(i)}\Big[-\infty \times \mathbf{1}_{\{x=0\}} + s_x - d, \\ & \qquad \infty \times \mathbf{1}_{\{x=M_1-1\}} + s_x + d\Big) \\ 0 & \text{otherwise} \end{cases}, \quad (29)$$

whereas for bit b_j, $j = 0, \ldots, k_2 - 1$, it is given by the following

$$
\mathbb{P}_k(i|j) = \begin{cases}
\mathcal{H}\left(-(s_{x'} - \alpha_k s_x + \beta_k s_y + d)\sqrt{\Theta_{j,k}}, -(s_{y'} - \beta_k s_x - \alpha_k s_y + d)\sqrt{\Theta_{j,k}}\right) & i = 0 \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y - d)\sqrt{\Theta_{j,k}}, -(s_{y'} - \beta_k s_x - \alpha_k s_y + d)\sqrt{\Theta_{j,k}}\right) & i = M_1 - 1 \\
\mathcal{H}\left(-(s_{x'} - \alpha_k s_x + \beta_k s_y + d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y - d)\sqrt{\Theta_{j,k}}\right) & i = M_1(M_2 - 1) \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y - d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y - d)\sqrt{\Theta_{j,k}}\right) & i = M_1 M_2 - 1 \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y - d)\sqrt{\Theta_{j,k}}, -(s_{y'} - \beta_k s_x - \alpha_k s_y + d)\sqrt{\Theta_{j,k}}\right) - & \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y + d)\sqrt{\Theta_{j,k}}, -(s_{y'} - \beta_k s_x - \alpha_k s_y + d)\sqrt{\Theta_{j,k}}\right) & i = 1, \ldots, M_1 - 2 \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y - d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y - d)\sqrt{\Theta_{j,k}}\right) - & \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y + d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y - d)\sqrt{\Theta_{j,k}}\right) & i - M_1(M_2 - 1) = 1, \ldots, (M_1 - 2) \\
\mathcal{H}\left(-(s_{x'} - \alpha_k s_x + \beta_k s_y + d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y - d)\sqrt{\Theta_{j,k}}\right) - & \\
\mathcal{H}\left(-(s_{x'} - \alpha_k s_x + \beta_k s_y + d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y + d)\sqrt{\Theta_{j,k}}\right) & i = M_1, 2M_1, \ldots, (M_2 - 2)M_1 \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y - d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y - d)\sqrt{\Theta_{j,k}}\right) - & \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y - d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y + d)\sqrt{\Theta_{j,k}}\right) & i - M_1 + 1 = M_1, 2M_1, \ldots, (M_2 - 2)M_1 \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y - d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y - d)\sqrt{\Theta_{j,k}}\right) - & \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y - d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y + d)\sqrt{\Theta_{j,k}}\right) - & x' = 1, 2, \ldots, M_1 - 2 \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y + d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y - d)\sqrt{\Theta_{j,k}}\right) + & y' = 1, 2, \ldots, M_2 - 2 \\
\mathcal{H}\left((s_{x'} - \alpha_k s_x + \beta_k s_y + d)\sqrt{\Theta_{j,k}}, (s_{y'} - \beta_k s_x - \alpha_k s_y + d)\sqrt{\Theta_{j,k}}\right) & i = x' + M_2 y'
\end{cases} \quad (24)
$$

disjoint union of intervals on the y-axis

$$
\widehat{b}_j = \begin{cases} 1 & \text{if } Z_Q \in \cup_{y \in Y_1(j)} \Big[-\infty \times \mathbf{1}_{\{y=0\}} + s_y - d, \\ & \qquad \infty \times \mathbf{1}_{\{y=M_2-1\}} + s_y + d\Big) \\ 0 & \text{otherwise.} \end{cases} \quad (30)
$$

Clearly, evaluating the average BEP for the constellation, using (29) and (30), requires knowledge of the CDF of Z_I and Z_Q, which are derived in (22) and (23), respectively, as a function of the source constellation symbol S_0. Upon using (22) and (23), the average BEP for bits a_i and b_j can be expressed as (31) and (32), respectively, shown at the top of the next page. The average BEP for the whole constellation is obtained by averaging over a_i and b_j as follows

$$
P_b = \frac{\sum_{i=1}^{\log_2(M_1)} P_b(a_i) + \sum_{j=1}^{\log_2(M_2)} P_b(b_j)}{\log_2(M_1) + \log_2(M_2)}. \quad (33)
$$

V. Results and Discussion

In this section, we present some numerical and simulation results on the average BEP performance of demodulate-and-forward relaying with multi-level modulations and imperfect receiver CSI. Note that the average BEP analysis in Section IV is valid for an arbitrary number of relays and rectangular QAM constellations with arbitrary size. However, for the sake of simplicity, we restrict the results presented in this section and the scope of our discussion to the case of a single relay node with 16- and 64-QAM square constellations.

Fig. 2 shows the impact of imperfect CSI on the average BEP performance of demodulate-and-forward relaying with Gray-coded 16-QAM as a function of the average received SNR on the $S \to D$ link. The average received SNR is set to 10 dB and 20 dB on the $S \to R$ and $R \to D$ links, respectively. The $S \to D$, $S \to R$ and $R \to D$ channels are estimated using pilot symbols with an average received pilot SNR of 10, 5 and 15 dB, respectively. We denote by p (perfect) and i (imperfect) the quality of the available CSI on a particular link. Hence, a total of 8 distinct possibilities exist describing the quality of the $(S \to D, S \to R, R \to D)$ links, ranging from (p, p, p) (all the links with perfect CSI) to (i, i, i) (all the links with imperfect CSI). The average BEP performance for the 8 distinct cases is shown in Fig. 2 along with the lower bound on the average BEP, which is obtained by assuming perfect CSI on all the links together with error-free demodulation at the relay. From Fig. 2, we conclude that even with perfect CSI on all the links, relay demodulation incurs approximately 25 dB loss in performance at 1 percent error rate. Among the $(S \to D, S \to R, R \to D)$ links, imperfect CSI on the $S \to D$ link causes maximum degradation in the average error performance.

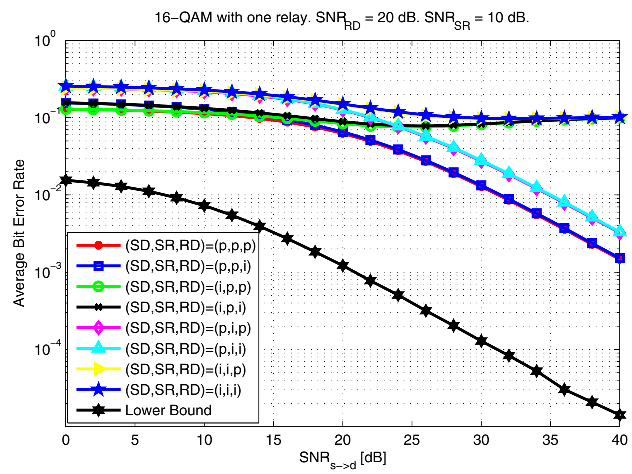

Fig. 2. Average BEP Performance of Gray-coded 16-QAM with single relay and imperfect CSI.

The average BEP performance for 64-QAM modulation is plotted in Fig. 3. In this figure, the average SNRs (in dB) on the $S \to R$ and $R \to D$ links are set to 20 and 30, respectively, whereas the average received pilot SNRs (in dB) on the $S \to D$, $S \to R$ and $R \to D$ links are set to 20, 10 and 20, respectively. Corroborating the trends observed in Fig. 2, it is apparent from Fig. 3 that, for the chosen set of parameters, even with perfect receiver CSI, relay demodulation errors alone lead to a significant loss in the average error performance. On top of this, imperfect CSI on the $S \to D$ link causes further degradation in error performance in terms of an irreducible error floor.

VI. Conclusion

In this paper, we studied the performance of a multi-relay demodulate-and-forward cooperation protocol with higher-order M-QAM modulation. Considering i.n.i.d. Rayleigh fading channels with imperfect receiver CSI, we derived closed-form expressions for the transition probability of error at the relays, the distribution of DTS and the average BEP at the destination. Our numerical and simulation results revealed that relay demodulation errors, even with perfect CSI, can lead to severe loss in receiver performance, whereas imperfect CSI introduces an irreducible error floor on the average error probability.

$$
\begin{aligned}
P_b(a_i) \;=\;& \frac{1}{M} \sum_{x_0 \in X_0(i)} \sum_{x_1 \in X_1(i)} \sum_{y \in \mathcal{Y}} \Big\{ F_{Z_I}\left(\infty \times \mathbf{1}_{\{x_1 = M_1 - 1\}} + (2x_1 + 2 - M_1)d; (2x_0 + 1 - M_1)d + \Im(2y + 1 - M_2)d\right) \\
& - F_{Z_I}\left(-\infty \times \mathbf{1}_{\{x_1 = 0\}} + (2x_1 - M_1)d; (2x_0 + 1 - M_1)d + \Im(2y + 1 - M_2)d\right) \Big\} + \\
& \frac{1}{M} \sum_{x_1 \in X_1(i)} \sum_{x_0 \in X_0(i)} \sum_{y \in \mathcal{Y}} \Big\{ F_{Z_I}\left(\infty \times \mathbf{1}_{\{x_0 = M_1 - 1\}} + (2x_0 + 2 - M_1)d; (2x_1 + 1 - M_1)d + \Im(2y + 1 - M_2)d\right) \\
& - F_{Z_I}\left(-\infty \times \mathbf{1}_{\{x_0 = 0\}} + (2x_0 - M_1)d; (2x_1 + 1 - M_1)d + \Im(2y + 1 - M_2)d\right) \Big\} \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
\text{and } P_b(b_j) \;=\;& \frac{1}{M} \sum_{y_0 \in Y_0(j)} \sum_{y_1 \in Y_1(j)} \sum_{x \in \mathcal{X}} \Big\{ F_{Z_Q}\left(\infty \times \mathbf{1}_{\{y_1 = M_2 - 1\}} + (2y_1 + 2 - M_2)d; (2x + 1 - M_1)d + \Im(2y_0 + 1 - M_2)d\right) \\
& - F_{Z_Q}\left(-\infty \times \mathbf{1}_{\{y_1 = 0\}} + (2y_1 - M_2)d; (2x + 1 - M_1)d + \Im(2y_0 + 1 - M_2)d\right) \Big\} + \\
& \frac{1}{M} \sum_{y_1 \in Y_1(j)} \sum_{y_0 \in Y_0(j)} \sum_{x \in \mathcal{X}} \Big\{ F_{Z_Q}\left(\infty \times \mathbf{1}_{\{y_0 = M_2 - 1\}} + (2y_0 + 2 - M_2)d; (2x + 1 - M_1)d + \Im(2y_1 + 1 - M_2)d\right) \\
& - F_{Z_Q}\left(-\infty \times \mathbf{1}_{\{y_0 = 0\}} + (2y_0 - M_2)d; (2x + 1 - M_1)d + \Im(2y_1 + 1 - M_2)d\right) \Big\}. \qquad (32)
\end{aligned}
$$

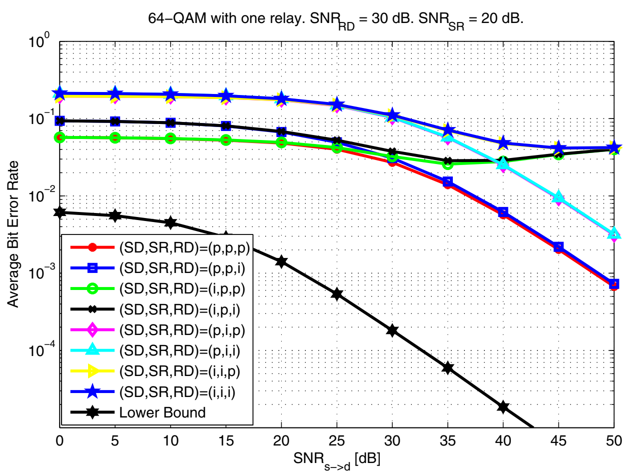

Fig. 3. Average BEP Performance of Gray-coded 64-QAM with single relay and imperfect CSI.

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