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A New Coupling Channel Estimator for Cross-Talk Cancellation at Wireless Relay Stations

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Abstract—**In this paper, we are concerned with cross-talk interference from the transmit to the receive antenna of a wireless *channel-reuse-relay-station* (CRRS) that forwards signals over the same channel as it receives signals. By estimating the coupling channel from the transmit to the receive antenna, the proposed scheme performs cross-talk reconstruction and cancellation at the RS. Different from the conventional coupling channel estimation schemes that require the RS to transmit dedicated pilots, the proposed scheme utilizes the random forwarded signals of the RS as pilots for coupling channel estimation, thus avoiding changing the structure of the RS's transmitted signal. For a general RS with any relay mechanism, we propose a least square coupling channel estimator; for an RS with the decode-and-forward mechanism, we further propose a minimum mean-square error coupling channel estimator. Also, we investigate the performance of the proposed cross-talk cancellation scheme with both analytical and numerical results.**

I. INTRODUCTION

In cellular communication systems, the quality of the received signal at a *mobile station* (MS) on the cell edge or in a severely shadowed region can not be guaranteed. To solve this problem, wireless relay techniques have been proposed. As a repeater, a *relay station* (RS) forwards data from the *base station* (BS) to the MS or vice versa. It has been demonstrated that wireless relay can effectively extend the coverage, decrease the overall transmit power, and enhance the capacity or reliability of the communication links [1]-[3].

Conventionally, the RS receives data from the BS (or the MS) over one channel, and forwards them to the MS (or the BS) over the other channel. In this way, interference between the BS-RS and the RS-MS hops is avoided, which, however, halves the spectral efficiency since the communication between the BS and the RS consumes a part of the time or frequency resource allocated to the MS. Alternatively, the RS may forward signals to the destination over the same channel as it receives signals. An RS working in this mode is called a *channel-reuse-relay-station* (CRRS) in this paper. In *Digital Video Broadcasting* (DVB), on-channel repeaters [4]-[6] have been applied as in-band RS's to extend the signal coverage. These on-channel repeaters amplify and forward their received signals without any channel translation, i.e., the same channel is utilized over both the BS-repeater and the repeater-MS links. In contrast with the conventional RS, a CRRS not only doubles the spectral efficiency, but also avoids changing the existing physical layer and link layer to support relay mechanism. However, a critical issue involved in a CRRS is severe co-channel cross-talk interference from its transmit antenna to its receive antenna. Since the transmit power of an RS is dramatically greater than its received desired signal power, such cross-talk interference, if not cancelled properly, will keep the RS from receiving signals from the BS and the MS. To mitigate cross-talk interference at an RS, a high isolation between the transmit and the receive antennas is required. To that end, directional transmission and reception can be applied at the RS; furthermore, a shield may be put between the transmit and the receive antennas of the RS. Besides high-isolation antennas, both analog and digital cancellers need to be implemented at the RS to further mitigate the cross-talk interference.

In this paper, we focus on developing digital cross-talk cancellation techniques for a CRRS. With the help of high isolation antennas and analog cancellers, the strength of cross-talk interference at the RS is already significantly reduced, paving the way for further digital cancellation. In the literature, various techniques for digital cross-talk cancellation based on the estimation of the coupling channel from the transmit to the receive antenna of the RS have been proposed [5]-[7]. However, all of these techniques require the RS to transmit dedicated pilots, such as the pseudo-noise sequences, for coupling channel estimation, which not only changes the original signal structure of the physical layer but also results in interference at the destination. In practice, cross-talk cancellation at the RS is supposed to be performed in a way completely transparent to the existing wireless standard, i.e., no modifications should be made to the signal structure of the physical layer, to the BS, and to the MS. Therefore, it is desired to develop a new cross-talk cancellation technique without requiring the RS to transmit any dedicated pilots.

The remainder of this paper is organized as follows. In Section II, cross-talk interference at a CRRS is modeled. Then we develop a new cross-talk canceller based on the *least square* (LS) coupling channel estimation without dedicated pilots in Section III. In Section IV, we develop a *minimum mean-square error* (MMSE) coupling channel estimator for cross-talk cancellation at a RS with the decode-and-forward mechanism. Then we present numerical and simulation results in Section V. Finally we conclude this paper in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Throughout this paper, we assume that the RS is working in a wireless network where OFDM modulation is utilized for broadband transmission. After receiving a whole OFDM symbol[1], the RS forwards it to the MS or the BS after appropriate processing according to the specific relay mechanism applied. To facilitate analysis, we assume that the duration of the signal processing at the RS is shorter than that of the OFDM cyclic extension. As a result, there is exactly one OFDM symbol delay at the RS and the received OFDM symbol of the RS is a summation of the desired symbol from the source and the cross-talk symbol from itself.

Without loss of generality, we consider an RS equipped with M_r receive antennas and M_t transmit antennas. Denote K as the total number of OFDM subcarriers, $\mathbf{s}(n,k)$ as the M_r-dimensional desired signal vector at the RS over the kth $(0 \le k \le K-1)$ subcarrier of the nth OFDM symbol, and $\mathbf{w}(n,k)$ as the white noise vector independent of $\mathbf{s}(n,k)$, then the received signal vector of the RS is given by

$$\mathbf{y}(n,k) = \mathbf{s}(n,k) + \mathbf{i}(n,k) + \mathbf{w}(n,k), \tag{1}$$

where $\mathbf{i}(n,k)$ denotes the cross-talk interference vector. Denote $\hat{\mathbf{x}}(n-1,k)$ as the M_t-dimensional transmitted signal vector of the RS, which is a forwarded version of $\mathbf{s}(n-1,k)$, and then $\mathbf{i}(n,k)$ is given by

$$\mathbf{i}(n,k) = \mathbf{H}_c(k)\,\hat{\mathbf{x}}(n-1,k), \tag{2}$$

where $\mathbf{H}_c(k)$ denotes the frequency-domain $M_r \times M_t$ coupling channel matrix over the kth OFDM subcarrier from the transmit to the receive antenna of the RS. Throughout this paper, we model the time-domain coupling channel as a multiple-tap one with the delay of each tap equal to integer OFDM sampling intervals. Suppose the maximum delay of the coupling channel is L OFDM sampling intervals and denote $\mathbf{H}_{c,l}$ as the time-domain coupling channel matrix over the lth $(0 \le l \le L-1)$ tap, and then $\mathbf{i}(n,k)$ can be expressed as

$$\begin{aligned}\mathbf{i}(n,k) &= \sum_{l=0}^{L-1} \mathbf{H}_{c,l} e^{-j\frac{2\pi lk}{K}} \hat{\mathbf{x}}(n-1,k)\\ &= \sum_{l=0}^{L-1} \left[e^{-j\frac{2\pi lk}{K}} \hat{\mathbf{x}}^T(n-1,k) \otimes \mathbf{I}_{M_r} \right] vec(\mathbf{H}_{c,l}),\end{aligned} \tag{3}$$

where $\mathbf{I}_{M_r}$ denotes the $M_r \times M_r$ identity matrix, $\otimes$ denotes the Kronecker product [8], and $vec(\mathbf{H}_{c,l})$ denotes the vectorization of $\mathbf{H}_{c,l}$ formed by stacking the columns of $\mathbf{H}_{c,l}$ into a single column vector. Define

$$\hat{\mathbf{X}}(n-1,k,l) = e^{-j\frac{2\pi lk}{K}} \hat{\mathbf{x}}^T(n-1,k) \otimes \mathbf{I}_{M_r}, \tag{4}$$

$$\begin{aligned}\hat{\mathbf{X}}(n-1,k) \;=\; & \Big(\hat{\mathbf{X}}(n-1,k,0), \hat{\mathbf{X}}(n-1,k,1),\\ & \cdots, \hat{\mathbf{X}}(n-1,k,L-1) \Big),\end{aligned} \tag{5}$$

and the composite coupling channel vector as

$$\mathbf{h}_c = \left(vec(\mathbf{H}_{c,0})^T, vec(\mathbf{H}_{c,1})^T, \cdots, vec(\mathbf{H}_{c,L-1})^T \right)^T, \tag{6}$$

and then $\mathbf{i}(n,k)$ can be expressed as

$$\mathbf{i}(n,k) = \hat{\mathbf{X}}(n-1,k)\,\mathbf{h}_c. \tag{7}$$

Further define the $KM_r \times LM_rM_t$ overall forwarded signal matrix of the RS as

$$\begin{aligned}\hat{\mathbf{X}}(n-1) \;=\; & \Big(\hat{\mathbf{X}}(n-1,0)^T, \hat{\mathbf{X}}(n-1,1)^T,\\ & \cdots, \hat{\mathbf{X}}(n-1,K-1)^T \Big)^T,\end{aligned} \tag{8}$$

and then the KM_r-dimensional overall cross-talk interference vector in the nth OFDM symbol is given by

$$\begin{aligned}\mathbf{i}(n) &= \left(\mathbf{i}(n,0)^T, \mathbf{i}(n,1)^T, \cdots, \mathbf{i}(n,K-1)^T \right)^T\\ &= \hat{\mathbf{X}}(n-1)\,\mathbf{h}_c.\end{aligned} \tag{9}$$

Correspondingly, the overall received signal vector of the RS in the nth OFDM symbol is given by

$$\mathbf{y}(n) = \mathbf{s}(n) + \mathbf{i}(n) + \mathbf{w}(n), \tag{10}$$

where

$$\mathbf{s}(n) = \left(\mathbf{s}(n,0)^T, \mathbf{s}(n,1)^T, \cdots, \mathbf{s}(n,K-1)^T \right)^T, \tag{11}$$

and

$$\mathbf{w}(n) = \left(\mathbf{w}(n,0)^T, \mathbf{w}(n,1)^T, \cdots, \mathbf{w}(n,K-1)^T \right)^T. \tag{12}$$

Based on (10), we define the *signal-to-interference ratio* (SIR) as $\mathrm{SIR} = \frac{P_s}{P_i}$ and the *signal-to-noise ratio* (SNR) as $\mathrm{SNR} = \frac{P_s}{P_w}$, where $P_s = E\left\{ \|\mathbf{s}(n)\|^2 \right\}$, $P_i = E\left\{ \|\mathbf{i}(n)\|^2 \right\}$, and $P_w = E\left\{ \|\mathbf{w}(n)\|^2 \right\}$ are the average sum powers of the desired signal vector, the cross-talk interference vector, and the noise vector, respectively. Since the transmit power of the RS is dramatically larger than its received desired signal power, the SIR at the RS is so low that the cross-talk interference must be cancelled before the RS can work properly.

III. CROSS-TALK CANCELLATION BASED ON LEAST SQUARE COUPLING CHANNEL ESTIMATION

Since cross-talk interference at the RS consists of the previously transmitted signal of the RS coupled with the channel from the transmit to the receive antenna, it can be reconstructed and cancelled if the coupling channel state information is available. Therefore, it is natural to perform cross-talk cancellation based on coupling channel estimation. Since the transmit and the receive antennas of the RS are collocated, the coupling channel at the RS is highly static in the sense that it does not change much either across OFDM subcarriers or with time, which greatly facilitates the coupling channel estimation and promises an excellent cross-talk cancellation performance at the RS.

In this paper, we propose a new coupling channel estimation scheme for cross-talk cancellation at the RS, a block diagram of which is shown in Figure 1. Different from the

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[1] Usually signal processing at the RS is performed frame by frame, which consists of one or more OFDM symbols. Without loss of generality, we assume that one frame of the physical layer consists of only one OFDM symbol and thus signal processing at the RS is performed symbol by symbol.

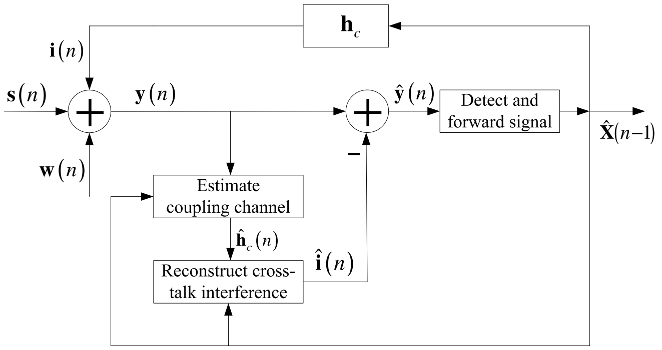

Fig. 1. Principle of cross-talk cancellation based on coupling channel estimation without dedicated pilots.

existing schemes, it does not require the RS to transmit any dedicated pilots. Specifically, we propose to utilize the random forwarded signal of the RS as an equivalent pilot for coupling channel estimation. Without inserting any dedicated pilots, the proposed scheme is especially attractive in practice because, firstly, no modification to the existing signal structure is incurred; secondly, no interference with the destination is caused; and thirdly, an accurate coupling channel estimate can be achieved as time goes by since every transmitted signal of the RS can be utilized as a pilot.

Mathematically, the proposed coupling channel estimation scheme tries to estimate $\mathbf{h}_c$ from

$$\mathbf{y}(n) = \hat{\mathbf{X}}(n-1)\,\mathbf{h}_c + \mathbf{s}(n) + \mathbf{w}(n), \tag{13}$$

where $\hat{\mathbf{X}}(n-1)$ works as the equivalent pilot matrix and the unknown desired signal, $\mathbf{s}(n)$, is regarded as noise just like $\mathbf{w}(n)$. Suppose that N recently received OFDM symbols of the RS are utilized to jointly estimate the coupling channel; denote the composite equivalent pilot matrix in the previous N OFDM symbols as

$$\hat{\mathbf{X}}_N = \left(\hat{\mathbf{X}}(n-1)^T, \hat{\mathbf{X}}(n-2)^T, \cdots, \hat{\mathbf{X}}(n-N)^T\right)^T, \tag{14}$$

and then the corresponding composite received signal vector of the RS is given by

$$\begin{aligned}\mathbf{y}_N &= \left(\mathbf{y}(n)^T, \mathbf{y}(n-1)^T, \cdots, \mathbf{y}(n-(N-1))^T\right)^T \\ &= \mathbf{i}_N + \mathbf{s}_N + \mathbf{w}_N,\end{aligned} \tag{15}$$

where $\mathbf{i}_N = \hat{\mathbf{X}}_N \mathbf{h}_c$ denotes the cross-talk interference and

$$\mathbf{s}_N = \left(\mathbf{s}(n)^T, \mathbf{s}(n-1)^T, \cdots, \mathbf{s}(n-(N-1))^T\right)^T, \tag{16}$$

$$\mathbf{w}_N = \left(\mathbf{w}(n)^T, \mathbf{w}(n-1)^T, \cdots, \mathbf{w}(n-(N-1))^T\right)^T. \tag{17}$$

From (15), we obtain the *least square* (LS) estimate of $\mathbf{h}_c$ in the nth OFDM symbol as [9]

$$\hat{\mathbf{h}}_{c,\mathrm{LS}}(n) = \left(\hat{\mathbf{X}}_N^H \hat{\mathbf{X}}_N\right)^{-1} \hat{\mathbf{X}}_N^H \mathbf{y}_N = \left[\mathbf{F}(n)\right]^{-1} \mathbf{g}(n), \tag{18}$$

where

$$\mathbf{F}(n) = \sum_{i=1}^{N} \hat{\mathbf{X}}^H(n-i)\,\hat{\mathbf{X}}(n-i), \tag{19}$$

and

$$\mathbf{g}(n) = \sum_{i=1}^{N} \hat{\mathbf{X}}^H(n-i)\,\mathbf{y}(n-i+1). \tag{20}$$

Based on the estimated coupling channel, the cross-talk interference at the RS is reconstructed as

$$\hat{\mathbf{i}}(n) = \hat{\mathbf{X}}(n-1)\,\hat{\mathbf{h}}_{c,\mathrm{LS}}(n), \tag{21}$$

and the estimate of the desired signal is obtained by subtracting $\hat{\mathbf{i}}(n)$ from the original received signal, i.e.,

$$\hat{\mathbf{y}}(n) = \mathbf{y}(n) - \hat{\mathbf{i}}(n) = \mathbf{s}(n) + \mathbf{w}(n) + \mathbf{e}(n), \tag{22}$$

where

$$\begin{aligned}\mathbf{e}(n) &= \hat{\mathbf{X}}(n-1)\left[\mathbf{h}_c - \hat{\mathbf{h}}_{c,\mathrm{LS}}(n)\right] \\ &= -\hat{\mathbf{X}}(n-1)\left(\hat{\mathbf{X}}_N^H \hat{\mathbf{X}}_N\right)^{-1} \hat{\mathbf{X}}_N^H \left(\mathbf{s}_N + \mathbf{w}_N\right),\end{aligned} \tag{23}$$

denotes the residual error vector after cross-talk cancellation. Equation (23) indicates that the amplitude of $\mathbf{e}(n)$ is independent of the average transmit power of the RS, namely the average amplitude of $\hat{\mathbf{X}}_N$ and $\hat{\mathbf{X}}(n-1)$, and is also independent of the amplitude of the coupling channel, $\mathbf{h}_c$. In other words, the amplitude of the residual error after cross-talk cancellation is independent of the average strength of cross-talk interference at the RS.

Based on (22), we define the post-SIR after cross-talk cancellation as[2]

$$\mathrm{PostSIR} = \frac{E\left\{\left\|\mathbf{s}(n)\right\|^2\right\}}{E\left\{\left\|\mathbf{e}(n)\right\|^2\right\}}, \tag{24}$$

which, according to the above analysis, is independent of the original SIR level before cross-talk cancellation. Intuitively, the specific value of the post-SIR for a given SNR depends on how autocorrelated the coupling channel is in the frequency-domain and in the time-domain; more precisely, it depends on the maximum delay spread of the coupling channel, L, and the maximum number of consecutive OFDM symbols during which the coupling channel remains unchanged, N.

As a final remark, cross-talk cancellation based on the LS coupling channel estimation proposed in this section is applicable to a wireless RS with the *decode-and-forward* (DF), the *amplify-and-forward* (AF), or any other relay schemes.

IV. COUPLING CHANNEL ESTIMATION AND CROSS-TALK CANCELLATION AT DF-BASED RS

In the last section, we have investigated cross-talk cancellation at a general RS based on the LS coupling channel estimation, which does not require any *a priori* knowledge on the coupling channel, the desired signal, and the noise. In this section, we will show that, when a DF-based RS is applied, the statistical characteristics of the coupling channel, the desired signal, and the noise can be conveniently utilized not only to

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[2] Note that $\mathbf{e}(n)$ and $\mathbf{s}(n)$ are generally statistically correlated because, as indicated in (16), $\mathbf{s}(n)$ constitutes $\frac{1}{N}$ part of $\mathbf{s}_N$ which works as noise during joint coupling channel estimation in N consecutive OFDM symbols. Since the coupling channel is highly static, N is usually a large number and therefore $\mathbf{e}(n)$ is approximately independent of $\mathbf{s}(n)$.

perform the *minimum mean-square error* (MMSE) coupling channel estimation, but also to analyze the performance of the proposed cross-talk canceller.

Recall that the composite received signal vector of the RS in the previous N OFDM symbols is given by

$$\mathbf{y}_N = \hat{\mathbf{X}}_N \mathbf{h}_c + \mathbf{s}_N + \mathbf{w}_N, \tag{25}$$

where $\mathbf{s}_N$ denotes the desired signal vector and $\hat{\mathbf{X}}_N$ denotes the RS's forwarded signal matrix, namely the equivalent pilot matrix for coupling channel estimation. Since $\hat{\mathbf{X}}_N$ is a forwarded version of $\mathbf{s}_N$ with one OFDM symbol delay, they are generally statistically correlated. However, determining the statistical distribution of $\mathbf{s}_N$ given $\hat{\mathbf{X}}_N$ is rather difficult since it involves imperfect cross-talk cancellation and desired signal detection at the RS in a noisy environment. As a result, the performance analysis of cross-talk cancellation based on coupling channel estimation at a general RS can not be delivered. However, when the DF scheme and constant power modulation are applied, the statistical correlation between $\mathbf{s}_N$ and $\hat{\mathbf{X}}_N$ disappears. To show this, we will first present the desired signal model in the following.

A. Desired Signal Model

Suppose there are M_s transmit antennas at the *source station* (SS), which may be the BS or the MS. Denote the transmitted signal vector of the SS over the kth subcarrier of the nth OFDM symbol as $\mathbf{x}(n,k)$, and the $M_r \times M_s$ channel matrix from the SS to the RS as $\mathbf{H}_r(n,k)$, and then the corresponding desired signal vector at the RS is given by

$$\mathbf{s}(n,k) = \mathbf{H}_r(n,k)\,\mathbf{x}(n,k)\,. \tag{26}$$

Similarly, the overall desired signal vector at the RS in the nth OFDM symbol given in (11) can be expressed as

$$\mathbf{s}(n) = \mathbf{H}_r(n)\,\mathbf{x}(n)\,, \tag{27}$$

where

$$\mathbf{x}(n) = \left(\mathbf{x}(n,0)^T, \mathbf{x}(n,1)^T, \cdots, \mathbf{x}(n,K-1)^T\right)^T, \tag{28}$$

and

$$\mathbf{H}_r(n) = \mathrm{diag}\left\{\mathbf{H}_r(n,0), \mathbf{H}_r(n,1), \cdots, \mathbf{H}_r(n,K-1)\right\}, \tag{29}$$

denotes the frequency-domain block diagonal channel matrix from the SS to the RS in the nth OFDM symbol.

Throughout this section, we assume that the DF scheme is applied at the RS and constant power modulation like *multiple phase-shift keying* (MPSK) is applied on each OFDM subcarrier. Then it can be shown [10] that $\mathbf{s}_N$ is statistically independent of $\hat{\mathbf{X}}_N$ when the wireless channel from the SS to the RS is with Rayleigh fading, thus enabling the MMSE coupling channel estimation and the performance analysis of the proposed cross-talk cancellation scheme.

B. MMSE Coupling Channel Estimation

Denote $\mathbf{R}_{\mathbf{h}_c} = E\left\{\mathbf{h}_c\mathbf{h}_c^H\right\}$, $\mathbf{R}_\mathbf{s} = E\left\{\mathbf{s}_N\mathbf{s}_N^H\right\}$, and $\mathbf{R}_\mathbf{w} = E\left\{\mathbf{w}_N\mathbf{w}_N^H\right\}$ as the correlation matrices of $\mathbf{h}_c$, $\mathbf{s}_N$, and $\mathbf{w}_N$, respectively. For the MMSE coupling channel estimation, $\mathbf{R}_{\mathbf{h}_c}$ is known to the RS. Denote σ_w^2 as the white noise power at the RS, and then $\mathbf{R}_\mathbf{w} = \sigma_w^2\mathbf{I}_{NKM_r}$. Since $\mathbf{s}_N$ is statistically independent of the equivalent pilot matrix $\hat{\mathbf{X}}_N$, $\mathbf{R}_\mathbf{s}$ does not vary with $\hat{\mathbf{X}}_N$. To provide insights with a simple expression for $\mathbf{R}_\mathbf{s}$, we assume that the transmitted signals of the SS are independent across different transmit antennas, subcarriers, and OFDM symbols with constant unit power, and that the multi-tap time-domain channel from the SS to the RS has independent channel gains over different taps with the $M_r \times M_s$ channel matrix over each tap consisting of independently and identically distributed elements. With these assumptions, it can be shown that $\mathbf{R}_\mathbf{s} = M_s\sigma_h^2\mathbf{I}_{NKM_r}$ where σ_h^2 denotes the average power gain over the channel from the SS to the RS. Then, from (25), we obtain the MMSE estimate of $\mathbf{h}_c$ in the nth OFDM symbol as [9]

$$\begin{aligned}\hat{\mathbf{h}}_{c,\mathrm{MMSE}}(n) &= \Big[\mathbf{R}_{\mathbf{h}_c}^{-1} + \hat{\mathbf{X}}_N^H\left(\mathbf{R}_\mathbf{s}+\mathbf{R}_\mathbf{w}\right)^{-1}\hat{\mathbf{X}}_N\Big]^{-1} \\ &\quad \cdot \hat{\mathbf{X}}_N^H\left(\mathbf{R}_\mathbf{s}+\mathbf{R}_\mathbf{w}\right)^{-1}\mathbf{y}_N \\ &= \left[\left(\sigma_w^2+M_s\sigma_h^2\right)\mathbf{R}_{\mathbf{h}_c}^{-1}+\mathbf{F}(n)\right]^{-1}\mathbf{g}(n)\,,\end{aligned} \tag{30}$$

where $\mathbf{F}(n)$ and $\mathbf{g}(n)$ are defined in (19) and (20), respectively. The corresponding MSE matrix of estimate $\hat{\mathbf{h}}_{c,\mathrm{MMSE}}(n)$ is given by [9]

$$\begin{aligned}\mathbf{\Phi}_{\mathrm{MMSE}}(n) &= E\left\{\Big[\mathbf{h}_c - \hat{\mathbf{h}}_{c,\mathrm{MMSE}}(n)\Big]\Big[\mathbf{h}_c - \hat{\mathbf{h}}_{c,\mathrm{MMSE}}(n)\Big]^H\right\} \\ &= \Big[\mathbf{R}_{\mathbf{h}_c}^{-1} + \hat{\mathbf{X}}_N^H\left(\mathbf{R}_\mathbf{s}+\mathbf{R}_\mathbf{w}\right)^{-1}\hat{\mathbf{X}}_N\Big]^{-1} \\ &= \left[\mathbf{R}_{\mathbf{h}_c}^{-1} + \frac{1}{\left(\sigma_w^2+M_s\sigma_h^2\right)}\mathbf{F}(n)\right]^{-1}.\end{aligned} \tag{31}$$

Utilize $\hat{\mathbf{h}}_{c,\mathrm{MMSE}}(n)$ for cross-talk cancellation at the RS, and then the corresponding residual error vector is given by

$$\mathbf{e}_{\mathrm{MMSE}}(n) = \hat{\mathbf{X}}(n-1)\Big[\mathbf{h}_c - \hat{\mathbf{h}}_{c,\mathrm{MMSE}}(n)\Big]\,. \tag{32}$$

Since $\hat{\mathbf{X}}(n-1)$ is exactly known to the RS, the correlation matrix of $\mathbf{e}_{\mathrm{MMSE}}(n)$ is given by

$$\begin{aligned}\mathbf{R}_{\mathbf{e},\mathrm{MMSE}}(n) &= E\left\{\mathbf{e}_{\mathrm{MMSE}}(n)\,\mathbf{e}_{\mathrm{MMSE}}^H(n)\right\} \\ &= \hat{\mathbf{X}}(n-1)\,\mathbf{\Phi}_{\mathrm{MMSE}}(n)\,\hat{\mathbf{X}}^H(n-1)\,.\end{aligned} \tag{33}$$

When the LS coupling channel estimation is applied, the correlation matrix of the residual error vector can be similarly obtained as

$$\mathbf{R}_{\mathbf{e},\mathrm{LS}}(n) = \hat{\mathbf{X}}(n-1)\,\mathbf{\Phi}_{\mathrm{LS}}(n)\,\hat{\mathbf{X}}^H(n-1)\,, \tag{34}$$

where $\mathbf{\Phi}_{\mathrm{LS}}(n)$ is the MSE matrix of estimate $\hat{\mathbf{h}}_{c,\mathrm{LS}}(n)$, i.e.,

$$\begin{aligned}\mathbf{\Phi}_{\mathrm{LS}}(n) &= E\left\{\Big[\mathbf{h}_c - \hat{\mathbf{h}}_{c,\mathrm{LS}}(n)\Big]\Big[\mathbf{h}_c - \hat{\mathbf{h}}_{c,\mathrm{LS}}(n)\Big]^H\right\} \\ &= \left(\hat{\mathbf{X}}_N^H\hat{\mathbf{X}}_N\right)^{-1}\hat{\mathbf{X}}_N^H\left(\mathbf{R}_\mathbf{s}+\mathbf{R}_\mathbf{w}\right)\hat{\mathbf{X}}_N\left(\hat{\mathbf{X}}_N^H\hat{\mathbf{X}}_N\right)^{-1} \\ &= \left(\sigma_w^2+M_s\sigma_h^2\right)\mathbf{F}^{-1}(n)\,.\end{aligned} \tag{35}$$

Since the residual error after cross-talk cancellation acts as noise, both $\mathbf{R}_{\mathbf{e},\mathrm{MMSE}}(n)$ and $\mathbf{R}_{\mathbf{e},\mathrm{LS}}(n)$ can be utilized to improve the detection of the desired signal at the RS [10].

C. Post-SIR after Cross-Talk Cancellation

According to (24), the post-SIR after cross-talk cancellation in the nth OFDM symbol can be expressed as

$$\mathrm{PostSIR(n)} = \frac{\mathrm{tr}\left\{\mathbf{R_s}\left(n\right)\right\}}{\mathrm{tr}\left\{\mathbf{R_e}\left(n\right)\right\}}, \tag{36}$$

where $\mathrm{tr}\{\cdot\}$ denotes the trace of a square matrix, $\mathbf{R_s}\left(n\right) = E\left\{\mathbf{s}\left(n\right)\mathbf{s}^H\left(n\right)\right\}$, and $\mathbf{R_e}\left(n\right)$ is given in (33) or (34). According to the desired signal model, $\mathbf{R_s}\left(n\right) = M_s\sigma_h^2\mathbf{I}_{KM_r}$ and therefore $\mathrm{tr}\left\{\mathbf{R_s}\left(n\right)\right\} = KM_rM_s\sigma_h^2$. Since $\mathbf{F}\left(n\right) = \hat{\mathbf{X}}_N^H\hat{\mathbf{X}}_N$, we see from (33) and (34) that $\mathrm{tr}\left\{\mathbf{R_e}\left(n\right)\right\}$ is generally a function of $\hat{\mathbf{X}}_N$, the random forwarded signal matrix of the RS in the previous N OFDM symbols. However, it is rather difficult to obtain a closed-form expression for the expectation of $\mathrm{tr}\left\{\mathbf{R_e}\left(n\right)\right\}$ with respect to $\hat{\mathbf{X}}_N$. As a result, the average post-SIR at a general DF-based RS can only be evaluated numerically based on (33) or (34).

To further investigate the post-SIR after cross-talk cancellation at a DF-based RS, we consider a special case that there is only one transmit antenna and one receive antenna at both the SS and the RS, i.e., $M_s = M_r = M_t = 1$. In this case, it can be shown from (4), (5), and (8) that

$$\hat{\mathbf{X}}\left(n-i\right) = \hat{\mathbf{X}}_d\left(n-i\right)\mathbf{F}_L, \quad 1 \le i \le N, \tag{37}$$

where $\mathbf{F}_L$ denotes the $K \times L$ truncated *discrete Fourier transform* (DFT) matrix defined as

$$\mathbf{F}_L = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j\frac{2\pi}{K}} & \cdots & e^{-j\frac{2\pi(L-1)}{K}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\frac{2\pi(K-1)}{K}} & \cdots & e^{-j\frac{2\pi(L-1)(K-1)}{K}} \end{bmatrix}, \tag{38}$$

and $\hat{\mathbf{X}}_d\left(n-i\right)$ denotes the diagonal forwarded signal matrix of the RS in the $(n-i)$th OFDM symbol defined as

$$\begin{aligned}\hat{\mathbf{X}}_d\left(n-i\right) &= \mathrm{diag}\Big\{\hat{x}\left(n-i,0\right), \hat{x}\left(n-i,1\right), \\ &\qquad \cdots, \hat{x}\left(n-i,K-1\right)\Big\}, \end{aligned}\tag{39}$$

in which $\hat{x}\left(n-i,k\right)$ denotes the forwarded signal over the kth subcarrier. Without loss of generality, we assume $\hat{x}\left(n-i,k\right)$ has constant unit power. Thus $\hat{\mathbf{X}}_d\left(n-i\right)$ is unitary and

$$\begin{aligned}\mathbf{F}\left(n\right) &= \mathbf{F}_L^H\left(\sum_{i=1}^{N}\hat{\mathbf{X}}_d^H\left(n-i\right)\hat{\mathbf{X}}_d\left(n-i\right)\right)\mathbf{F}_L \\ &= NK\mathbf{I}_L. \end{aligned}\tag{40}$$

Substitute (40) in (33), and then we have

$$\begin{aligned}\mathbf{R}_{\mathbf{e},\mathrm{MMSE}}\left(n\right) = \hat{\mathbf{X}}_d\left(n-1\right)\mathbf{F}_L\bigg[\mathbf{R}_{\mathbf{h}_c}^{-1} + \frac{NK}{\left(\sigma_w^2+\sigma_h^2\right)}\mathbf{I}_L\bigg]^{-1} \\ \cdot\mathbf{F}_L^H\hat{\mathbf{X}}_d^H\left(n-1\right). \end{aligned}\tag{41}$$

Suppose $\mathbf{R}_{\mathbf{h}_c} = \mathrm{diag}\left\{\lambda_0, \lambda_1, \cdots, \lambda_{L-1}\right\}$, where λ_l denotes the average power gain over the lth tap of the time-domain coupling channel, and then it can be shown that

$$\mathrm{tr}\left\{\mathbf{R}_{\mathbf{e},\mathrm{MMSE}}\left(n\right)\right\} = K\left(\sigma_w^2+\sigma_h^2\right)\sum_{l=0}^{L-1}\frac{1}{NK + \frac{\sigma_w^2+\sigma_h^2}{\lambda_l}}. \tag{42}$$

Therefore, the post-SIR after cross-talk cancellation based on the MMSE coupling channel estimation can be obtained as

$$\mathrm{PostSIR_{MMSE}} = \frac{1}{\sum\limits_{l=0}^{L-1}\frac{1}{NK+\frac{\left(\sigma_w^2+\sigma_h^2\right)}{\lambda_l}}}\frac{\sigma_h^2}{\sigma_w^2+\sigma_h^2}. \tag{43}$$

When the LS coupling channel estimation is applied, the corresponding post-SIR can be obtained similarly as

$$\mathrm{PostSIR_{LS}} = \frac{NK}{L}\frac{\sigma_h^2}{\sigma_w^2+\sigma_h^2} = \frac{NK}{L}\frac{1}{1+\frac{1}{\mathrm{SNR}}}, \tag{44}$$

where $\mathrm{SNR} = \frac{\sigma_h^2}{\sigma_w^2}$. Equation (44) indicates that $\mathrm{PostSIR_{LS}}$ is independent of the original SIR level[3] and increases with the SNR. In contrast, $\mathrm{PostSIR_{MMSE}}$ increases with both the the original SIR and the SNR. Comparison between (43) and (44) indicates that the MMSE coupling channel estimation achieves a higher post-SIR than the LS estimation. However, the performance gap between them diminishes as the original SIR decreases. When the original SIR is low enough where the MMSE estimation reduces to the LS estimation, they have the same performance. Furthermore, Equations (43) and (44) indicate that, for a given K, both $\mathrm{PostSIR_{MMSE}}$ and $\mathrm{PostSIR_{LS}}$ increase with N but decrease with L, i.e., the performance of cross-talk cancellation improves as the coupling channel turns more and more autocorrelated in the frequency-domain and in the time-domain.

V. NUMERICAL AND SIMULATION RESULTS

In this section, we present numerical and simulation results to demonstrate the performance of the proposed cross-talk cancellation scheme at an RS with AF or DF mechanism. While the proposed scheme and its performance analysis throughout this paper are applicable to a general *multi-input-multi-output* (MIMO) relay system, we consider a basic *single-input-single-output* (SISO) one in our simulation where the BS, the RS, and the MS are all equipped with one transmit antenna and one receive antenna. In this system, 16-subcarrier ($K = 16$) OFDM modulation is utilized for broadband transmission and independent QPSK modulated symbols are loaded on different subcarriers. The maximum delay of the multiple-tap coupling channel at the RS is L OFDM sampling intervals; the channel gains over these taps are with independent Rayleigh fading while their average powers decay with the delay exponentially with the exponent factor one. The coupling channel is jointly estimated from the N recently received OFDM symbols during which the coupling channel remains unchanged. The Rayleigh fading channel between the SS and the RS is simulated as a 4-tap one also with the power decay factor one. Without loss of generality, the SNR at the RS is set to be 40 dB.

Figure 2 shows the post-SIR versus SIR curves when $L = 2$ and $N = 4$. Both an AF-based RS and a DF-based RS are considered. For an AF-based RS, only the LS coupling channel estimation is applied and the post-SIR after cross-talk cancellation is obtained experimentally. For a DF-based

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[3] For the single antenna case, it can be shown that $\mathrm{SIR} = \frac{\sigma_h^2}{\sum_{l=0}^{L-1}\lambda_l}$.

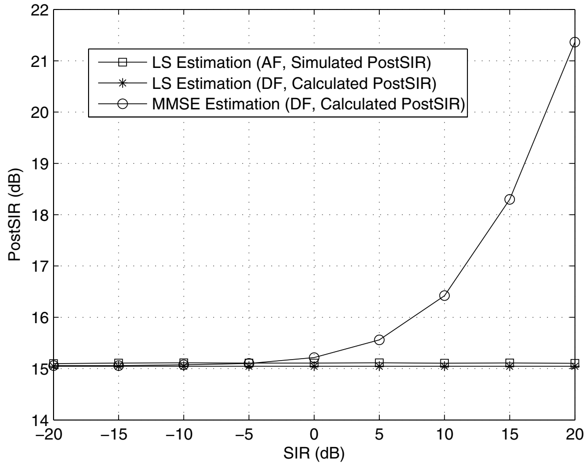

Fig. 2. Post-SIR versus SIR curves when the 2-tap ($L = 2$) coupling channel is jointly estimated from 4 ($N = 4$) recently received OFDM symbols.

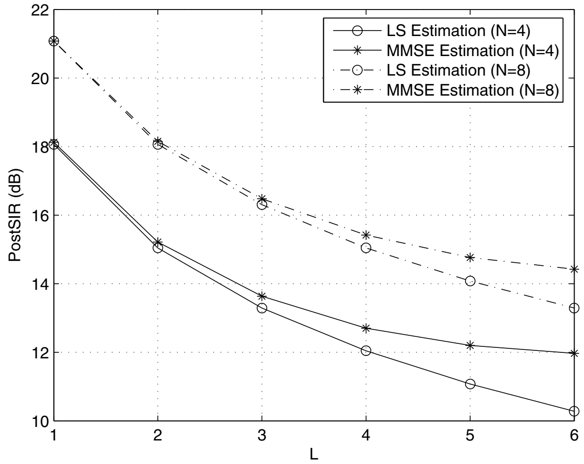

Fig. 3. Post-SIR versus the number of taps of the coupling channel (L) when SIR $= 0$ dB.

RS, both the LS and the MMSE coupling channel estimation are applied and their post-SIR's are calculated based on (44) and (43), respectively. These calculated post-SIR's perfectly match the simulation results which are omitted in the figures. Figure 2 indicates that, whether for an AF-based RS or a DF-based RS, the post-SIR after cross-talk cancellation with the LS coupling channel estimation does not vary with the original SIR level before cross-talk cancellation, which coincides with our analysis. Specifically, the post-SIR is always about 15 dB for both the AF-based RS and the DF-based RS. In other words, when the original SIR is 0 dB, a 15 dB improvement gain is achieved by cross-talk cancellation with the LS coupling channel estimation; when the original SIR is -20 dB, a 35 dB improvement gain is achieved. In contrast, the post-SIR after cross-talk cancellation with the MMSE coupling channel estimation at a DF-based RS increases with the original SIR level. Figure 2 indicates that when the SIR is lower than 0 dB, cross-talk cancellation based on the LS and the MMSE coupling channel estimation achieve almost the same improvement gain; when the SIR is greater than 0 dB but less than 15 dB, the MMSE coupling channel estimation achieves a higher improvement gain than the LS estimation; when the SIR is higher than 15 dB, the MMSE coupling channel estimation still achieves an improvement gain while the LS estimation can not.

Figure 3 shows the post-SIR versus L curves at a DF-based RS when $\mathrm{SIR} = 0$ dB, which verifies that the performance of cross-talk cancellation improves as the coupling channel turns more and more autocorrelated in the frequency-domain and in the time-domain. For example, as L decreases from 2 to 1 when $N = 4$, the improvement gain achieved by cross-talk cancellation increases from 15 to 18 dB; if N increases from 4 to 8, the improvement gain further increases from 18 to 21 dB. Therefore, a higher and higher improvement gain can be achieved by jointly estimating the coupling channel in multiple consecutive OFDM symbols if only the coupling channel remains unchanged during this period of time.

VI. CONCLUSION

In this paper, we have proposed a new digital cross-talk canceller for a wireless channel-reuse-relay-station. By utilizing the random forwarded signal of the RS as pilots for estimation of the coupling channel from the transmit to the receive antenna, the proposed scheme performs cross-talk reconstruction and cancellation without requiring the RS to transmit any dedicated pilots. While the LS coupling channel estimation is applicable to an RS with any relay mechanism, the MMSE coupling channel estimation can be applied to a DF-based RS when constant power modulation is utilized. Both analytical and numerical results have demonstrated the performance of the proposed cross-talk canceller.

REFERENCES

[1] J. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[2] J. Laneman and G. W. Wornell, “Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks,” *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415-2425, Oct. 2003.

[3] A. Wittneben and B. Rankov, “Impact of cooperative relays on the capacity of rank-deficient MIMO channels,” in *IST Summit*, 2003.

[4] K. Salehian, M. Guillet, B. Caron, and A. Kennedy, “On-channel repeater for digital television broadcasting service,” *IEEE Trans. Broadcasting*, vol. 48, no. 2, pp. 97-102, Jun. 2002.

[5] K. M. Nasr, J. P. Cosmas, M. Bard, and J. Gledhill, “Performance of an echo canceller and channel estimator for on-channel repeaters in DVB-T/H networks,” *IEEE Trans. Broadcasting*, vol. 53, no. 3, pp. 609-618, Sep. 2007.

[6] S. I. Park, S. R. Park, H. Eum, J. Y. Lee, Y. T. Lee, and H. M. Kim, “Equalization on-channel repeater for terrestrial digital multimedia broadcasting system,” *IEEE Trans. Broadcasting*, vol. 54, no. 4, pp. 752-760, Dec. 2008.

[7] M. Mazzotti, F. Zabini, D. Dardari, and O. Andrisano, “Performance of an echo canceller based on pseudo-noise training sequences,” in *Proc. 58th Annual IEEE Broadcast Symposium*, Oct. 2008.

[8] P. Lancaster and M. Tismenetsky, *The Theory of Matrices with Applications*, Second ed., Academic Press, New York, 1985.

[9] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Vol I, Prentice Hall, 1993.

[10] J. Ma, G. Y. Li, and J. Y. Zhang, “A new cross-talk cancellaion scheme for wireless relay,” *in preparation*, Mar. 2009.