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***Abstract*—In this paper, we are concerned with a two-hop *multi-input-multi-output* (MIMO) *amplify-and-forward* (AF) relay system consisting of a source node (SN), a relay node (RN), and a destination node (DN). Since the simple RN in this system is unaware of the structure of its received signal and incapable of performing complicated signal processing, the interim channels over the SN-RN and the RN-DN hops can not be estimated directly. Therefore, we develop a novel interim channel estimation approach in this paper. Furthermore, we find necessary and sufficient conditions for the pilot amplifying matrix sequence at the RN to ensure successful interim channel estimation at the DN, and present rules to design low-complexity pilot amplifying matrices meeting these conditions.**

I. INTRODUCTION

Relay-based cooperative communication has become a hot topic in wireless communication. A typical single-user relay-based cooperative communication system consists of a *source node* (SN), one or multiple *relay nodes* (RN's), and a *destination node* (DN). As a repeater, the RN relays the signal from the SN to the DN after appropriate processing. It has been shown that relay techniques can expand the communication coverage, decrease the overall transmit power, and increase the capacity or reliability of the communication links [1]- [3].

Depending on how much signal processing is performed at the RN, the existing relay techniques can be broadly categorized as *decode-and-forward* (DF) and *amplify-and-forward* (AF) [1]. In the DF scheme, the RN detects and demodulates its received signals, decodes the encoded data, re-modulates and forwards them to the DN. In contrast, the RN operating in AF mode only amplifies and forwards its received signals without any further processing and hence has much simpler implementation than that in DF mode. Furthermore, the RN in AF mode does not need any *a priori* information of its received signal and thus can be applied to any scenario.

In this paper, we are concerned with a narrowband two-hop *multi-input-multi-output* (MIMO) AF relay system in which there is no direct communication link between the SN and the RN and data are conveyed from the SN to the DN via two orthogonal channels by time-division or frequency-division. With the help of pilots inserted at the SN, the overall channel between the SN and the DN can be obtained at the DN through conventional channel estimation algorithms proposed for MIMO systems [4], [5]. While the overall channel state information ensures successful data reception at the DN, the interim channel state information, if available, can be utilized to further improve the system performance. In [6] and [7], the interim channel state information has been utilized to determine the optimal amplifying matrix at the RN and power allocation at and between the SN and the RN so as to maximize the instantaneous capacity of the two-hop MIMO AF relay system. Conventionally, the interim channels over the SN-RN and the RN-DN hops can be estimated directly at the RN and the DN, respectively, with the help of pilots. However, such direct interim channel estimation is based on the assumption that the RN is aware of the structure of its received signal and capable of performing complicated signal processing, which does not hold in a practical two-hop MIMO AF relay system with a low-complexity RN. In light of this, we propose the indirect estimation of the interim channels at the DN based on a known pilot amplifying matrix sequence at the RN and the corresponding overall channel sequence obtained at the DN through conventional channel estimation algorithms. Furthermore, we find necessary and sufficient conditions for such a pilot amplifying matrix sequence to ensure successful interim channel estimation. Based on these conditions, we present rules to design pilot amplifying matrices so that the interim channels can be successfully estimated with minimum complexity at the RN.

The rest of this paper is organized as follows. In Section II, we present the principle of interim channel estimation. In Section III, we find necessary and sufficient conditions for successful interim channel estimation and present rules to design simple pilot amplifying matrices meeting these conditions. In Section IV, we extend interim channel estimation to a general two-hop MIMO AF relay system. In Section V, we present simulation results on interim channel estimation in a noisy environment. Finally we conclude this paper in Section VI.

II. PROBLEM FORMULATION

In this paper, we are concerned with a narrowband two-hop MIMO AF relay system, the block diagram of which is shown in Figure 1. As indicated in the figure, there is no direct communication link between the SN and the DN and data are conveyed from the SN to the DN via two orthogonal channels by time-division or frequency-division.

A. System Model

Suppose there are N_s transmit antennas at the SN, N_r receive and transmit antenna pairs at the RN, and N_d receive

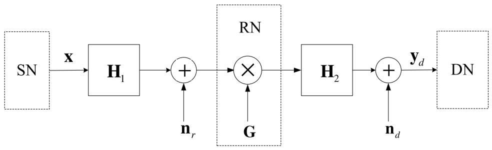


Fig. 1. Two-hop MIMO AF relay system model

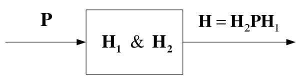


Fig. 2. Equivalent system for interim channel estimation

antennas at the DN. Denote $\mathbf{H}_1$ to be the $N_r \times N_s$ channel matrix between the SN and the RN, $\mathbf{H}_2$ to be the $N_d \times N_r$ channel matrix between the RN and the DN, both of which are assumed nonsingular throughout this paper, and $\mathbf{G}$ to be the $N_r \times N_r$ amplifying matrix at the RN, then the received signal vector at the DN is given by

$$\mathbf{y}_d = \mathbf{H}_2\mathbf{G}\mathbf{H}_1\mathbf{x} + \mathbf{H}_2\mathbf{G}\mathbf{n}_r + \mathbf{n}_d = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{x}$ is the transmitted signal vector of the SN, $\mathbf{n}_r$ and $\mathbf{n}_d$ denote the local noise vectors at the RN and the DN, respectively, $\mathbf{H} = \mathbf{H}_2\mathbf{G}\mathbf{H}_1$ denotes the $N_d \times N_s$ overall channel matrix between the SN and the DN, and $\mathbf{n} = \mathbf{H}_2\mathbf{G}\mathbf{n}_r + \mathbf{n}_d$ denotes the overall noise vector at the DN. In order for the overall channel to be nonsingular, we assume $N_r \geq \min\{N_s, N_d\}$ throughout this paper. Furthermore, to maintain a constant power amplifying gain at the RN, we let $\mathbf{G} = g\mathbf{P}$ where g is the fixed amplifying gain of the RN and $\mathbf{P}$ is a unitary matrix. Without loss of generality, we further assume $g = 1$ throughout this paper for notational convenience and thus $\mathbf{H} = \mathbf{H}_2\mathbf{P}\mathbf{H}_1$.

In a practical two-hop MIMO AF relay system, the low-complexity RN is unaware of the structure of its received signal and incapable of performing complicated signal processing. As a result, the interim channels over the SN-RN and the RN-DN hops, $\mathbf{H}_1$ and $\mathbf{H}_2$, can not be estimated directly. However, as demonstrated in [6] and [7], the performance of a two-hop MIMO AF relay system can be significantly improved if the interim channel state information is exploited. Therefore, the indirect estimation of the interim channels at the DN is very important in improving the system performance.

B. Principle of Interim Channel Estimation

Figure 2 shows an equivalent system for interim channel estimation. Our objective is to estimate the interim channels, $\mathbf{H}_1$ and $\mathbf{H}_2$, based on the amplifying matrix at the RN, $\mathbf{P}$, and the corresponding overall channel, $\mathbf{H} = \mathbf{H}_2\mathbf{P}\mathbf{H}_1$, which is assumed available at the DN through conventional channel estimation algorithms. Since the input $\mathbf{P}$ is known and works like a pilot, it is called the pilot amplifying matrix in this paper. By varying the pilot amplifying matrix within several consecutive time slots*, multiple equations *with respect to* (w.r.t.) $\mathbf{H}_1$ and $\mathbf{H}_2$ can be established and once the number of independent equations is large enough, successful interim channel estimation will be guaranteed.

Denote $\mathbf{P}_1, \mathbf{P}_2, \cdots, \mathbf{P}_L$ as the pilot amplifying matrices at the RN for L consecutive time slots. Throughout this paper, we assume $\mathbf{P}_1 = \mathbf{I}_{N_r}$ where $\mathbf{I}_{N_r}$ denotes the $N_r \times N_r$ identity matrix since, from the perspective of interim channel estimation, the pilot amplifying matrix sequence, $(\mathbf{P}_1, \mathbf{P}_2, \cdots, \mathbf{P}_L)$, is equivalent to $\left(\mathbf{I}_{N_r}, \mathbf{P}_1^{-1}\mathbf{P}_2, \cdots, \mathbf{P}_1^{-1}\mathbf{P}_L\right)$. Denote $\mathbf{H}_{o,1}, \mathbf{H}_{o,2}, \cdots, \mathbf{H}_{o,L}$ as the corresponding overall channel sequence, then

$$\mathbf{H}_{o,l} = \mathbf{H}_2\mathbf{P}_l\mathbf{H}_1, \quad 1 \leq l \leq L. \tag{2}$$

Obviously we can only obtain $\alpha\mathbf{H}_2$ and $\frac{1}{\alpha}\mathbf{H}_1$ from (2), where α denotes an unknown complex number, no matter how many different pilot amplifying matrices are utilized at the RN. Since $\mathbf{H}_2$ (or $\mathbf{H}_1$) and $\alpha\mathbf{H}_2$ (or $\frac{1}{\alpha}\mathbf{H}_1$) share the same matrix structure, it is still meaningful to estimate the interim channels even if there is a constant ambiguity.

Given the framework of interim channel estimation, it is our interest to investigate what and how many pilot amplifying matrices are needed at the RN to guarantee successful interim channel estimation at the DN. In the rest of this paper, we will first assume perfect overall channels at the DN and find necessary and sufficient conditions for successful interim channel estimation. Based on these conditions, rules for designing pilot amplifying matrices will be given accordingly.

III. Necessary and Sufficient Conditions for Successful Interim Channel Estimation

In this section, we will investigate the feasibility of interim channel estimation based on the matrix equation w.r.t. $\mathbf{H}_1$ and $\mathbf{H}_2$ in (2). Since (2) is nonlinear and difficult to analyze directly, we will first transform it into a linear matrix equation. For analytical convenience, we will first consider a special case, $N_r = \min\{N_s, N_d\}$. The extension to the general case that $N_r \geq \min\{N_s, N_d\}$ will be discussed in Section IV.

A. From Nonlinear to Linear Matrix Equation

When $N_r = \min\{N_s, N_d\}$, transformation of (2) into a linear equation is straightforward. We first express $\mathbf{H}_2$ in terms of $\mathbf{H}_1$ or vice versa; then a linear matrix equation w.r.t. one of the two interim channels can be established by eliminating the other. To be brief, the original nonlinear matrix equation in (2) can be transformed into a linear one as

$$\mathbf{P}_l\mathbf{H}_x = \mathbf{H}_x\mathbf{Q}_l, \quad 2 \leq l \leq L, \tag{3}$$

where

$$\mathbf{H}_x = \begin{cases} \mathbf{H}_1, & \text{if } N_r = N_s \leq N_d, \\ \mathbf{H}_2^{-1}, & \text{if } N_r = N_d \leq N_s, \end{cases} \tag{4}$$

denotes the $N_r \times N_r$ channel matrix to estimate, and

$$\mathbf{Q}_l = \begin{cases} \left(\mathbf{H}_{o,1}^H\mathbf{H}_{o,1}\right)^{-1}\mathbf{H}_{o,1}^H\mathbf{H}_{o,l}, & \text{if } N_r = N_s \leq N_d, \\ \mathbf{H}_{o,l}\mathbf{H}_{o,1}^H\left(\mathbf{H}_{o,1}\mathbf{H}_{o,1}^H\right)^{-1}, & \text{if } N_r = N_d \leq N_s, \end{cases} \tag{5}$$

denotes the observed $N_r \times N_r$ matrix when the input is $\mathbf{P}_l$. Figure 3 shows a new equivalent system for the estimation of

*Here a *time slot* is defined as a period of time it takes to estimate the overall channel, $\mathbf{H}$. Throughout this paper, we assume slow fading channel so that the interim channels remain constant during the whole estimation process.

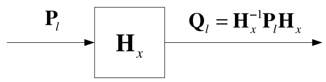


Fig. 3. New equivalent system for interim channel estimation

the interim channel, $\mathbf{H}_x$. In this equivalent system, we need to estimate $\mathbf{H}_x$ based on the input pilot matrix sequence, $\{\mathbf{P}_l, 1 \leq l \leq L\}$, and the observed matrix sequence, $\{\mathbf{Q}_l, 1 \leq l \leq L\}$.

B. First Necessary and Sufficient Condition

The matrix equation in (3) can be rewritten as [8]

$$\left(\mathbf{I}_{N_r} \otimes \mathbf{P}_l - \mathbf{Q}_l^T \otimes \mathbf{I}_{N_r}\right) \cdot \mathbf{h}_x = 0, \quad 2 \leq l \leq L, \tag{6}$$

where $\mathbf{h}_x = \text{vec}(\mathbf{H}_x)$ denotes the vectorization of $\mathbf{H}_x$ formed by stacking the columns of $\mathbf{H}_x$ into a single column vector, and $\otimes$ denotes the Kronecker product [8], [9].

Define

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_2 \\ \mathbf{A}_3 \\ \vdots \\ \mathbf{A}_L \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{N_r} \otimes \mathbf{P}_2 - \mathbf{Q}_2^T \otimes \mathbf{I}_{N_r} \\ \mathbf{I}_{N_r} \otimes \mathbf{P}_3 - \mathbf{Q}_3^T \otimes \mathbf{I}_{N_r} \\ \vdots \\ \mathbf{I}_{N_r} \otimes \mathbf{P}_L - \mathbf{Q}_L^T \otimes \mathbf{I}_{N_r} \end{bmatrix}, \tag{7}$$

then (6) can be rewritten as

$$\mathbf{A} \cdot \mathbf{h}_x = 0. \tag{8}$$

As the interim channel to estimate, $\mathbf{h}_x \neq 0$. Consequently, $\mathbf{A}$ is not of full column rank, i.e., rank $(\mathbf{A}) \leq N_r^2 - 1$. As a result, Equation (8) has an infinite number of solutions. In other words, the exact value of $\mathbf{h}_x$ can not be determined no matter how many different pilot amplifying matrices are utilized at the RN. For interim channel estimation, our objective is to obtain $\alpha\mathbf{h}_x$ from (8), where α is an unknown complex number, so as to learn about the matrix structure of $\mathbf{H}_x$. Therefore, the necessary and sufficient condition for successful interim channel estimation is rank $(\mathbf{A}) = N_r^2 - 1$, i.e., the solutions to (8) have only one *degree of freedom* (DoF) embodied in the unknown α. Since $\mathbf{A}$ depends on not only the pilot amplifying matrix sequence, $\{\mathbf{P}_l, 2 \leq l \leq L\}$, but also the interim channel to estimate, $\mathbf{H}_x$, it is inconvenient to control its rank directly to meet this necessary and sufficient condition. To deal with this problem, we further define

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_2 \\ \mathbf{B}_3 \\ \vdots \\ \mathbf{B}_L \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{N_r} \otimes \mathbf{P}_2 - \mathbf{P}_2^T \otimes \mathbf{I}_{N_r} \\ \mathbf{I}_{N_r} \otimes \mathbf{P}_3 - \mathbf{P}_3^T \otimes \mathbf{I}_{N_r} \\ \vdots \\ \mathbf{I}_{N_r} \otimes \mathbf{P}_L - \mathbf{P}_L^T \otimes \mathbf{I}_{N_r} \end{bmatrix}. \tag{9}$$

It can be shown that rank $(\mathbf{A}) =$ rank $(\mathbf{B})$ [10]. Thus we obtain the following proposition.

Proposition 1. *The necessary and sufficient condition for successful interim channel estimation is*

$$\textit{rank}\,(\boldsymbol{B}) = N_r^2 - 1. \tag{10}$$

Different from $\mathbf{A}$, $\mathbf{B}$ depends only on the pilot amplifying matrix sequence, independent of the interim channel to estimate. Therefore, given $\{\mathbf{P}_l, 2 \leq l \leq L\}$, it is convenient to decide on the identifiability of the interim channels by checking the rank of $\mathbf{B}$.

Although Proposition 1 enables exhaustive search of a feasible pilot amplifying matrix sequence, it does not provide any clue on rules to design feasible pilot matrices directly since it is still rather difficult to establish an explicit relationship between the pilot matrix sequence and the rank of $\mathbf{B}$. Therefore, we will define a dual problem that is equivalent to the original primary problem of interim channel estimation, based on which rules for designing pilot amplifying matrices will be given.

C. Second Necessary and Sufficient Condition

So far, we have described the primary problem of interim channel estimation, the definition of which is given below.

Primary problem: Solve

$$\mathbf{H}_{o,l} = \mathbf{H}_2\mathbf{P}_l\mathbf{H}_1, \quad 1 \leq l \leq L, \tag{11}$$

to get $\mathbf{H}_1$ and $\mathbf{H}_2$.

To facilitate further analysis of the primary problem, we define its dual problem as follows.

Dual problem: Solve

$$\mathbf{P}_l = \mathbf{T}_2\mathbf{P}_l\mathbf{T}_1, \quad 1 \leq l \leq L, \tag{12}$$

to get $N_r \times N_r$ square matrices $\mathbf{T}_1$ and $\mathbf{T}_2$.

Obviously, $\mathbf{T}_1 = \alpha\mathbf{I}_{N_r}$ and $\mathbf{T}_2 = \frac{1}{\alpha}\mathbf{I}_{N_r}$, where α is an arbitrary complex number, are always solutions to the dual problem. Furthermore, it can be shown that the solutions to the primary problem and those to the dual problem have the same DoF [10]. As mentioned, the necessary and sufficient condition for successful interim channel estimation is that the solutions to the primary problem have only one DoF. Thus we obtain the second necessary and sufficient condition for successful interim channel estimation as follows.

Proposition 2. *Given $\{\boldsymbol{P}_l, 2 \leq l \leq L\}$, successful interim channel estimation is guaranteed if and only if $\boldsymbol{T}_1 = \alpha\boldsymbol{I}_{N_r}$ and $\boldsymbol{T}_2 = \frac{1}{\alpha}\boldsymbol{I}_{N_r}$, where α is an arbitrary complex number, are the sole solutions to the dual problem.*

D. Third Necessary and Sufficient Condition

Considering the equivalence between the primary and dual problems indicated in Proposition 2, we will reinvestigate the identifiability of the interim channels for a given pilot amplifying matrix sequence by examining the DoF of the solutions to the dual problem.

Since $\mathbf{P}_1 = \mathbf{T}_2\mathbf{P}_1\mathbf{T}_1$ and $\mathbf{P}_1 = \mathbf{I}_{N_r}$, $\mathbf{T}_1 = \mathbf{T}_2^{-1}$. Thus Equation (12) can be rewritten as

$$\mathbf{T}_2 = \mathbf{P}_l\mathbf{T}_2\mathbf{P}_l^{-1}, \quad 2 \leq l \leq L. \tag{13}$$

Perform eigenvalue decomposition on $\mathbf{T}_2$ to get $\mathbf{T}_2 = \mathbf{U}\Lambda\mathbf{U}^{-1}$, where $\mathbf{U}$ and Λ denote the eigenvector and the eigenvalue matrices of $\mathbf{T}_2$, respectively. Then (13) can be rewritten as

$$\mathbf{T}_2 = \mathbf{P}_l\mathbf{U} \cdot \Lambda \cdot (\mathbf{P}_l\mathbf{U})^{-1}, \quad 2 \leq l \leq L, \tag{14}$$

which indicates that, for any $2 \leq l \leq L$, $\mathbf{P}_l\mathbf{U}$ is also an eigenvector matrix of $\mathbf{T}_2$. According to Proposition 2, successful interim channel estimation is guaranteed if and only if $\mathbf{T}_2$ in (14) has only one DoF. To further find the necessary and sufficient condition for that, we first perform eigenvalue decomposition on each pilot matrix to get

$$\mathbf{P}_l = \mathbf{V}_l \Lambda_l \mathbf{V}_l^{-1}, \quad 2 \leq l \leq L, \tag{15}$$

where $\mathbf{V}_l = (\mathbf{v}_{l1}, \mathbf{v}_{l2}, \cdots, \mathbf{v}_{lN_r})$ and $\mathbf{v}_{ln}, 1 \leq n \leq N_r$, denotes the nth eigenvector of $\mathbf{P}_l$. Then we group the N_r † eigenvectors of $\mathbf{P}_l$ into K clusters with n_k eigenvectors in the kth cluster $V_{lk}, 1 \leq k \leq K$, such that V_{lk}'s for different l's span the same n_k-dimension space S_k. Given the above eigenvector grouping rule, the following third necessary and sufficient condition for successful interim channel estimation can be proved [10].

Proposition 3. *Given $\{\mathbf{P}_l, 2 \leq l \leq L\}$, successful interim channel estimation is guaranteed if and only if the maximum number of their eigenvector clusters is one, i.e., $K_{max} = 1$.*

According to Proposition 3, it is impossible to successfully estimate the interim channels within two time slots since, in this case, $L = 2$ and the maximum number of eigenvector clusters of $\mathbf{P}_2$ alone is N_r. On the other hand, for appropriately designed $\mathbf{P}_2$ and $\mathbf{P}_3$, the maximum number of their eigenvector clusters can be as small as one. In other words, the minimum number of time slots required for successful interim channel estimation is 3, i.e., $L_{min} = 3$.

E. Design Rules

In practice, it is attractive to use a diagonal matrix or a permutation of a diagonal matrix, also called a quasi-diagonal matrix, as the pilot amplifying matrix at the RN since, in this case, the complexity of the RN can be reduced significantly. In light of this, we come up with the following design rules.

- Let $\mathbf{P}_1 = \mathbf{I}_{N_r}$.
- Generate N_r distinct complex numbers with unit magnitude to get a diagonal $\mathbf{P}_2$.
- Get $\mathbf{P}_3$ by cyclically shifting the rows of $\mathbf{P}_2$.

Given the above design rules, it can be shown that the maximum number of eigenvector clusters of $\mathbf{P}_2$ and $\mathbf{P}_3$ is one [10]. Thus, according to Proposition 3, successful interim channel estimation is guaranteed. The above design rules are especially attractive in practice because, firstly, the interim channels can be successfully estimated within only three time slots, secondly, the application of diagonal and quasi-diagonal pilot amplifying matrices maintains a minimum complexity at the RN, and thirdly, the orthogonality of these pilot amplifying matrices ensures a constant power amplifying gain at the RN.

F. Least-Square Estimation of Interim Channels

So far, we have assumed perfect overall channels and focused on the design of pilot amplifying matrices at the RN for interim channel estimation. In practice, the overall channel between the SN and the DN, $\mathbf{H}_{o,l}$, has to be estimated and therefore is always accompanied with estimation error, i.e.,

$$\mathbf{H}_{o,l}^N = \mathbf{H}_{o,l} + \mathbf{N}_{o,l}, \quad 1 \leq l \leq L, \tag{16}$$

where $\mathbf{H}_{o,l}$ is the actual overall channel, $\mathbf{H}_{o,l}^N$ denotes its estimate at the DN, and $\mathbf{N}_{o,l}$ denotes an additive noise matrix. Accordingly, the original linear matrix in (6) is revised as

$$\left(\mathbf{I}_{N_r} \otimes \mathbf{P}_l - \left(\mathbf{Q}_l^N\right)^T \otimes \mathbf{I}_{N_r}\right) \cdot \mathbf{h}_x = 0, \quad 2 \leq l \leq L, \tag{17}$$

where $\mathbf{Q}_l^N = \mathbf{Q}_l + \mathbf{N}_{Q,l}$ denotes the corrupted version of $\mathbf{Q}_l$ and $\mathbf{N}_{Q,l}$ denotes an equivalent additive noise matrix.

As mentioned, we can only obtain $\alpha \mathbf{h}_x$, where α is an unknown number, through interim channel estimation. Consequently, we estimate the value of $\mathbf{h}_x$ normalized to one of its elements as a *least-square* (LS) solution to the inconsistent equation (17) [10], the details of which are omitted here.

IV. Extension to General Case

In order for the overall channel to be nonsingular, we have assumed $N_r \geq \min\{N_s, N_d\}$, which consists of Case I: $N_r = \min\{N_s, N_d\}$ and Case II: $N_r > \min\{N_s, N_d\}$. In the previous sections, we have investigated interim channel estimation based on a pilot amplifying matrix sequence at the RN under Case I; in this section, we will extend it to Case II.

Under Case II, we propose to estimate the interim channels via multiple steps. In each step, the interim channel estimation procedure under Case I is applied. Specifically, we divide the receive and transmit antenna pairs at the RN into multiple groups so that the number of antenna pairs in each group, $N_r^{'}$, equals $\min\{N_s, N_d\}$. Obviously there will be overlapping antenna pairs in different groups if $I = \frac{N_r}{N_r^{'}}$ is not an integer. Without loss of generality, we assume I is an integer to facilitate the explanations. By grouping the receive and transmit antenna pairs at the RN, the original two-hop MIMO AF relay system is decomposed into I parallel subsystems with still N_s and N_d antennas at the SN and the DN, respectively, but $N_r^{'}$ receive and transmit antenna pairs at the RN. Let $\mathbf{H}_{1,i}$ and $\mathbf{H}_{2,i}, 1 \leq i \leq I$, denote the interim channels of the ith subsystem, then the overall interim channels can be expressed as

$$\mathbf{H}_1 = \left(\mathbf{H}_{1,1}^T, \mathbf{H}_{1,2}^T, \cdots, \mathbf{H}_{1,I}^T\right)^T, \tag{18}$$

and

$$\mathbf{H}_2 = \left(\mathbf{H}_{2,1}, \mathbf{H}_{2,2}, \cdots, \mathbf{H}_{2,I}\right). \tag{19}$$

Since $N_r^{'} = \min\{N_s, N_d\}$, $\mathbf{H}_{1,i}$ and $\mathbf{H}_{2,i}$ can be estimated following the procedure under Case I by switching off all of the antenna pairs of the RN that are outside the ith group. Then the overall interim channels, $\mathbf{H}_1$ and $\mathbf{H}_2$, can be obtained by combining $\mathbf{H}_{1,i}$'s and $\mathbf{H}_{2,i}$'s, respectively. Since we can only get $\frac{1}{\alpha_i}\mathbf{H}_{1,i}$ and $\alpha_i \mathbf{H}_{2,i}$ through interim channel estimation, where α_i is an unknown complex number, J extra antenna groups are needed to obtain the amplitude and phase relationship between the interim channels of different subsystems, where the specific value of J depends on I and $N_r^{'}$. Suppose it takes L time slots to estimate the interim channels of each subsystem, then altogether $(I + J) \times L$ time slots are needed to estimate the overall interim channels.

V. Simulation Results

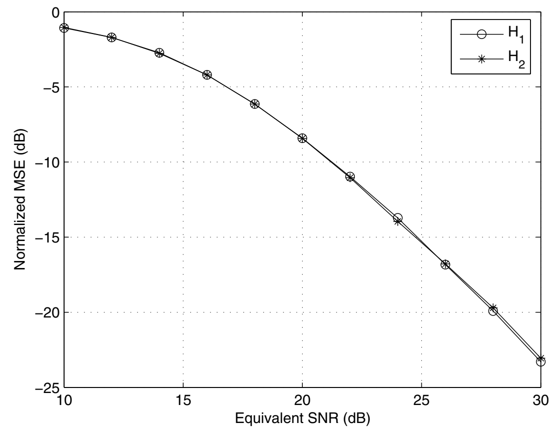
In this section, we will present simulation results on interim channel estimation. In our simulation, a $4/4/4$ ($N_s = 4$,

† In the case that $\mathbf{P}_l$ has an n-dimension ($n > 1$) eigenspace associated with an n-order repeated eigenvalue, arbitrary n eigenvectors are chosen from this eigenspace for grouping.

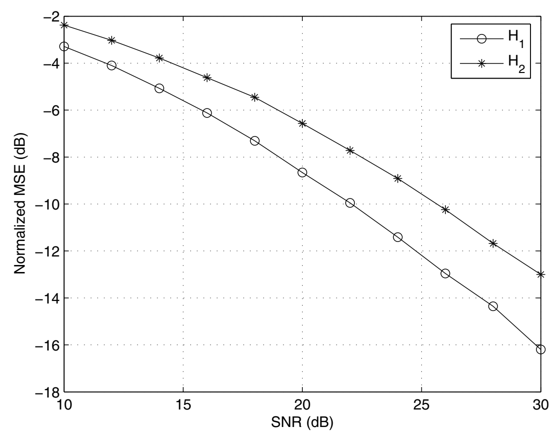
$N_r = 4$, and $N_d = 4$) two-hop MIMO AF relay system is considered. The elements of $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{n}_r$, and $\mathbf{n}_d$ are all modeled as i.i.d. complex Gaussian random variables with zero mean and unit variance. As mentioned, the power amplifying gain of the RN is set to one, i.e., $g = 1$. Denote P_s to be the transmit power of each SN antenna, then it also represents the equivalent SNR of the overall system. Three pilot amplifying matrices are designed for interim channel estimation according to the rules given in Section III-E. The normalized interim channels, $\overline{\mathbf{H}}_1 = \frac{1}{\mathbf{H}_1(1,1)}\mathbf{H}_1$ and $\overline{\mathbf{H}}_2 = \frac{1}{\mathbf{H}_2(1,1)}\mathbf{H}_2$, where $\mathbf{H}_i(1,1)$ denotes the element at the first row and the first column of $\mathbf{H}_i, 1 \le i \le 2$, are estimated through the linear LS estimation procedure given in Section III-F. The corresponding normalized MSE of estimate $\overline{\mathbf{H}}_{i,E}$ is defined as

$$\xi_i = \frac{\|\overline{\mathbf{H}}_i - \overline{\mathbf{H}}_{i,E}\|^2}{\|\overline{\mathbf{H}}_i\|^2}, \quad 1 \le i \le 2. \tag{20}$$

Figure 4 shows the normalized MSE curves of the estimated interim channels, $\overline{\mathbf{H}}_{1,E}$ and $\overline{\mathbf{H}}_{2,E}$. In Figure 4(a), we apply the imperfect overall channel model in (16) and define its equivalent SNR as the ratio of the average power of the overall channel to that of the additive white Gaussian noise. In contrast, the imperfect overall channel in Figure 4(b) is obtained by employing the LMMSE-based channel estimation algorithm under a certain system SNR. Figure 4 indicates that an acceptable estimation performance, such as a normalized MSE smaller than -5 dB, can be achieved when the equivalent SNR of the overall channel (in Figure 4(a)), or the system SNR (in Figure 4(b)), is greater than 15 dB. This suggests that the accuracy of the overall channels is significant to the estimation of the interim channels. While $\overline{\mathbf{H}}_{1,E}$ and $\overline{\mathbf{H}}_{2,E}$ have the same normalized MSE under the imperfect overall channel model in (16), it is interesting to notice that $\overline{\mathbf{H}}_{1,E}$ has a smaller normalized MSE than $\overline{\mathbf{H}}_{2,E}$ when the overall channel is obtained through the conventional channel estimation algorithm. This is actually reasonable since the overall system model in (1) indicates that $\mathbf{H}_1$ and $\mathbf{H}_2$ are not playing interchangeable roles in the original system. To be more specific, $\mathbf{H}_1$ is independent of the overall noise at the DN while $\mathbf{H}_2$ is not since it conveys $\mathbf{n}_r$, the noise vector at the RN, to the DN. While the focus of this paper is on the identifiability of the interim channels by the use of perfect overall channels, it will be an interesting future research topic to jointly estimate $\mathbf{H}_1$, $\mathbf{H}_2$, and the overall channel in a two-hop MIMO AF relay system.



(a) Normalized MSE versus equivalent SNR of the overall channel



(b) Normalized MSE versus system SNR

Fig. 4. Normalized MSE curves of the estimated interim channels

VI. Conclusion

In this paper, we have proposed to estimate the interim channels of a two-hop MIMO AF relay system based on the known pilot amplifying matrices at the RN and the corresponding overall channels obtained at the DN through conventional channel estimation algorithms. We have found necessary and sufficient conditions for successful interim channel estimation, and presented rules to design diagonal or quasi-diagonal pilot amplifying matrices so that the interim channels can be successfully estimated with minimum complexity at the RN. Furthermore, initial simulation results have demonstrated the performance of such an estimation scheme.

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