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Progressive Accumulative Routing: Fundamental Concepts and Protocol

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Abstract— This paper considers a multi-hop network in which relay nodes cooperate to minimize the total energy consumed in transmitting a (unicast) packet from a source to a destination. We propose the *Progressive Accumulative Routing (PAR)* algorithm, which progressively performs relay discovery, relay ordering and relay power allocation in a distributed manner, such that each relay node only needs local information. We assume *Destination Energy Accumulation*, in which the destination accumulates the energy of multiple received copies of a packet, each of which is too weak to be reliably decoded by itself, while the lower-complexity relay nodes use a decode-and-forward approach. We also provide a closed-form analysis of the energy-savings achieved by the PAR when a relay node is added to an already existing DEA route. Simulations verify that the algorithm considerably reduces the total energy consumption, and can be implemented efficiently.

Index Terms—Communication system routing, distributed algorithms, energy accumulation, radio networks

I. INTRODUCTION

M ULTI-HOP routing is often used in traditional wireless relay networks to reduce the total energy required to deliver a unicast message [1], [2]. In these networks, a source sends a packet to a destination through many intermediate relays along a pre-determined energy-efficient route. When a packet cannot be decoded successfully by an intermediate relay or the destination, it is discarded and needs to be retransmitted [3]–[6]. This approach is not energy efficient, as a node completely discards the information contained in the corrupted packets.

Energy accumulative routing has been recently proposed to improve the energy efficiency of wireless relay networks [7]–[9]. In energy accumulative routing, a node *stores* a received signal of a packet that is too weak for decoding and combines it with another copy of the same packet that arrives later. After successfully decoding the packet, the relay node transmits it and propagates it to the destination(s).

While current and next generation wireless systems do have mechanisms in place to implement energy accumulation, doing so at each and every node is challenging. The accumulationbased techniques proposed so far work on the idealized premise that every node stores each and every received copy of a packet that is transmitted from multiple nodes in the network

until it can successfully decode it. Typically, the source sends multiple packets one after the other. The relays will then have to store multiple "soft" copies of not one but many packets that are transmitted by all the nodes (and the source) that have already decoded these packets. To make matters worse, relay nodes can act as relays for different sources, so that their storage effort is proportional to the total number of distinct packets "in transit" in the network. Since the nodes acting as relays do not directly benefit from transmitting a packet from a source to a destination, it is difficult to justify their dedicating the significant resources required by energy accumulative routing. Finally, finding the optimal energyaccumulative route in a given wireless network consisting of many relay nodes and jointly determining the transmit power levels of the nodes in the route is extremely difficult: the work in [9] showed that for unicast transmission, finding the Minimum Energy Accumulative Route (MEAR) is an NP-Complete problem. Thus, no scalable optimum mechanism exists, though a heuristic algorithm has been proposed in [9].

In this paper, we focus on an intermediate case, which we call destination energy accumulation (DEA), that fills the gap between the two extremes considered in the literature, namely (i) a traditional network, which requires simple decode-andforward relays that do not benefit from energy accumulation, and (ii) a complete energy-accumulation network, which requires highly complex decode-and-forward relays that can accumulate energy to the greatest possible extent. In our setup, only the destination node uses energy accumulation to decode the packet, while the intermediate relays do not. Energy accumulation at the destination is justifiable for the following reasons: (i) in many sensor network applications, the message sink (destination node) can have the higher complexity and large memory storage required to process the received transmissions, (ii) the additional effort of accumulation occurs at the node that benefits from it, and (iii) the number of packets that need to be accumulated and stored is limited. As we shall see, energy accumulation at the destination reduces the aggregate energy consumption in the network.

In this paper, we propose, develop the fundamentals of, and analyze the Progressive Accumulative Routing (PAR) protocol. This protocol determines the energy-efficient DEA route and sets the node powers. It has a number of key properties that make it suitable for practical implementation in ad-hoc networks: (i) *Progressive addition of nodes*: It adds, *in an incremental fashion*, new nodes to an established DEA route to realize additional energy savings. Thus, an established route does not have to be torn down every time. (ii) *Distributed*

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computation of route nodes and powers: As a distributed algorithm, PAR establishes energy-efficient energy accumulative routes based on only the local channel knowledge available at each relay node, and uses a single field in the protocol to keep track of the remaining signal energy required for the destination to successfully decode the message in transit. (iii) Simple protocol structure: Adding nodes to further reduce energy consumption is achieved by the transmission of simple "request for cooperation" packets, which are always received by nodes that can help. (iv) Large energy savings: While the relay nodes selected by PAR are optimum in a progressive sense, the resulting route need not be the optimum route obtained by an exhaustive centralized search with global knowledge. Still, energy savings achieved by the PAR can be comparable to those achieved by optimum centralized DEA routes.

When the nodes are uniformly distributed geographically, we also derive closed-form expressions for the energy savings achieved by PAR when it adds an additional relay to any pre-specified DEA route. Through simulations, we verify our analysis and show that the PAR protocol improves the energy-efficiency compared to traditional non-accumulative networks. We also see that the PAR algorithm shows performance close to the optimal complete energy accumulation in a number of randomly generated scenarios. Finally, the results developed in this paper also lay the foundation for a more elaborate routing algorithm that allows the use of more powerful relays – should they be available – as intermediate destinations [10].

While many algorithms have been proposed for energyaccumulative routing in the literature, none of them is directly applicable to the scenario we consider. The heuristic algorithm suggested in [9] is intended for full energy accumulation, and is completely centralized, i.e., every node needs to be aware of the states of all the links in the network. The methods developed in [7], [8], [11] are designed for broadcast and not unicast. While [12] considered energy-accumulative routing for multicast, of which unicast can be considered a special case, the objective of maximizing the network lifetime is different from ours. The PAR algorithm we develop in Section IV is thus the first distributed algorithm suitable for unicast with DEA.

The remainder of this paper is organized as follows. In Section II, we present the network model. Section III lays the theoretical foundations of the PAR algorithm, followed by Section IV, which describes the PAR algorithm in detail. Section V presents an analytical investigation of the power savings achieved by PAR. Simulation results in Section VI are followed by our conclusions in Section VII.

II. NETWORK MODEL

We consider the problem of unicast traffic in a wireless network that consists of a source node, s, a destination node, t, and intermediate decode-and-forward relay nodes. All nodes use a single omni-directional antenna for transmission and reception, and operate in half-duplex mode, i.e., they can either transmit or receive, but not do both simultaneously. The network is quasi-static, in which occasional link updates reflect the possible changes of the channel state of the network. Let V be the set of nodes in a network. For nodes $u, v \in V$, let h_{uv} be the channel (power) gain between u and v. In general, h_{uv} is a random variable, but we assume that it remains unchanged in the duration in which the algorithm operates. A node only knows its channel gain to other nodes – it does not know the phase of any channel gain, nor does it know any other link gain. Finally, an additive white Gaussian noise (AWGN) channel is assumed between any two nodes.

A node can forward a packet only after having reliably decoded that packet. As discussed in the introduction, we assume that the destination accumulates energy, while the intermediate relays do not. The destination receives multiple "soft" copies of the same packet (at different times) from multiple nodes and stores all of them. The packet can be successfully decoded at the destination once the total energy accumulated from the multiple received copies exceeds the threshold $\bar{\gamma}$, which depends on the modulation and coding used for transmission [7], [8].¹

If the destination receives one copy of the packet from each of the nodes u_1, u_2, \dots, u_n , then it can decode the packet successfully if $\sum_{k=1}^n p_k h_{u_k t} \ge \bar{\gamma}$, where p_k is the transmit power of node u_k . An intermediate node v can successfully decode the packet transmitted by node u with power p if and only if $ph_{uv} \ge \bar{\gamma}$; otherwise, it discards the undecodable packet. Without loss of generality, the packet duration is normalized to unity; we, therefore, interchangeably use the terms energy and power.

III. FUNDAMENTALS OF PROGRESSIVE ACCUMULATIVE ROUTING

We consider a single message source, s, and a single message destination, t. We first derive the general necessary and sufficient conditions for power saving when (i) a single relay is introduced between s and t, and (ii) when a second relay is introduced in an energy-accumulative route that contains one relay. All the cases considered ensure that the current route is not torn down, given that this makes a distributed implementation impractical. We shall see that very limited information is often needed to determine the relay that can maximally reduce the total transmission required when it is added to the existing route. We then extend the result to a general energy-accumulative route that contains an arbitrary number of relays. We shall again see that additional energy savings can be achieved using the local information at the relays and limited additional information.

A. Adding the First Relay Between Source and Destination

Lemma 1: An accumulative route from s to t through a relay r can reduce the total power consumption if and only if

$$h_{st} < \min\{h_{sr}, h_{rt}\}.$$
(1)

¹This is akin to the Chase combining technique used in third generation cellular receivers [13], [14]. A repetition coding based justification for wideband regime was provided by [7], and Maximum Ratio Combining was used by [8] to justify this.

The maximum total power saving, $P_s^{\rm sav}(r)$, by having r as a relay is given by

$$P_s^{\rm sav}(r) = \left(1 - \frac{h_{st}}{h_{sr}}\right) \left(1 - \frac{h_{st}}{h_{rt}}\right) \frac{\bar{\gamma}}{h_{st}},\tag{2}$$

and is achieved when s and r set their transmission powers P_s and P_r , respectively, at

$$P_s = \frac{1}{h_{sr}}\bar{\gamma}, \text{ and } P_r = \frac{1}{h_{rt}}\left(1 - \frac{h_{st}}{h_{sr}}\right)\bar{\gamma}.$$
 (3)

Proof: Clearly, a node that does not satisfy (1) cannot help as it is then more energy-efficient for the source to directly transmit to the destination.

Let there exist at least one node, r, such that $h_{st} < \min\{h_{sr}, h_{rt}\}$. In DEA, if r is a relay, then the source first transmits a packet with power P_s so that r can decode it successfully. Then, r transmits the packet to t with power P_r . The destination decodes the packet using the energy accumulated from the transmissions of both s and r. Hence, the optimal power allocation problem is the following:

$$\min_{P_s,P_r} \left(P_s + P_r \right) \text{ subject to } \begin{bmatrix} h_{sr} & 0\\ h_{st} & h_{rt} \end{bmatrix} \begin{bmatrix} P_s\\ P_r \end{bmatrix} \ge \begin{bmatrix} \bar{\gamma}\\ \bar{\gamma} \end{bmatrix}.$$
(4)

The first inequality in the constraint in (4) ensures that r decodes the packet sent by s. Once r decodes the packet, it is more energy-efficient to let r deliver the remaining energy for t to decode the packet, since $h_{rt} > h_{st}$. This leads to the optimal power allocation in (3), which satisfies the constraint in (4) with equality. The total power savings with the power setting in (3), compared to the minimum power, $\bar{\gamma}/h_{st}$, required for a direct transmission from s to t, is then given by (2). This power saving is positive iff (1) is satisfied.

Lemma 1 shows that only nodes that satisfy (1) are eligible candidates for reducing total energy consumption.² Note that for the source to determine which node is the best relay, it only needs to know h_{rt} in addition to the local information it already has. And, if s is sending a packet directly to t, all the eligible candidates can already decode the packet because $h_{sr} > h_{st}$.

B. Adding the Second Relay

We consider algorithms that progressively add relays into an existing route. Specifically, given an existing route, we are interested in finding the relay that leads to a maximum reduction in total power consumption when it is added to an existing route.

Let r denote the optimal first relay already present in the DEA route. As shown in Fig. 1, the second relay can be added to one of the three links: s-t, s-r, and r-t. The following Lemma shows that the first possibility is always sub-optimal and need not be considered.

Lemma 2: If the relay r is the optimal single relay for cooperating in the transmission from s to t, adding an additional



Fig. 1. The three ways to add a second relay to a one relay configuration (a): (b) Second relay is added in parallel to the established DEA route. (c) Second relay is added between the source and the first relay. (d) Second relay added between the first relay and the destination.

node, q, in parallel between s and t (as in Fig. 1b) cannot reduce the total transmission power in DEA.

Lemma 2 shows that we only need to consider adding a new relay between the s-r and r-t links in the established DEA route, as per Fig. 1c or Fig. 1d.³

Lemma 3: Let r be the optimal single relay in an established DEA route. If and only if there exists a node $q \in V - \{s, r, t\}$, such that $h_{sq} > h_{sr}$, $h_{qt} < \min\{h_{qr}, h_{rt}\}$, and

$$h_{qr}\left(\frac{1}{h_{sr}} - \frac{1}{h_{sq}}\right) > \frac{h_{rt} - h_{qt}}{h_{rt} - h_{st}},\tag{5}$$

does adding q between s and r, as in Fig. 1c, save total power. The progressively optimal power saving, $P_s^{sav}(q)$, is

$$P_s^{\rm sav}(q) = \frac{\bar{\gamma}}{h_{rt}} \left[\left(h_{rt} - h_{st} \right) \left(\frac{1}{h_{sr}} - \frac{1}{h_{sq}} \right) + \frac{h_{qt} - h_{rt}}{h_{qr}} \right],\tag{6}$$

when the source and the relays set their respective transmission powers, P_s , P_q , and P_r , to

$$P_{s} = \frac{1}{h_{sq}}\bar{\gamma}, P_{q} = \frac{1}{h_{qr}}\bar{\gamma}, \text{ and } P_{r} = \frac{1}{h_{rt}}\left(1 - \frac{h_{st}}{h_{sq}} - \frac{h_{qt}}{h_{qr}}\right)\bar{\gamma}.$$
(7)

Proof: See Appendix.

From the power settings above, it can be seen that all eligible nodes that can reduce the total power can successfully decode the packet sent out by r.

Lemma 4: Let r be the optimal single relay in an established DEA route. If and only if there exists a node $q \in V - \{s, r, t\}$, such that

$$h_{qt} > h_{rt}$$
 and $\frac{h_{rt}}{h_{rq}} < 1 - \frac{h_{st}}{h_{sr}}$, (8)

³Lemma 2 is valid under the assumption that only the magnitude of the channel gain is available. If phase information of all nodes is globally available, and all the nodes are synchronized, then cooperative beamforming is optimal [16], [17].

 $^{^{2}}$ Lemma 1 applies to the case of energy accumulation, when only the magnitude of the channel gain is known. If additional information is present, even when the nodes are further away from the source than the destination, they can still positively impact the transmission efficiency [15].

then adding q between r and t, as in Fig. 1d, leads to an optimal power saving, $P_r^{\rm sav}(q),$ of

$$P_r^{\rm sav}(q) = \left(\frac{1}{h_{rt}} - \frac{1}{h_{qt}}\right) \left(1 - \frac{h_{st}}{h_{sr}} - \frac{h_{rt}}{h_{rq}}\right) \bar{\gamma},\qquad(9)$$

when the source and the relays set their transmission powers P_s , P_q , and P_r , respectively, at

$$P_s = \frac{1}{h_{sr}}\bar{\gamma}, P_r = \frac{1}{h_{rq}}\bar{\gamma}, P_q = \frac{1}{h_{qt}}\left(1 - \frac{h_{st}}{h_{sr}} - \frac{h_{rt}}{h_{rq}}\right)\bar{\gamma}.$$
(10)

Proof: See Appendix.

Notice that before the second relay is added, the first relay r transmits the packet with power $\frac{1}{h_{rt}} \left(1 - \frac{h_{st}}{h_{sr}}\right) \bar{\gamma}$. From the necessary and sufficient condition in (8), it can be seen that all eligible nodes that can reduce the total power can successfully decode the packet sent out by r. This fact shall be exploited when we design the PAR protocol to progressively add relays to reduce the total power consumption.

C. Multiple relays

In the previous subsection, we saw that two relays in parallel cannot reduce the total power consumption over an optimal single relay DEA route. This result can be generalized to the case where multiple relays are present. Therefore, we only need to consider the cases where new nodes are inserted in between two adjacent relays or between relay and destination, as was done in Fig. 1c and Fig. 1d. We refer to such a route the *serial DEA route*. We will consider an algorithm that progressively adds relays into an existing route without removing any of the previously selected relays from the route.

To consider adding a node, w, in a serial DEA route that already contains multiple relays, we first define the following terminology. If u and v are two relays in a serial DEA route, and u successfully decodes the packet before the relay v, then we say that u is *before* v and v is *after* u. We say that v is *immediately after* or *next to* u if v is after u and there is no relay that is after u and before v. The relay immediately after u in the serial DEA route is denoted by N(u). A relay u is called the *last relay* in the serial DEA route if N(u) = t.

The backward relay set, B(u), is the ordered set of relays before u in the route. $A(u) = \sum_{r \in B(u)} \frac{h_{rt}}{h_{rN(r)}}$ denotes the fraction of the total energy, which is required to successfully decode a packet at the destination, that accumulates at the destination due to transmissions from the relays in the set B(u).

Theorem 1: Let u be a relay in the serial DEA route, with v = N(u) being the relay immediately after it. If u is not the last relay, l, in the route, then adding the node w as a relay immediately after u reduces the total power consumption if w satisfies the following two sufficient conditions:

$$h_{uw} > h_{uv}$$
 and $h_{wv} \left(\frac{1}{h_{uv}} - \frac{1}{h_{uw}}\right) > \frac{h_{lt} - h_{wt}}{h_{lt} - h_{ut}}$. (11)

A total power saving of

$$P_{u}^{\text{sav}}(w) = \frac{1}{h_{lt}} \left[(h_{lt} - h_{ut}) \left(\frac{1}{h_{uv}} - \frac{1}{h_{uw}} \right) + \frac{h_{wt} - h_{lt}}{h_{wv}} \right] \bar{\gamma}$$
(12)

is achieved when the transmit powers of u and l are changed to

$$P_u = \frac{\bar{\gamma}}{h_{uw}}, P_l = \frac{1}{h_{lt}} \left(1 - A(l) + \frac{h_{ut}}{h_{uv}} - \frac{h_{ut}}{h_{uw}} - \frac{h_{wt}}{h_{wv}} \right) \bar{\gamma},$$
(13)

where A(l) refers to the fraction of accumulated energy at t before w is added. The transmit power of the new relay, w, is $P_w = \bar{\gamma}/h_{wv}$. The transmit powers of all the other relays in the route are unchanged.

Proof: Using an argument analogous to that in Lemma 3, the power allocation after w is added as a relay corresponds to that in (13). The condition for power saving in (11) can be derived in a fashion similar to (5).

While (11) provides a general condition to achieve power savings, it requires that every relay in the serial DEA route knows h_{lt} , which is not conducive to a distributed implementation. The following Corollary provides a weaker sufficient condition that guarantees power savings without the need for every relay knowing h_{lt} .

Corollary 1: When u is not the last relay in a serial DEA route, adding a node w immediately after u results in power savings if

$$h_{wt} > h_{ut}$$
 and $\frac{1}{h_{uw}} + \frac{1}{h_{wv}} < \frac{1}{h_{uv}}$. (14)

Theorem 2: When u is the last relay in a serial DEA route, adding a node w immediately after u can reduce power consumption if w satisfies the two conditions:

$$h_{wt} > h_{ut}$$
 and $\frac{h_{ut}}{h_{uw}} < 1 - A(u).$ (15)

A total power saving of

$$P_u^{\text{sav}}(w) = \left(\frac{1}{h_{ut}} - \frac{1}{h_{wt}}\right) \left(1 - A(u) - \frac{h_{ut}}{h_{uw}}\right) \bar{\gamma} \quad (16)$$

is achieved when the transmit power of u is changed to $P_u = \bar{\gamma}/h_{uw}$, and the transmit power of the new node w is

$$P_w = \frac{1}{h_{wt}} \left(1 - A(u) - \frac{h_{ut}}{h_{uw}} \right) \bar{\gamma}.$$
 (17)

The transmit powers of all the other relays in the route are unchanged.

Proof: Using an argument analogous to that in Lemma 4, the power allocation after w is added corresponds to that in (17). The condition for power saving in (15) can be derived in a similar fashion as (8).

Both Theorem 2 and Corollary 1 show that all potential relays (the nodes that lead to power savings) can already successfully decode the transmissions from the relay that they will be immediately after. As a result, local CSI and minimal feedback from the potential relays can be used to progressively increment the serial DEA route to save total power.



Fig. 2. Overview of the PAR protocol. Shown are the fields of the data packet transmitted by relay u, and the fields of the ready to cooperate (RTC) packet transmitted by a potential relay, w.

IV. THE PAR ALGORITHM: PROTOCOL

Based on the analysis in Sec. III, we now propose the PAR algorithm. Initially, a basic route is established between the source and the destination.⁴ The PAR algorithm then progressively and distributively adds relays to improve the energy-efficiency of a serial DEA route. This relay discovery process is done via two types of packets: a *data* packet that contains the data to be sent from *s* to *t*, and a *ready-to-cooperate (RTC)* packet for feedback of the limited additional information required for modifying the route.

The source transmits data to the destination through an already established serial DEA route. It transmits a new packet to its next relay, N(s), with power $\bar{\gamma}/h_{sN(s)}$. Neighboring nodes that overhear a transmission from a currently transmitting relay in an established serial DEA route check, using only the local information available with them and the information in the data packet, whether their participation as a relay can lead to further power savings. If so, they feedback an RTC packet to the relay. The structure of the data and RTC packets is shown in Fig. 2. The meaning of each field in the packet is shown below:

The fields that are common to both data and RTC packets:

- MSrc: The source, s, where data originates.
- MDest: The destination, t, of data. RSrc: The relay, u, that transmits the packet.
- RDest: The relay, v, immediately after u.

The fields that are specific to the data packet are:

- GainD: The channel gain, h_{ut} , from the current relay to the destination.
- GainR: The channel gain, h_{uv} , from u to the relay immediately after u.
- FracDelivered: The fraction of total energy, which is required to successfully decode a message at the destination, that has been accumulated at the destination before u transmits: $A(u) = \frac{h_{st}}{h_{sq}} + \frac{h_{qt}}{h_{qu}}$.



assign $\mathbf{q} \leftarrow (\mathbf{p}.\mathsf{MSrc}, \mathbf{p}.\mathsf{MDest}, \mathbf{p}.\mathsf{RSrc}, \mathbf{p}.\mathsf{RDest}, w, h_{wt}, h_{wv})$ Transmit \mathbf{q} using power $\bar{\gamma}/h_{uw}$ when possible

Fig. 3. Pseudo code of the PAR algorithm.

The fields that are specific to the RTC packet are:

- GainD: The channel gain, h_{wt} , from the node generating the RTC packet to the destination.
- GainR: The channel gain, h_{wv} , from the node generating the RTC packet to the relay immediately after u.
- RelayID: The identity of the node transmitting the RTC packet.

The pseudo code of the PAR algorithm is shown in Fig. 3. The field FracDelivered is used to keep track of how much energy the destination has accumulated *after* a relay sends the packet, so that the last relay can adjust its transmission power in a distributive manner. When a relay u (that is not the source) successfully decodes the data packet p, it acts upon it only if p.RDest = u. It then knows that the final destination is p.MDest, and the total power that has accumulated at the destination after p was transmitted is p.FracDelivered + p.GainD/p.GainR. If u is not the last relay, it transmits the packet to its next relay with power $\bar{\gamma}/h_{uN(u)}$. If it is the last relay, it transmits the packet to the destination with power $(1 - A(u))\bar{\gamma}/h_{ut}$.

It is important to point out that our protocol structure is designed to support DEA. If additional intermediate relays were also allowed to accumulate energy, the amount of overhead required to achieve energy saving also increases significantly, as one would need to have additional FracDelivered field to keep track of the status of each energy accumulating relay, before energy saving can truly be attained.

The relay u updates the route after a sufficient time, minTime, has elapsed since it last updated the route. minTime depends on the multiple access protocol, and is used to ensure

⁴Traditional routing algorithms can be used to discover a route between s and t in larger networks when a direct link from s to t does not exist. This is considered in detail in [10].

that a relay has sufficient time to receive many RTC feedback packets before it decides on an additional relay. It updates the next relay to be the node, denoted by bestCandidate, that leads to maximum power savings. The RTC packets enable u to find bestCandidate. When u receives an RTC packet from a node w, the fields of the packet enable u to compute the power savings if w is made its next relay as follows:⁵

If u is not the last relay

$$\tilde{P}_u^{\text{sav}}(w) = \left(\frac{1}{h_{uv}} - \frac{1}{h_{uw}} - \frac{1}{h_{wv}}\right)\bar{\gamma},\tag{18}$$

If u is the last relay,

$$\tilde{P}_{u}^{\text{sav}}(w) = \left(\frac{1}{h_{ut}} - \frac{1}{h_{wt}}\right) \left(1 - A(u) - \frac{h_{ut}}{h_{uw}}\right) \bar{\gamma}, \quad (19)$$

where v is the relay immediately after u: v = N(u). If $\tilde{P}_u^{\text{sav}}(w)$ exceeds the power savings achievable by the current best candidate, we update bestCandidate to be w.

When a node w overhears a data packet, p, from the relay u, the fields of the data packet enable it to check, using (14) or (15), whether its becoming a relay can reduce total power. If so, it stores N(w) = p.RDest in memory, and generates and sends an RTC packet to u when possible (according to the multiple access protocol). The pseudo code for a node is given in bottom portion of Fig. 3. The problem of using suitable multiple access mechanisms to make the nodes send RTC packets in a distributed, time- and power-efficient manner such that the relay selects the best candidate node is discussed in [10]. For the purpose of this paper, we may assume that the relay has received the RTC packets from the potential relays if and when it updates the serial DEA route.

The route converges when no RTC feedback is received by any relay in the route. It must be noted that while the PAR algorithm does guarantee power savings in every progressive step, it may be not be optimal when many relays are present.

V. PERFORMANCE ANALYSIS OF BASIC PAR

We now derive expressions for the expected power saving when a relay is added to a given serial DEA route, when the nodes are uniformly distributed spatially with density ρ . We assume that the channel gains are given by $h_{uv} = K/d(\mathbf{x}_u, \mathbf{x}_v)^{\alpha}$, where $d(\mathbf{x}_u, \mathbf{x}_v)$ is the Euclidean distance between \mathbf{x}_u and \mathbf{x}_v , and $\alpha \ge 2$ is the channel decay exponent.⁶ Without loss of generality, we assume K = 1 (otherwise, Kcan be absorbed into the threshold, $\bar{\gamma}$). We take the origin to be $(\mathbf{x}_u + \mathbf{x}_v)/2$. The power saving is a random variable, as it depends on the location of the new relay, which itself is a random variable.

The analysis, which utilizes the geometry and stochastic nature of the node layout, has three major steps. First, we find the probability distribution function (pdf) of the power saving when an arbitrary node that leads to power savings is added to the serial DEA route as a relay. As we show using asymptotic analysis, the power saving is a uniform random variable and lies between 0 and a maximum value, which depends on the relay to which the node is added and on the energy accumulation at the destination until then. Finally, we find the expected power saving when the algorithm can choose the best node from a random number of potential relays.

Let the normalized power saving metric, $M_u(\mathbf{x})$, denote the ratio of the power saving when a node, $w(\mathbf{x})$, located at \mathbf{x} , is added to the serial DEA route immediately after u, to the transmit power, P_u , of relay u in the existing DEA route:

$$M_u(\mathbf{x}) = \frac{P_u^{\text{sav}}(w(\mathbf{x}))}{P_u}.$$
(20)

Note that the power saving and the metric $M_u(\mathbf{x})$ are linearly related, and one can be calculated given the other. The evaluation of $M_u(\mathbf{x})$ depends on whether or not u is the last relay currently in the DEA route. We consider the two cases separately below.

A. Expected power saving when u is not the last relay

When u is not the last relay, let v = N(u). Using (18), the relationship between the channel gain and the distance between nodes, and $P_u = \bar{\gamma}/h_{uv}$, we can show that

$$M_u(\mathbf{x}) = 1 - \frac{1}{d(\mathbf{x}_u, \mathbf{x}_v)^{\alpha}} \left(d(\mathbf{x}_u, \mathbf{x})^{\alpha} + d(\mathbf{x}_v, \mathbf{x})^{\alpha} \right).$$
(21)

We define the conditional complementary cumulative distribution function (CCDF) of M_u as

$$F_{M_u}(\mu) = \Pr\{M_u(\mathbf{x}) \ge \mu | M_u(\mathbf{x}) \ge 0\}, \quad \mu \ge 0.$$
 (22)

Let $\Upsilon_u(\mu)$ denote the region in which adding a relay immediately after u makes M_u exceed μ , i.e., $\Upsilon_u(\mu) = \{\mathbf{x} | M_u(\mathbf{x}) \ge \mu\}$. Let $\|\Upsilon_u(\mu)\|_A$ denote the area of $\Upsilon_u(\mu)$. Then,

$$F_{M_u}(\mu) = \frac{\|\Upsilon_u(\mu)\|_A}{\|\Upsilon_u(0)\|_A}.$$
(23)

By characterizing the area $\|\Upsilon_u(\mu)\|_A$ for all $\mu \ge 0$, we can characterize $F_{M_u}(\mu)$. Let \mathcal{L}_{uv} be the line segment connecting u and v. The region $\Upsilon_u(\mu)$ is symmetrical with respect to \mathcal{L}_{uv} (see Fig. 4(left)). We define the width, $\xi_u(\mu)$, to be the length of the portion of \mathcal{L}_{uv} that lies within the region $\Upsilon_u(\mu)$. Notice that $\xi_u(0) = d(\mathbf{x}_u, \mathbf{x}_v)$. We first show that the area $\|\Upsilon_u(\mu)\|_A$ scales asymptotically as $\xi_u(\mu)^2$.

Lemma 5: When u is not the last relay of a serial DEA route, then as $d(\mathbf{x}_u, \mathbf{x}_v) \to \infty$ and $1 - \xi_u(\mu)/d(\mathbf{x}_u, \mathbf{x}_v) \ll 1$, $\|\Upsilon_u(\mu)\|_A$ scales as $\xi_u(\mu)^2$.

Proof: See Appendix.

Technically, the asymptotic results applies only when $\xi_u(\mu)$ is very large, i.e., when μ is very small. However, we will relax this condition in the analysis that follows.

Lemma 6: If u is not the last relay in a serial DEA route, then asymptotically in $\xi(\mu)$,

$$\mu = 1 - \frac{1}{2^{\alpha}} \left[\left(1 - \sqrt{F_{M_u}(\mu)} \right)^{\alpha} + \left(1 + \sqrt{F_{M_u}(\mu)} \right)^{\alpha} \right].$$
(24)

⁵Notice that $\tilde{P}_{u}^{\text{sav}}(w)$ in (18) is obtained from $P_{u}^{\text{sav}}(w)$, defined in (12), by assuming that $h_{lt} \gg h_{ut}$ and $h_{lt} \gg h_{wt}$. This is justifiable because h_{lt} is not available at u and the last relay is often much closer to the destination than to the other relays. This ensures that w does not have to also know h_{lt} .

⁶Typically, this assumption is valid only when $d(\mathbf{x}_u, \mathbf{x}_v)$ is sufficiently large.



Fig. 4. The boundary of region $\Upsilon_u(\mu)$ when $\alpha = 4$. The circle denotes the relay u, and the diamond denotes either the original next relay v, or the destination node t. (left) u is not the last relay. (right) u is the last relay, and A(u) = 0.5.

Proof: The area $\|\Upsilon_u(0)\|_A$ is linearly proportional to $\xi_u(0)^2$, which is equal to $d(\mathbf{x}_u, \mathbf{x}_v)^2$, and $\|\Upsilon_u(\mu)\|_A$ is linearly proportional to $\xi_u(\mu)^2$. From (23), it follows that

$$\xi_u(\mu) = d(\mathbf{x}_u, \mathbf{x}_v) \sqrt{F_{M_u}(\mu)}.$$
(25)

Finally, due to symmetry of (40) with respect to u and v, the portion of \mathcal{L}_{uv} that lies inside $\Upsilon_u(\mu)$ must be centered at the origin. Hence, the width, $\xi_u(\mu)$, can be written in terms of μ as $\mu = M_u(\xi_u(\mu)/2, 0)$. Combining this relationship between μ and $\xi_u(\mu)$ with (21) and (25) leads to (24).

Corollary 2: If u is not the last relay in a serial DEA route, and if $\alpha \leq 4$, the conditional CCDF $M_u(\mu)$ is well approximated by a uniform distribution.

Proof: The Taylor series expansion of (24) with respect to $F_{M_u}(\mu)$ is

$$\mu = 1 - \frac{1}{2^{\alpha - 1}} \left(1 + \frac{1}{2} \alpha (\alpha - 1) F_{M_u}(\mu) + O(\alpha^4) \right).$$
 (26)

The higher order terms $O(\alpha^4)$ are insignificant as long as $(\alpha - 2)(\alpha - 3)/12 \ll 1$, which is true when $\alpha \leq 4$. The linear relationship between μ and $F_{M_u}(\mu)$ implies a uniform distribution.

 $M_u(x)$ is thus approximated as a uniformly distributed random variable, with pdf $p_{M_u}(\mu) = 1/\mu_u^{\max}$, $0 \le \mu \le \mu_u^{\max}$. The largest power saving occurs at $\mathbf{x} = (0,0)$ and equals $\mu_u^{\max} = 1 - \frac{1}{2^{\alpha-1}}$. We now have a closed-form analytical expression for the pdf

We now have a closed-form analytical expression for the pdf of $M_u(\mathbf{x})$. As PAR selects the best node among the eligible candidates, the power saving depends also on the number of available nodes, κ , in the region $\Upsilon_u(0)$. Given that the nodes are uniformly distributed with density ρ , κ follows a Poisson distribution $\Pr(\kappa) = e^{-\lambda} \lambda^{\kappa} / \kappa!$, with mean $\lambda = \rho || \Upsilon_u(0) ||_A$. Using the theory of order statistics of uniformly distributed random variables, the expected maximum value, $M_u^{(\kappa)}$, of $\kappa \ge$ 1 realizations of $M_u(\mathbf{x})$, can be shown to be [18]

$$M_u^{(\kappa)} = \frac{\kappa}{\kappa+1} \mu_u^{\max}.$$
 (27)

Lemma 7: If u is not the last relay in a serial DEA route, the expected maximum power saving metric is

$$E_{\kappa}[M_{u}^{(\kappa)}] = \left(1 - \frac{1 - e^{-\rho \|\Upsilon_{u}(0)\|_{A}}}{\rho \|\Upsilon_{u}(0)\|_{A}}\right) \left(1 - \frac{1}{2^{\alpha - 1}}\right).$$
(28)

Proof: Using (27), the expected maximum power saving

is

$$E_{\kappa}[M_u^{(\kappa)}] = \left(1 - E_{\kappa}\left[\frac{1}{\kappa+1}\right]\right) \left(1 - \frac{1}{2^{\alpha-1}}\right).$$
(29)

Furthermore, since κ follows Poisson distribution with mean $\lambda = \rho \|\Upsilon_u(0)\|_A$, we have

$$E_{\kappa}\left[\frac{1}{\kappa+1}\right] = \sum_{k=0}^{\infty} \frac{1}{k+1} \frac{e^{-\lambda}\lambda^k}{k!} = \frac{1-e^{-\lambda}}{\lambda}.$$
 (30)

The last equality can be obtained by integrating the identity $\sum_{k=0}^{\infty} \lambda^k / k! = e^{\lambda}$ with respect to λ . Combining the above equations, we obtain the expression in (28).

We can now state the desired expression for the expected power savings:

Theorem 3: If relay u is not the last node in a serial DEA route, then the expected power saving, $E_{\mathbf{x}}[\tilde{P}_u^{\text{sav}}(w(\mathbf{x}))]$, by adding a node immediately after u is

$$E_{\mathbf{x}}[P_{u}^{\text{sav}}(w(\mathbf{x}))] = \frac{\bar{\gamma}}{h_{uv}} \left(1 - \frac{1 - e^{-\rho \|\Upsilon_{u}(0)\|_{A}}}{\rho \|\Upsilon_{u}(0)\|_{A}}\right) \left(1 - \frac{1}{2^{\alpha - 1}}\right), \quad (31)$$

where v = N(u) is the relay that is immediate after u in the existing DEA route, and

$$\frac{\pi}{4} d(\mathbf{x}_u, \mathbf{x}_v)^2 < \|\Upsilon_u(0)\|_A \simeq \frac{\pi \sqrt{4^{1-1/\alpha} - 1} \, d(\mathbf{x}_u, \mathbf{x}_v)^2}{4} < \sqrt{4^{1-\frac{1}{\alpha}} - 1} \, d(\mathbf{x}_u, \mathbf{x}_v)^2.$$
(32)

Proof: Using the definition of $\tilde{P}_u^{\text{sav}}(w(\mathbf{x}))$, (31) follows from Lemma 7. In the proof of Lemma 5, the area $\|\Upsilon_u(0)\|_A$ is lower bounded by a circle of diameter $d(\mathbf{x}_u, \mathbf{x}_v)$, and is upper bounded by a rectangle of width $d(\mathbf{x}_u, \mathbf{x}_v)$ and height $\sqrt{4^{1-1/\alpha} - 1} d(\mathbf{x}_u, \mathbf{x}_v)$ (which follows from (41) in the appendix). This leads to the bounds in (32). The approximation assumes the area is an ellipse with major and minor axis of length $\sqrt{4^{1-1/\alpha} - 1} d(\mathbf{x}_u, \mathbf{x}_v)$ and $d(\mathbf{x}_u, \mathbf{x}_v)$, respectively.

It must be noted that the upper bound is strict only when $F_{M_u}(\mu)$ is uniform. Otherwise, it is an upper bound of an approximation of $F_{M_u}(\mu)$.

B. Expected power saving when u is the last relay

When u is the last relay, the key difference is that the power saving metric depends on A(u). The transmission power of the last relay is $P_u = \bar{\gamma}(1 - A(u))/h_{ut}$. Thus, from (19),

$$M_{u}(\mathbf{x}) = 1 - \frac{d(\mathbf{x}, \mathbf{x}_{t})^{\alpha}}{d(\mathbf{x}_{u}, \mathbf{x}_{t})^{\alpha}} - \frac{1}{1 - A(u)} \left(\frac{d(\mathbf{x}_{u}, \mathbf{x})^{\alpha}}{d(\mathbf{x}_{u}, \mathbf{x}_{t})^{\alpha}} - \frac{d(\mathbf{x}_{u}, \mathbf{x})^{\alpha} d(\mathbf{x}, \mathbf{x}_{t})^{\alpha}}{d(\mathbf{x}_{u}, \mathbf{x}_{t})^{2\alpha}} \right).$$
(33)

A closed-form analytical expression for $F_M(\mu)$ is intractable in this case. However, when $F_{M_u}(\mu) = \|\Upsilon_u(\mu)\|_A / \|\Upsilon_u(0)\|_A$ is numerically evaluated for different values of A(u) and α , the CCDF again turns out to be relatively linear over all parameter values, as was the case when u was not the last relay. This is shown in Fig. 5. Therefore, the pdf of $M_u(\mathbf{x})$ is again given



Fig. 5. The conditional complementary cumulative distribution function, $F_{M_u}(\mu)$, for different values of α and A(u).

by: $p_{M_u}(\mu) = 1/\mu_u^{\max}$, where μ_u^{\max} is given in closed-form by the following Lemma.

Lemma 8: If u is the last node in a serial DEA route, then the maximum value of the power saving metric $M_u(\mathbf{x})$ when a relay is added immediately after u is

$$\mu_u^{\max} = 1 - \left(\frac{\beta - 1}{\beta}\right)^{\alpha} - \frac{1}{1 - A(u)} \left(\frac{1}{\beta^{\alpha}} - \frac{(\beta - 1)^{\alpha}}{\beta^{2\alpha}}\right),\tag{34}$$

where β is given by

$$A(u) = 1 - \frac{1}{(\beta - 1)^{\alpha - 1}} - \frac{1}{\beta^{\alpha - 1}} + \frac{2(\beta - 1)}{\beta^{\alpha}}.$$
 (35)

Proof: See Appendix.

Lemma 9: If u is the last node in a serial DEA route, the expected maximum power saving metric is

$$E_{\kappa}[M_{u}^{(\kappa)}] = \left(1 - \frac{1 - e^{-\rho \|\Upsilon_{u}(0)\|_{A}}}{\rho \|\Upsilon_{u}(0)\|_{A}}\right) \mu_{u}^{\max}, \quad (36)$$

where $\|\Upsilon_u(0)\|_A$ is given by (37), and μ_u^{\max} is given by (34).

Proof: The proof is similar to that in Lemma 7. The area $\|\Upsilon_u(0)\|_A$, in (36), can be computed exactly using the following Lemma.

Lemma 10: If u is the last node in a serial DEA route, then

$$\|\Upsilon_{u}(0)\|_{A} = d(\mathbf{x}_{u}, \mathbf{x}_{t})^{2} (1 - A(u))^{2/\alpha} \phi^{*} + 2d(\mathbf{x}_{u}, \mathbf{x}_{t})^{2} \left(\frac{\pi}{2} - \phi^{*} - \frac{\sin(2\phi^{*})}{2}\right), \quad (37)$$

where $\phi^{\star} = \cos^{-1}\left(\frac{(1-A(u))^{1/\alpha}}{2}\right)$. *Proof:* See Appendix.

We can now provide the expression for the expected power saving.

Theorem 4: If u is the last node in a serial DEA route, then the expected power saving by adding a node immediately after the relay u is

$$E_{\mathbf{x}}[\tilde{P}_{u}^{\text{sav}}(w(\mathbf{x}))] \simeq \frac{\bar{\gamma}}{h_{ut}} (1 - A(u)) \times \left(1 - \frac{1 - e^{-\rho \|\Upsilon_{u}(0)\|_{A}}}{\rho \|\Upsilon_{u}(0)\|_{A}}\right) \mu_{u}^{\max}, \quad (38)$$

where v = N(u) is the relay that is immediate after u in the existing DEA route, $\|\Upsilon_u(0)\|_A$ is given in (37), and μ_u^{\max} is given in (34).

As a corollary, we now have an analytical expression for the expected power saving of DEA when the *first relay* is added.

Corollary 3: The expected power saving in DEA by introducing a single relay between the source and destination a distance d apart is

$$E_{\mathbf{x}}[\tilde{P}_{s}^{\text{sav}}(w(\mathbf{x}))] \simeq \bar{\gamma} d^{\alpha} \left(1 - \frac{1 - e^{-\rho\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)d^{2}}}{\rho\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)d^{2}} \right) \times \left(1 - \frac{1}{2^{\alpha-1}} + \frac{1}{2^{2\alpha}} \right). \quad (39)$$

Proof: By definition, we have A(s) = 0. Hence, (34) gives $\mu_s^{\max} = 1 - \frac{1}{2^{\alpha-1}} + \frac{1}{2^{2\alpha}}$. And, $\|\Upsilon_s(\mathbf{x})\|_A = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) d^2$. The result then follows from Theorem 4.

VI. SIMULATION RESULTS

We simulate a wireless network with 100 nodes that are uniformly distributed in a cartesian grid of size 20×20 units bounded between (0,0) and (20,20). The source is located at (5,10), and the destination is at (15,10). The SNR threshold, $\bar{\gamma}$, is set to unity. The channel gain h_{uv} between any two nodes u and v that are a distance d(u,v) apart is $h_{uv} = 1/d(u,v)^{\alpha}$. At every progressive step, we assume that the relays have sufficient time to determine, using the RTC packets of PAR, the best candidate to add to the DEA route after them.

We first verify the analysis in Section V using the aforementioned simulation parameters, and $\alpha = 4$. The total power consumption is reduced by 86.4% after the first relay is added into the route, which is very close to the 85.1% predicted by Corollary 3. We also consider the more general DEA route s-r-t, which already has the first relay, r, at (10,10). From simulations, the average power saving by adding a relay between s and r (Sec. V-A) is 483, which matches well with Theorem 3, which returns an approximate value of 465, a lower bound of 436, and an upper bound of 482. Note that the upper bound uses the *approximation* that $F_u(\mu)$ is linear, which is why it turns out to be slightly lower than the value from the simulations. Similarly, the power saving for adding a relay between r and t (Sec. V-B) is 454. This matches well with Theorem 4, which predicts it to be 474.

Fig. 6 illustrates how PAR works (with 13 nodes and $\alpha = 2$). The circles show the distance over which nodes can successfully decode (without accumulating energy) the transmitted message. Also shown is the RTC feedback from potential relays. After 3 iterations, the number of relays increases from 0 to 3, and the total power decreases to



Fig. 6. Demonstration of the PAR algorithm. Crosses (\times) represent the nodes in the network. Triangles represent the chosen best candidates to be added to the existing route. Circles represent the relays in the established route. The larger dashed circle represents the range up to which the node can be heard given the transmission power set by the PAR algorithm. RTC packets arriving at a relay are shown by straight lines.

just 24.0% of the original value. It can be seen that the transmission from the last relay (- - circle line) does not include the destination because the destination has accumulated energy from transmissions of relays before the last relay.

We now study the statistics of the total power consumed by the routes established by PAR over 2000 random node placements, and $\alpha = 4$. Figure 7 shows, using a box plot, the probability distribution of the total transmit power as a function of the number of iterations of the PAR algorithm. The PAR algorithm considerably decreases the total power consumption after only 5 iterations. In the first five iterations, the median total power consumption decreases from 100% to 13.6% to 2.84% to 1.47% to 1.35%. The last column in the figure also shows the probability distribution of the total power consumed when *all* relays accumulate energy using the same route. It can be seen that DEA using PAR is within 0.44 dB of complete energy accumulation (with the same relays).

While Fig. 7 allowed for complete energy accumulation, it did not optimize its route. This is dealt with in detail in Fig. 8, which compares PAR with (i) MEAR that uses full energy accumulation and route optimization, and (ii) optimal DEA route that uses global information to set up the route. We generate results for 5, 10 and 15 nodes in the network, each with 5000 random placements and $\alpha = 4$. The networks operate over a geographical grid of size 10×20 units with bounding corners at (5,0) and (15,20).⁷ The other simulation parameters are the same as before. The figure shows the CCDF of the ratios of the total power usage of the PAR algorithm and that of the above two benchmarks. For 5, 10 and 15 nodes, with a probability of 50%, the PAR routes are less than 0.034, 0.167 and 0.269 dB away, respectively, from MEAR. Finally, PAR is less than 0.5 dB away from MEAR with a probability of 83%, 73%, and 66%, respectively. Similarly, PAR is less than 0.5 dB away from optimal DEA with a probability of 90%, 81%, and 75%, respectively. In all cases, PAR performs as well as the optimal DEA with a probability of 60%. Furthermore, when we consider a smaller area of size 10×4 units (with bounding corners at (5,8) and (15,12), but with the same 10 nodes, PAR is less than 0.25 dB away from MEAR with a probability of 50%, and is less than 0.5 dB away from MEAR with a



Fig. 7. Distribution of total power consumption for sending a packet as a function of the number of iterations of the PAR algorithm. The top, middle, and bottom lines of the box represent 75 percentile, median (50 percentile), and 25 percentile, respectively. The dashed-lines (- -) extending from each end of the boxes show the extent of the rest of data. The distribution of the total power consumed if all of the same relays (not just the destination) accumulate energy is shown in the last column ('Full coop').

probability of 77%. Note that these results show the relative performance between PAR and two optimal algorithms, and they do not provide an absolute measure of the energy saving. This result shows that the routes selected by PAR are further away from the optimal as the node density increases. This is expected since having more nodes allows for more chances for PAR to get stuck by selecting a relay that is suboptimal.

Not shown in the figure is a comparison of the total energy consumption of PAR and the optimal route chosen by shortest path algorithm when the weight of the link between nodes uand v is equal to $d(u, v)^{\alpha}$, and the destination is enabled to accumulate energy. With a probability of 50%, PAR is better than the DEA-enabled shortest path route by 0.019, 0.013, and 0.008 dB, respectively, for 5, 10, and 15 nodes.. Similarly, PAR is less than 0.5 dB away from DEA-enabled shortest path route with a probability of 90%, 80%, and 74%, respectively. While the shortest path algorithm may sometimes result in routes that consume less power than those found by PAR, the former does require distance vectors to be exchanged between each pair of nodes, which can be overhead intensive. In contrast, the PAR algorithm can set up the route progressively with very feedback from each node.

We also compare the total powers consumed by PAR and conventional relaying (no DEA) as a function of the node density. For this, 5, 10, and 15 nodes were randomly placed over an area of size 10×20 units. In all three cases, PAR reduces the total power consumption by more than 0.2 dB in over 50% of the scenarios, and by more than 0.6 dB in over 10% of the scenarios for $\alpha = 3$. These numbers increase to 0.7 dB and 1.2 dB, respectively, for $\alpha = 2$. In general, the benefits from DEA decrease as α increases or as the node density increases, since the transmission power of each relay decreases much faster than the number of relays from which the destination accumulates energy.

Finally, Fig. 9 considers the impact of slow fading on the performance of the PAR algorithm (with $\alpha = 4$). In a slow fading environment, the PAR algorithm can update the DEA route when the channels occasionally change. We consider

⁷Considering more nodes is computationally cumbersome given the NP-complete nature of the MEAR problem [9].



Fig. 8. The CCDF of the ratio of the total power usage of PAR algorithm, and MEAR with full energy accumulation, or optimal DEA route. Large and small areas refer to grid sizes of 10×20 and 10×4 , respectively. The lines without circles shows the comparison to MEAR with full energy accumulation, while the lines with circles shows the comparison to the optimal DEA route.

200 uniformly distributed node placements in a grid of size 20×20 units, with 1000 independent channel realizations simulated for each one. The figure plots the pdf of the ratio of the total power consumption with fading and without fading (i.e., the channel gains are determined entirely by the distance between the nodes) when the total number of nodes is 20 and 100. In most cases, we see that the total power decreases in the presence of fading. On an average, *additional* power savings of 0.76 dB and 0.09 dB are obtained with 20 and 100 nodes, respectively. This result is expected since the algorithm picks up the better path whenever possible, which leads to a performance improvement in a way similar to multiuser diversity in cellular networks.

VII. CONCLUSIONS

We considered energy-efficient unicast networks in which the destination accumulates energy, but the relay nodes do not. We showed that such networks, despite requiring considerably simpler relays, have energy efficiency comparable to those in which energy accumulation occurs at every node. Destination energy accumulative networks are more energy-efficient than traditional multi-hop networks that do not accumulate energy. We developed an algorithm called PAR that exploits the local information about the channel gains and discovers a DEA route progressively without undoing the previous route. It determines the relay transmission powers in a distributed manner. The route discovery in PAR has a very low complexity, and requires very limited feedback from nodes that can be added to the route as relays. Using PAR, the nodes listen to (and can easily decode) the packets currently being transmitted in the DEA route, and determine for themselves whether they will be useful as relays. We developed closed-form expressions for the power saving by PAR, and verified their accuracy using simulations. The route setup latency of PAR is low as a basic connectivity between source and destination is established right from the beginning, and improved routes that progressively add more relays are established over time. Thus, PAR is well suited for reducing the energy consumption in practical sensor networks with low complexity nodes.



Fig. 9. The pdf of the ratio of the total power usage (in dB) for a slow fading channel and a static channel. The PAR algorithm is used over 20 and 100 nodes with $\alpha = 4$.

Appendix

Proof of Lemma 2

Proof: This follows directly from geometry. In order for both relays q and r to successfully decode the packet from s, s must transmit with a minimum power $P_s = \bar{\gamma} / \max\{h_{sq}, h_{sr}\}$. Once q and r successfully decode the packet, it is optimal to add power only to the node with the best channel to t. Thus, two relays in parallel are useful only if $h_{qt} = h_{rt}$.

Now, assume $h_{qt} = h_{rt}$. If $h_{sq} > h_{sr}$, it implies that $P_s^{sav}(q) > P_s^{sav}(r)$, which contradicts the assumption that relay r is the optimal single relay. If $h_{sq} < h_{sr}$, then only r should be used as the relay. If $h_{sq} = h_{sr}$, then the total power consumption is the same as the single relay case.

Proof of Lemma3

Proof: In an energy-efficient DEA route, each relay transmits the packet with the minimum power required to reach its next relay, while the last relay sends the packet to the destination with a power that is just enough for the destination to decode the packet using the energy accumulated from the transmissions by previous relays. This can be shown to lead to the power allocation in (7) for the DEA route s-q-r-t. The power saving in (6) is the difference between the total transmit powers for s-q-r-t and s-r-t.

The DEA route s-q-r-t cannot save power if $h_{sq} > h_{sr}$; otherwise, r itself can successfully decode the packet transmitted by s. Similarly, r can be dropped from the route if $h_{qt} > \min\{h_{qr}, h_{rt}\}$. But this contradicts the assumption that r is the optimal single relay. Finally, the total power saving in (6) is positive if and only if the condition in (5) is satisfied.

Proof of Lemma 4

Proof: The power allocation in (10) follows from an argument similar to that in Lemma 3. Also, q can be dropped from the DEA route s-r-q-t if $h_{qt} \leq h_{rt}$. Finally, the total power saving in (9) is the difference between the total powers consumed by s-r-q-t and s-r-t. It is positive if and only if (8) is satisfied.

Proof of Corollary 1

Proof: Since $\frac{1}{h_{uw}} + \frac{1}{h_{wv}} > \frac{1}{h_{uv}}$ and $h_{wv} > 0$, it follows that $h_{uw} > h_{uv}$, which is the first condition in (11). Also, $\frac{1}{h_{uw}} + \frac{1}{h_{wv}} > \frac{1}{h_{uv}}$ implies that $h_{wv} \left(\frac{1}{h_{uv}} - \frac{1}{h_{uw}}\right) > 1$. But since $h_{wt} > h_{ut}$, it follows that $1 > \frac{h_{lt} - h_{uu}}{h_{lt} - h_{ut}}$. Combining the two inequalities, we obtain $h_{wv} \left(\frac{1}{h_{uv}} - \frac{1}{h_{uw}}\right) > \frac{h_{lt} - h_{wt}}{h_{lt} - h_{ut}}$. Hence, both conditions in (11) in Theorem 1 are satisfied, and power saving is guaranteed.

Proof of Lemma 5

Proof: The symmetry of the expression in (21) with respect to u and v implies that $M_u(\mathbf{x})$ is symmetric about the perpendicular bisector of \mathcal{L}_{uv} . Consider a polar coordinate system, $\mathbf{x} = (r, \phi)$, with the origin at $(\mathbf{x}_u + \mathbf{x}_v)/2$, and the 0° azimuth is oriented along \mathcal{L}_{uv} . We then have

$$M_u(r,\phi) = 1 - \frac{\left(r^2 + \frac{d(\mathbf{x}_u, \mathbf{x}_v)^2}{4} - rd(\mathbf{x}_u, \mathbf{x}_v)\cos(\phi)\right)^{\alpha/2}}{d(\mathbf{x}_u, \mathbf{x}_v)^{\alpha}} - \frac{\left(r^2 + \frac{d(\mathbf{x}_u, \mathbf{x}_v)^2}{4} + rd(\mathbf{x}_u, \mathbf{x}_v)\cos(\phi)\right)^{\alpha/2}}{d(\mathbf{x}_u, \mathbf{x}_v)^{\alpha}}.$$
 (40)

We first show that $\|\Upsilon_u(\mu)\|_A$ contains a circle with diameter $\xi_u(\mu)$. In (40), for a fixed $d(\mathbf{x}_u, \mathbf{x}_v)$, r, and $\alpha \ge 2$, we can verify that $M_u(r, \phi) \ge M_u(r, 0) = M_u(r, \pi)$ for all $\phi \in (-\pi, \pi]$. Then, the region, $\Upsilon_u(\mu)$, covered by the contour $\mu = M_u(r, 0)$ must include the region covered by the circle of radius $r = \xi_u(\mu)/2$. Hence, $\|\Upsilon_u(\mu)\|_A \ge \pi \xi_u(\mu)^2/4$.

Next, we show that the area $\|\Upsilon_u(\mu)\|_A$ is encapsulated by a rectangle whose area scales no faster than $\xi_u(\mu)^2$. In (40), for a given $d(\mathbf{x}_u, \mathbf{x}_v)$ and r, $M_u(r, \phi)$ attains its largest value at $\phi = \pm \pi/2$. Let $\mathbf{x}_1 = (r_1, \pi/2)$ and $\mathbf{x}_2 = (\xi_u(\mu)/2, 0)$ be the two points at which $M_u(\mathbf{x}_1) = M_u(\mathbf{x}_2) = \mu$. Then, $\|\Upsilon_u(\mu)\|_A \leq 2r_1\xi_u(\mu)$. Upon equating the expressions for $M_u(r, \phi)$ in (40) at \mathbf{x}_1 and \mathbf{x}_2 , we obtain

$$r_{1} = \left[\frac{1}{2^{2/\alpha}} \left(\left(\frac{d(\mathbf{x}_{u}, \mathbf{x}_{v})}{2} - \frac{\xi_{u}(\mu)}{2}\right)^{\alpha} + \left(\frac{d(\mathbf{x}_{u}, \mathbf{x}_{v})}{2} + \frac{\xi_{u}(\mu)}{2}\right)^{\alpha} \right)^{2/\alpha} - \frac{d(\mathbf{x}_{u}, \mathbf{x}_{v})^{2}}{4} \right]^{1/2}.$$
 (41)

If $1 - \xi_u(\mu)/d(\mathbf{x}_u, \mathbf{x}_v) \ll 1$, we have $r_1 \simeq \frac{1}{2}d(\mathbf{x}_u, \mathbf{x}_v)(\frac{1}{2^{2/\alpha}}(1 + \frac{\xi_u(\mu)}{d})^2 - 1)^{1/2}$, and $d(\mathbf{x}_u, \mathbf{x}_v) \simeq \xi_u(\mu)$. Hence, r_1 scales like $\xi_u(\mu)$, and $\|\Upsilon_u(\mu)\|_A$ scales no faster than $\xi_u(\mu)^2$.

Proof of Lemma 8

Proof: By contradiction, it can be shown that the maximum value of $M_u(\mathbf{x})$ always occurs on the line segment, \mathcal{L}_{ut} , connecting u and t.

Let $\beta = d(\mathbf{x}_u, \mathbf{x}_t)/d(\mathbf{x}_u, \mathbf{x})$. Since the maximum value for $M_u(\mathbf{x})$ occurs on the line segment \mathcal{L}_{ut} , say at \mathbf{x}' , it follows that $d(\mathbf{x}_u, \mathbf{x}_t) = d(\mathbf{x}_u, \mathbf{x}') + d(\mathbf{x}', \mathbf{x}_t)$. Using these relationships, we can express $M_u(\mathbf{x}')$ as a function of β as follows:

$$M_u(\beta) = 1 - \left(\frac{\beta - 1}{\beta}\right)^{\alpha} - \frac{1}{1 - A(u)} \left(\frac{1}{\beta^{\alpha}} - \frac{(\beta - 1)^{\alpha}}{\beta^{2\alpha}}\right).$$
(42)

Then the maximum value of $M_u(\beta)$, shown in (34), is obtained by equating the derivative of (42) to zero, which leads to (35).

Proof of Lemma 10

Proof: From Theorem 2 and the definition of $\Upsilon_u(0)$, we know that $\Upsilon_u(0) = \{\mathbf{x} | d(\mathbf{x}_u, \mathbf{x}) \leq d(\mathbf{x}_u, \mathbf{x}_t)(1 - A(u))^{1/\alpha} \text{ and } d(\mathbf{x}, \mathbf{x}_t) \leq d(\mathbf{x}_u, \mathbf{x}_t)\}$. Consider a polar coordinate system with center at the \mathbf{x}_u such that t lies on the 0° azimuth line. The region enclosed by $\Upsilon_u(0)$ is the intersection of the area of two circles, one centered at the origin with radius $d(\mathbf{x}_u, \mathbf{x}_t)(1 - A(u))^{1/\alpha}$ and the other centered at $(d(\mathbf{x}_u, \mathbf{x}_t), 0)$ with radius $d(\mathbf{x}_u, \mathbf{x}_t)$. These two circles intersect at the two points: $(d(\mathbf{x}_u, \mathbf{x}_t)(1 - A(u))^{1/\alpha}, \phi^*)$ and $(d(\mathbf{x}_u, \mathbf{x}_t)(1 - A(u))^{1/\alpha}, -\phi^*)$, such that

$$d(\mathbf{x}_u, \mathbf{x}_t)^2 = d(\mathbf{x}_u, \mathbf{x}_t)^2 + d(\mathbf{x}_u, \mathbf{x}_t)^2 (1 - A(u))^{2/\alpha} - 2d(\mathbf{x}_u, \mathbf{x}_t)^2 (1 - A)^{1/\alpha} \cos \phi^\star.$$
 (43)

After simplifying, we obtain the desired expression for ϕ^* .

The area $\|\Upsilon_u(0)\|_A$ is the sum of Ψ_1 , which is the area of a circle of radius $d(\mathbf{x}_u, \mathbf{x}_t)(1 - A(u))^{1/\alpha}$ within the sector $[-\phi^*, \phi^*]$, and Ψ_2 , which is the area covered within the sector angle $(\phi^*, \pi/2]$ and $[-\pi/2, -\phi^*)$ and within the circle centered at $(d(\mathbf{x}_u, \mathbf{x}_t), 0)$ with radius $d(\mathbf{x}_u, \mathbf{x}_t)$. Thus,

$$\Psi_1 = \pi d(\mathbf{x}_u, \mathbf{x}_t)^2 (1 - A(u))^{2/\alpha} \frac{2\phi^*}{2\pi}$$
(44)

$$= d(\mathbf{x}_u, \mathbf{x}_t)^2 (1 - A(u))^{2/\alpha} \phi^\star, \tag{45}$$

and,
$$\Psi_2 = 2 \int_{\phi^*}^2 \int_0^{2a(\kappa_u,\kappa_f) \cos\phi} r \, dr \, d\phi$$
 (46)

$$= 2d(\mathbf{x}_u, \mathbf{x}_t)^2 \left(\frac{\pi}{2} - \phi^* - \sin\phi^* \cos\phi^*\right).$$
(47)

The final expression of $\|\Upsilon_u(0)\|_A = \Psi_1 + \Psi_2$ is given in (37).

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