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Stark Draper, Lingjia Liu, Andreas Molisch, Jonathan Yedidia

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Routing in Cooperative Wireless Networks with Mutual-Information Accumulation

Stark C. Draper, *Member, IEEE*, Lingjia Liu, *Student Member, IEEE*
Andreas F. Molisch, *Fellow, IEEE*, and Jonathan S. Yedidia, *Member, IEEE*

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I. Introduction

Multihop relay networks are one of the most active research topics in wireless communications. The use of relays enables a number of performance improvements. Energy efficiency can be improved since the distances over which each node must transmit are often reduced significantly. Improved robustness to fading results from the increased number of possible transmission paths connecting source and destination. Greater system reliability to the failure of individual nodes need also results from the availability of multiple paths, reducing the probability of loss of session connectivity.

The most basic form of relaying consists of routing information along a single path. Data packets are passed from one node to the next in a manner akin to a bucket brigade. For example, this principle underlies the widely used Zigbee standard [1] for low-rate, low-power networking. More sophisticated approaches that require tighter synchronization between nodes at the physical and MAC layer can lead to much larger performance gains. See, e.g., [2]–[6] and the references therein.

[1]S. C. Draper is with the University of Wisconsin, Madison, WI, USA. L. Liu is with Texas A&M University, College Station, TX, USA. A. F. Molisch and J. S. Yedidia are with Mitsubishi Electric Research Labs (MERL), Cambridge, MA, USA. A. F. Molisch is also at Lund University, Sweden. Emails: {draper, lliu, molisch, yedidia}@merl.com. This work was done while the authors were at MERL.

At a high level multihop relaying can be broken down into two distinct sub-problems. The first is the design of physical-layer and MAC techniques for relaying information from one set of nodes to the next. The second is routing, i.e., identifying which of the available nodes should participate in the transmission and what system resources (time, energy, bandwidth) should be allocated to each. These two sub-problems are connected. As we see in this paper the physical-layer technique used strongly impacts the optimum route.

Most of the routing papers in the literature are based on physical layer techniques that either use virtual beamforming or energy accumulation. In virtual beamforming the amplitude and phases of the signals at transmitting nodes are adjusted to interfere constructively at the receiver [7]–[9]. On the other hand, energy accumulation is performed at the receiving nodes, enabled, e.g., through space-time coding or repetition coding [10]–[12]. A different approach based on mutual-information accumulation is proposed in [22], [24]. Note that at low signal-to-noise ratios (SNRs) the energy accumulation is actually equivalent to mutual-information accumulation. However, as SNR increases, mutual-information accumulation gives better results than the either virtual beamforming or energy accumulation. Therefore, mutual-information accumulation is the physical-layer technique used in this paper; it can be realized using rateless codes of which Fountain and Raptor codes [17]–[19] are two prominent examples. To our knowledge, there has been little prior work investigating routing in networks consisting of nodes using mutual-information accumulation. In [22] only a single-relay network is considered. The analysis of [24] assumes network “flooding”, i.e., all nodes transmit all the time; this is not an optimum use of energy. In [10], [11] the authors also formulated the problem in a linear-program under energy accumulation but the outcomes of the linear-program is not further explored to improve the selected route. Another heuristic algorithm for routing with energy accumulation was proposed in [12]. Ref. [13] derived a heuristic algorithm for relaying information with hybrid ARQ (automatic repeat request) with mutual information accumulation, which is related to, but different from, our setting.

The contributions of the current paper are threefold

- First, we present a mathematical formulation of the routing problem with mutual-information accumulation under a system-wide bandwidth constraint (the case of per-node bandwidth constraints is discussed in [29]). The case of energy minimization under an end-to-end delay constraint can be treated in a completely analogous manner.
- Second, under the assumption of centrally available and complete channel state information (CSI), we detail a method to iteratively optimize the route based on the

outputs of the linear program which solves for the optimal resource allocation within the route. By leveraging our solution to revise the route, the proposed algorithm can find a "good" route very efficiently.
- Finally, taking inspiration from our centralized solution, we provide two distributed solutions that require only local CSI. Simulations show that the resulting solutions require only less than 5% additional energy for the same end-to-end delay as the centralized solution.

An outline of the paper is as follows. We present the system model in Sec. II. The centralized routing and resource allocation algorithm, and its constituent parts, are developed in Sec. III. In Sec. IV we describe the two distributed algorithms. We provide simulations in Sec. V and conclude in Sec. VI.

II. System model

We consider a uni-cast network consisting of $N+1$ nodes, out of which $L+1$ nodes participate in transmission. Without loss of generality, we label the source as node 0, and the destination as node L. The network's objective is to convey a data packet composed of B bits from source to destination in the minimum time under sum-energy and bandwidth constraints.[2] Intermediary nodes $(1, 2, \dots L-1)$ are the relays. They may take an active role in the transmission, or may remain silent for the duration of communication. Relays either transmit or receive but cannot do both simultaneously.

All nodes are assumed to use ideal rateless codes. Rateless codes encode information bits into a potentially infinite-length codestreams.[3] Each transmitter uses an *independently generated* rateless code. This design aspect is key to mutual-information accumulation. If the *same* rateless code were used by each transmitter, the receiver would get multiple looks at each codeword symbol. This is "energy-accumulation". By getting looks at different codes (generated from the same information bits) the receiver accumulates mutual information rather than energy. The receiver combines receptions from multiple transmitters. The requirement for decoding is that the total received mutual information (summing over all transmitting nodes) exceeds B bits [24]. By "ideal" rateless codes we mean that the codes are assumed to perform at the Shannon limit and fully exploit all available degrees-of-freedom. The energy costs of reception and decoding and the non-ideal nature of real-world rateless codes can be incorporated into the optimization framework without undue trouble.

The ith node operates at a fixed transmit power spectral density (PSD) P_i (joules/sec/Hz), uniform across the transmission band. The bandwidth of the transmission band is W. When multiple nodes are scheduled to transmit simultaneously, the bandwidth is allocated among them with the bandwidth assigned node i being denoted W_i.

The propagation channel between each pair of nodes is modeled as frequency-flat and block-fading. The channel power gain between the ith and the jth nodes is denoted $h_{i,j}$. Under these assumptions, and a uniform transmit PSD, the spectral efficiency of data transmission from node i to node j can be expressed as

$$C_{i,j} = \log_2\left[1 + \frac{h_{i,j} P_i W_i}{N_0 W_i}\right] = \log_2\left[1 + \frac{h_{i,j} P_i}{N_0}\right] \text{ bits/s/Hz}, \tag{1}$$

where $N_0/2$ denotes the PSD of the (white) noise process.

Due to the block-fading nature of the channel, our routing algorithms can be executed for arbitrary channel coefficients. If the channels are time-varying (fading), the routing can be done for each channel realization separately.

[2] Multiple messages can be transmitted in parallel over (quasi-) orthogonal channels. See the discussion in [25] and [24].

[3] The use of rateless codes isn't required by the centralized solution, but is needed for the distributed solutions presented in Sec. IV.

III. Centralized Algorithm

To find the system parameters (routing and resource allocations) that yield the minimum end-to-end delay under sum-energy and a bandwidth constraints we decompose the problem into two sub-problems:

1) The first sub-problem is the determination of the subset of nodes that should participate in relaying the message and the order in which they should decode. Since a node cannot start to transmit until it has decoded the message, the decoding order is equivalent to the order in which nodes come on-line as available transmitters.
2) The second sub-problem is the determination of the optimal transmission parameters for a given ordering. These include the timing of each node's transmissions and the bandwidth allocated. (Recall that the transmission power of each node is fixed.)

In Sec. III-A we show that given a particular decoding order the problem of finding the optimal transmission parameters can be expressed as a linear program. In Section III-B we show how to use the solution of the LP to get a new ordering that has a lower-delay solution. Our final route and resource allocation algorithm, presented in Sec. III-C, iterates between (a) re orderings based on the LP solution of the last ordering, and (b) running a new linear programs based on the new ordering. This iterative procedure finds a very good (often globally optimal – as we have verified on small networks) route selection and the corresponding resource allocations efficiently, even for very large networks.

A. Optimizing resource allocation for fixed decoding order

Without loss of generality nodes are labeled according to their (current) decoding order: $0, 1, 2, \dots, L$. The objective is to find the transmission times of the nodes such that a single message of B bits is transmitted with minimum delay under a sum energy constraint $E = \sum_{i=0}^{L} E_i$. The time at which node i decodes the message is denoted T_i with $T_0 = 0$ and the total source-destination transmission duration T_L. Instead of working with the T_i it turns out to be more useful to work with the inter-node decoding delays, Δ_i, where $\Delta_i = T_i - T_{i-1}$ for $1 \le i \le L$. Message transmission can be thought of as consisting of L phases. The ith phase is of duration Δ_i and is characterized by the fact that by the *end* of the phase the first i nodes have all decoded the message. We refer to each phase as a "time-slot". It should be noted that these time-slots are not of predetermined and equal length, being the result of the optimization problem stated next.

We now state the objective function of the routing problem (for a given decoding order) as a linear function of the Δ_i. Our objective is to minimize

$$\sum_{i=1}^{L} \Delta_i \tag{2}$$

subject to: (i) $\Delta_i \geq 0$ for all i, (ii) the sum energy constraint, (iii) node i must decode by time $T_i = \sum_{l=1}^{i} \Delta_l$, and (iv) the constraint on degrees-of-freedom in each transmission phase. We state constraints (ii)–(iv) in turn.

First, consider the energy constraint. Define $A_{i,j} \geq 0$ to be the degrees-of-freedom, i.e., the time-bandwidth product (or "area" in sec $\cdot$ Hz), used by the ith node in the jth time slot where $i \in \{0, 1, \ldots, L-1\}$ and $j \in \{1, 2, \ldots, L\}$. Note that $A_{i,j} = 0$ for $j \leq i$ since node i has decoded and therefore can only transmit (and therefore would be allocated positive bandwidth) in slots $i+1, \ldots, L$. The energy constraint is

$$\sum_{i=0}^{L-1} \sum_{j=1}^{L} A_{i,j} P_i = \sum_{i=0}^{L-1} \sum_{j=i+1}^{L} A_{i,j} P_i \leq E. \tag{3}$$

We next express each of the L decoding constraints as

$$\sum_{i=0}^{k-1} \sum_{j=i+1}^{k} A_{i,j} C_{i,k} \geq B \quad \text{for all} \quad k \in \{1, 2, \ldots, L\}, \tag{4}$$

where the kth constraint ensures that the kth node can decode by $T_k = \sum_{i=1}^{k} \Delta_i$. To see (4) recall that $A_{i,j}$ is the degrees-of-freedom (sec $\cdot$ Hz) allocated to the ith transmitter in the jth phase of transmission and $C_{i,k}$ is the spectral efficiency (bits/sec/Hz) of the channel connecting the ith transmitter to the kth receiver. The kth node is required (by definition) to decode by the end of the kth transmission phase. Therefore, the total mutual information flow to the kth node must exceed B bits by the end of the kth time slot. Only the first $k-1$ nodes can contribute to this sum. The remaining nodes have not yet decoded so cannot yet transmit.

An important comment regards the constraints (4). Not all $N+1$ nodes in the network must decode, but only a subset of cardinality $L+1$. If, for instance, one node (neither source nor destination) is far from the rest (or masked by a building), then including its decoding constraint in the set (4) would increase the solution's objective (2). As we discuss when we present the swapping algorithm that improves the decoding order, nodes can be swapped out of the order. Such nodes are then no longer treated as part of the network, and the total number of nodes $L+1$ is decreased by one in the next LP.

Finally, we limit the allocations of time-bandwidth area to the total available in each time slot. We constrain the sum allocation of $A_{i,j}$ not to exceed the total available in slot j,

$$\sum_{i=0}^{j-1} A_{i,j} \leq \Delta_j W \quad \text{for all} \quad j \in \{1, 2, \ldots, L\}. \tag{5}$$

A couple aspects of (5) are valuable to note. First, the specific time-bandwidth allocation to each user within each transmission phase is not specified. This is because we model the fading as block-fading and frequency-flat. Therefore, within the transmission band, each transmitter is agnostic as to what is its exact time-bandwidth allocation. Degrees-of-freedom are treated like a fluid. Only the allocated time-bandwidth product is important. Our ideal rateless codes are assumed to be able to use optimally whatever region of the spectrum is allocated each node for transmission.

We conclude the section by showing that in the special case $P_i = P$ the solution that minimizes delay also minimizes the sum energy. The sum energy expended E_{used} is the left-hand side of (3). Set $P_i = P$. Substitute in (5) with $j = L$. Then since by definition $\sum_{j=1}^{L} \Delta_j = T_L$ we have

$$E_{\text{used}} = \sum_{i=0}^{L-1} \sum_{j=1}^{L} A_{i,j} P_i \leq \sum_{j=1}^{L} \Delta_j W P = T_L W P \tag{6}$$

At the delay-minimizing optimum, the inequality must hold with equality. If it did not some degrees of freedom A go unallocated in some time slot. If this is the case T_L can be strictly decreased by moving up all subsequent decoding times by A/W. Thus T_L is proportional to E_{used} and minimizing one minimizes the other.

B. Optimizing decoding order

While the LP formulation of Sec. III-A tells us how to allocate resources given a decoding order, it leaves open the question of how to determine the best decoding order. In a network of $L+1$ nodes there are $\sum_{i=0}^{L-1} \frac{(L-1)!}{(L-1-i)!}$ distinct decoding orders. Exhaustive search of all orderings quickly exceeds computational capabilities.

In this section, we introduce a novel algorithms that iteratively improves the decoding order by exploiting the characteristics of the LP solution obtained in Section III. While in general we obtain a local minimum, for small networks (of, say, 15 nodes, where we can exhaustively search all orders) we almost always reach the global optimum. In addition, since the algorithm is very efficient, we can try a number of different initializations to avoid particularly bad local minima.

An arbitrary decoding order is chosen to initialize the algorithm.[4] Without loss of generality we label the nodes in the initial decoding order as $[0, 1, \ldots, L]$. Define

$$\mathbf{x}^* = \left[\Delta_1^*, \ldots, \Delta_L^*, A_{0,1}^*, A_{0,2}^*, \ldots A_{0,L}^*, A_{1,2}^*, \ldots, A_{L-1,L}^*\right]$$

to be the optimum solution obtained by the linear program for the initial decoding order. Denote the optimum decoding delay as $T_L^* = \sum_{i=1}^{L} \Delta_i^*$.

Lemma 1. *If $\Delta_i^* = 0$, use T_L^{**} to denote the optimum decoding delay (under the same energy and bandwidth constraints) of the "swapped" decoding order:*

$$\begin{array}{lll} [0, \ldots, i-2, i, i-1, i+1, \ldots, L] & \textit{if} & i \leq L-1 \\ [0, \ldots, L-2, L] & \textit{if} & i = L \end{array} . \tag{7}$$

Then $T_L^{**} \leq T_L^*$.

Note that in the case $i = L$ the number of nodes in the order has decreased from $L+1$ to L. Node $L-1$ has been dropped from the order. This was discussed when we first discussed the decoding constraints (4) of the linear program.

[4]A clever initial choice, e.g., the solution of the distributed algorithm of Sec. IV, can accelerate convergence; see Sec. V.

Case 1: $(i=1)$ Combine node 1's decoding constraint (4) with the total degrees-of-freedom in time slot 1 to get

$$\frac{B}{C_{0,1}} \leq A_{0,1} \leq \Delta_1^* W. \tag{8}$$

Eq. (8) demonstrates the intuitive fact that no node can decode the message before the source. Thus, $\Delta_1^* > 0$ is true for any ordering and we need only consider $2 \leq i \leq L$.

Case 2: $(2 \leq i \leq L–1)$ We show that $\tilde{\mathbf{x}}$, a "swapped" version of $\mathbf{x}^*$, is a feasible solution for the swapped ordering that has a decoding delay equal to the optimal decoding delay of the original ordering. Define

$$\tilde{\mathbf{x}} = \left[\tilde{\Delta}_1, \ldots, \tilde{\Delta}_L, \tilde{A}_{0,1}, \tilde{A}_{0,2}, \ldots \tilde{A}_{0,L}, \tilde{A}_{1,2}, \ldots, \tilde{A}_{L-1,L}\right],$$

where

$$\begin{array}{rcll} \tilde{\Delta}_i &=& \Delta_i & \text{for all} \quad i \\ \tilde{A}_{k,l} &=& A^*_{k,l} & \text{for all} \quad k,j \text{ s.t. } k \neq i-1, k \neq i \\ \tilde{A}_{i-1,i} &=& 0 & \\ \tilde{A}_{i-1,j} &=& A^*_{i,j} & \text{for all} \quad j \in \{i+1, \ldots, L\} \\ \tilde{A}_{i,j} &=& A^*_{i-1,j} & \text{for all} \quad j \in \{i+1, \ldots, L\}. \end{array}$$

We immediately see $\sum_{i=1}^L \tilde{\Delta}_i = \sum_{i=1}^L \Delta_i^*$. We now show that $\tilde{\mathbf{x}}$ satisfies all problem constraints.

First note that the degree-of-freedom allocations $A_{i,j}$ made to each node in each time slot are almost all identical in $\mathbf{x}^*$ and $\tilde{\mathbf{x}}$. There are two exceptions. The first, $A_{i-1,i}$ doesn't appear in $\tilde{\mathbf{x}}$, but $A_{i-1,i} = 0$ since $\Delta_i = 0$. The second, $\tilde{A}_{i-1,i} = 0$.

From this we immediately get that the energy, decoding, and degrees-of-freedom constraints remain satisfied for $\tilde{\mathbf{x}}$. First, since the non-zero degree-of-freedom allocations are identical for $\mathbf{x}^*$ and $\tilde{\mathbf{x}}$, the energy usage remains the same. For the same reason the decoding ability of nodes $1, \ldots, i-2$, nodes $i+1, \ldots, L$, and the "old" (pre-swapped) node $i-1$ remain unchanged. The old node i doesn't benefit from the old node $i-1$'s transmissions any longer since the order is swapped in $\tilde{\mathbf{x}}$. However, because $\Delta_i = 0$, $A_{i-1,i} = 0$ and it didn't accumulated any mutual information in the old order in any case. Finally, since the positive degree-of-freedom allocations remain the same, and the time-slot durations $\tilde{\Delta}_i$ remain the same, the degree-of-freedom constraints all remain satisfied.

Case 3: $(i = L)$ For the same reasoning as in case 2, if we define the same vector $\tilde{\mathbf{x}}$, the decoding delay remains the same and all constraints remain satisfied. Now, if we drop the (new) node L from the problem completely (the destination is the new node $L-1$) the reduced solution is still feasible since none of the other nodes relied on the dropped nodes transmission. (It was the last in the order). □

C. Algorithm for route & resource allocation optimization

We now state the iterative route optimization algorithm. (1) Start with an initial decoding order. The order can be arbitrary, but all nodes are in the order. (2) Use the LP of Section III to solve for the parameters of the minimum-delay solution. (3) Based on Lemma 1 adapt the decoding order to find an ordering whose minimum-delay solution is upper bounded by the delay of the current solution. (3a) If $\Delta_i = \Delta_j = 0$ for $i < j-1$ swap both nodes i and $i-1$ and j and $j-1$. If $\Delta_i = \Delta_{i+1} = 0$ we swap only i and $i-1$. (3b) If the $L-1$th node is swapped with the L node, drop that node from the order entirely. That is, the new order has only $L-1$ nodes in it. (5) Repeat steps (2)–(4) until one finds an ordering with an associated set of parameters $\mathbf{x}^*$ satisfying $\Delta_i^* > 0$ for all i. At this point the algorithm terminates.

The joint routing-plus-resource-allocation optimization problem is extremely complicated to solve exactly. Our algorithm finds an approximate solution by iteratively solving simpler problems. Due to the non-convexity of the optimization problem, our algorithm may terminate at a local optimum. For networks of small size (less than about fifteen nodes) where we can test all orderings exhaustively, we often find the global minimum. However, this depends on the shape of the optimization space. For example, networks can be constructed that trick the algorithm to descend in a direction that leads to a local (non-global) optimum. Examples are given in [29].

The number of constraints in the LP is linear in the network size. The algorithm can thus be applied to very large networks.

In the discussion surrounding (6) we showed that for the special case $P_i = P$ the minimum energy and minimum delay transmission schemes are the same. Additionally, when $P_i = P$ for all i, at the optimum allocations only one node's transmitter is broadcasting at any particular time, giving

Lemma 2. *Let*

$$\mathbf{x}^* = \left[\Delta_1^*, \ldots, \Delta_L^*, A^*_{0,1}, \ldots, A^*_{0,L}, \ldots, A^*_{L-1,L}\right] \tag{9}$$

be the optimum solution of the LP proposed in Section III. If $\Delta_i > 0$ there is a unique $j \leq i-1$ such that $A_{j,i} > 0$.

Lemma 2 (see [29] for proof) says that one-node-at-a-time broadcasting strategy is optimal. Note that the strategy is still quite different from traditional multi-hop due to the mutual information accumulation at each node. Because of the broadcast nature of the wireless medium, until it decodes, each node is accumulating useful information from all transmissions. The decoding process thus has a very long memory. The memory makes it impossible to solve for the best route efficiently through, e.g., dynamic programming. In contrast, in traditional multi-hop each node listens only to its immediate predecessor. The route generated by our algorithm is also quite different from that of the multi-hop systems. This is seen in the examples presented in Section V.

IV. Distributed algorithms

It is often not desirable or possible to require centralized routing. In centralized solutions all CSI must be aggregated centrally. The resulting routing information is then dispersed throughout the system. Limitations on centralized solutions are particularly constraining in the following circumstances:

- *Large networks*: Since the number of possible links (and thus CSI that has to be distributed) increases as $(L+1)!$, aggregating the CSI of all links can incur an unacceptable overhead if L is large.
- *Temporally varying networks*: Even in small networks time-slotting and other restrictions can cause the CSI to be outdated by the time it arrives at the central location.

To address these issues we provide two distributed algorithms inspired by the characteristics of our centralized solution.

A. Distributed algorithm 1

Our first distributed algorithm commences with a direct source-to-destination transmission. In an iterative fashion intermediate nodes are added to the route.[5] Specifically, the source transmits a sounding signal. All nodes estimate their channel from the source. The destination replies with a second sounding signal. Nodes then estimate their channel to the destination. Given this CSI each node determines the potential energy savings if they were to join the path according to

$$\frac{B}{W}\frac{(C_{i,L}-C_{0,L})(C_{0,i}-C_{0,L})}{C_{0,i}C_{0,L}C_{i,L}}.$$

Each node then broadcasts this information to the rest of the network using a contention multiple access scheme. The node with the highest energy saving is chosen to participate. In the next step, the CSI from that node to all other nodes in the network is determined. Again, all nodes analyze whether they can save energy by joining the route. The process continues until no further energy savings are possible.

The algorithm is simple and, as we see in Sec. V, very effective. It does has one drawback. The initial setup takes a long time because the algorithm starts with a direct source-to-destination transmission. If the source-to-destination pathloss is high, a long sounding signal is required (noise averaging over a long time results in a good estimate of the channel strength). Adding nodes progressively shortens the transmission delay. Once a route is set up, changes (due to changing channel conditions) can be done efficiently, since the route can be modified without tearing down and rebuilding from scratch.

B. Distributed algorithm 2

A somewhat simpler algorithm can be implemented as follows. The destination broadcasts a sounding signal and all nodes estimate their channels to the destination. They broadcast this information to all other nodes. The source then starts to transmit the information packet. The first node that can decode and has a better channel to the destination takes over and the source node turns off. New nodes continue to replace previous nodes until the message reaches the destination.

V. Simulation results

We now present simulation results for energy consumption and delay. We compare the performances of the centralized and distributed solutions. We set $B = 28.9$ (i.e., 20 nats), $N_0/2 = 1$, $W = 1$, $P = 1$ (where $P_i = P$ for all i), so that delay and energy have the same numerical value. In order to give a strong sense of the relationship between geometry and channel strength, we present the case where the channel gain $h_{i,j}$ between node i and node j is related to the distance $d_{i,j}$ between them as $h_{i,j} = (d_{i,j})^{-2}$.

Consider the two-dimensional wireless network depicted in Fig. 1. The network consists of fifty nodes. The source node 0 is located at $[0.2, 0.2]$ and the destination node 49 is located at $[0.8, 0.8]$. The remaining nodes are placed randomly according to a uniform distribution in the unit square. For this example we find that the optimum decoding order of the subset of nodes that also transmit is $[0, 16, 33, 9, 47, 14, 43, 22, 38, 49]$ and the

[5]In principle the algorithm is somewhat similar to the PAR algorithm [26].

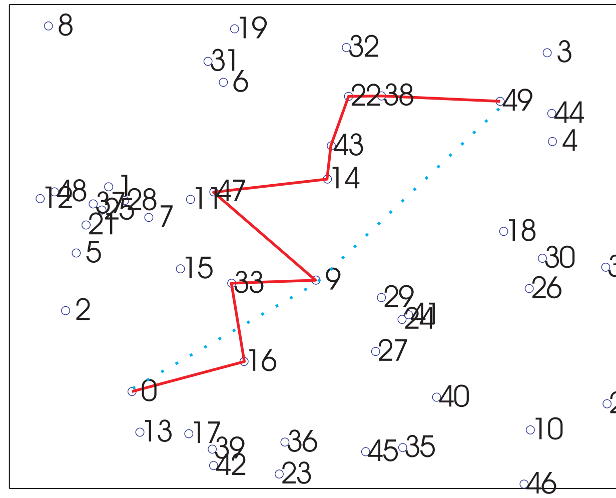

Fig. 1. Location of nodes in the fifty node network. Route for mutual-information accumulation (solid) and shortest-path route (dashed).

destination decodes after 13.09 seconds. Due to the fact that channel gain is inversely proportional to the distance squared, the nodes that are active in the minimum delay (and therefore minimum energy) solution are those that lie closest to the direct source-to-destination path. As is shown in Lemma 2 only one node is transmitting at any specific time. That is, the source node 0 transmits until node 16 decodes. Then the source node turns off and node 16 keep transmits until node 9 decodes, and so on, until the destination decodes. For the same source and destination locations we generated another 499 independently generated placements of the other 48 nodes. The mean delay across all 500 simulations is 12.54 seconds.

We now compare these results to a non-cooperative multi-hop routing example. In the non-cooperative case, the delay accrued by the hop from node i to node j is $B/W\,C_{i,j} = B/W \log_2[1 + h_{i,j}P/N_0]$. The optimum path is the shortest path through the network which can be computed with Dijkstra's algorithm [28]. For the node placements in Fig. 1 the shortest path is $[0, 9, 49]$ while the source-to-destination delay is 21.47 seconds.[6] Interestingly, the set of nodes that transmit in the shortest path problem is a proper subset of those that transmit in the cooperative protocol. Furthermore, the only relay node participating in the optimal (shortest-path) route is the one closest to the direct path connecting source to destination. Over the 500 simulated placements the average source-to-destination delay for multihop was 21.52 seconds. On average conventional non-cooperative multihop transmission incurs additional delay and energy usage on the order of 70 % as compared to cooperative transmission.

Finally, in Fig. 2 we compare the delay distribution of the four algorithms: the centralized router, shortest path, and the two distributed algorithms of Sec. IV. We plot the cumulative distribution function of the delay incurred by each algorithm

[6]If we use information-accumulating nodes in conjunction with the route obtained from Dijkstra's algorithm (instead of our optimum route), then the delay is 16.51. Thus, to a very rough approximation, the advantages of our scheme are half due to the use of mutual-information accumulation, and half due to the routing that is optimized for these types of nodes.

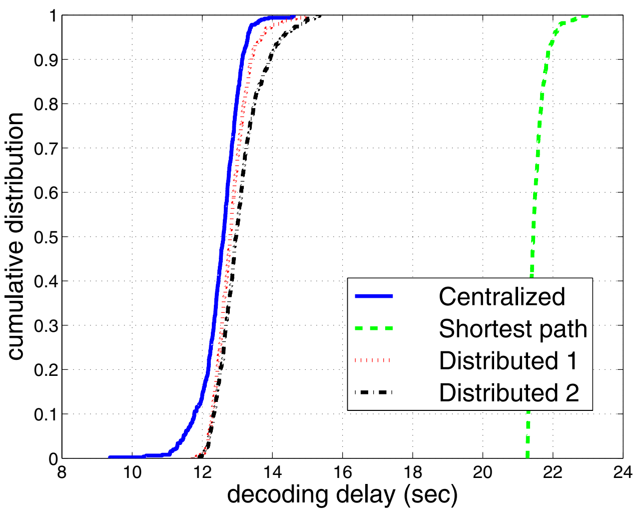

Fig. 2. Cumulative distribution of excess delay of distributed solutions as compared to centralized algorithm.

(under the same BW constraint) as calculated over the 500 placements discussed above. The figure demonstrates that the penalty for using the distributed algorithms in terms of delay (or, equivalently, energy) is small, while the penalty for not using mutual information accumulation is large. On average the first distributed algorithm incurs less than 2.5% excess delay as compared to the centralized solution. The excess delay of the second is less than 4.2%. The distributed algorithms therefore relax the need for centralized CSI at the cost of modest increases in delay. Further simulation results and comparisons with other transmission strategies, omitted here for space reasons, are presented in [29].

VI. Summary and Conclusions

We consider the problem of routing in cooperative relay networks that use mutual-information accumulation. Our model assumes ideal codes, fixed per-node transmit PSD, and a system-wide bandwidth constraint. We split the problem into one of finding the best decoding order and one of finding the best resource allocation given a decoding order. Our solution is based on solving a sequence of LPs. It is computationally efficient and can be applied to large networks. Under equal per-node PSDs, we show that the minimum-delay solution also minimizes energy consumption. Further, this solution yields a schedule that calls for only a single node to transmit at any specific time. All the same, the scheme is markedly different from conventional shortest-path routing. The delay (and energy usage) of the latter is about 70 % higher in typical examples. We also present distributed algorithms that retain most of these performance gains without requiring centralized CSI.

The approach presented in this paper is a step towards practically realizing cooperative communications in large networks. Future work will focus on algorithms that are suitable for imperfect CSI and the impact of code and hardware non-idealities. We note that in the case where bandwidth is constrained on a per-node basis (e.g., when each node can only transmit on a fixed channel) rather than on a system-wide basis, the solution differs in a number of important ways, including a trade-off between energy consumption and delay. Details are reported in [29].

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