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Abstract

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Optimal Signaling for Single Transmit Antenna Selection with Erroneous Feedback

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***Abstract*— We consider a MIMO system where error-prone feedback from the receiver is used by the transmitter to select a single optimum antenna to transmit data. Such error-prone feedback is common in the bandwidth-limited real systems, and is in marked contrast with the idealizations assumed in the selection literature. We show how the *signaling assignment*, which maps the antenna indices to the codewords that are fed back to indicate the index of the best transmit antenna, affects the performance of transmit antenna selection. The impact is intimately coupled with the receiver design. We derive approximate closed-form expressions for the average symbol error probability of QPSK modulated data in a spatially correlated channel, and then systematically find the optimal signaling assignment. Performance improvements are demonstrated for different antenna topologies without introducing *any* additional redundancy.**

I. Introduction

While multiple-antenna systems promise important improvements in the reliability of transmission over wireless channels, their widespread adoption has been inhibited by their increased hardware and signal processing complexity. Antenna selection is a low-complexity technique that reduces the hardware complexity of such systems [1]–[4]. In this paper, we focus on single transmit antenna selection. The general case in which multiple antenna subsets are selected is beyond the scope of this paper. Selecting a single antenna is interesting because it minimizes the hardware effort while still retaining the full diversity order in a frequency-flat block-fading channel, and enables the use of conventional single-input-single-output transmission schemes.[1]

In transmit antenna selection, feedback is critical to ensure that the best antenna is selected at the transmitter, which often does not have complete channel state information (CSI). A majority of papers in the literature on selection assume that the feedback is ideal, i.e., it is received error-free and without significant delays. However, this assumption fails in practical systems, which employ feedback channels that are *severely* bandwidth limited. This is done to reduce the feedback overhead and its latency, and to optimize overall spectral efficiency. For example, in third generation (3G) cellular systems, the feedback rate is just 1.5 kbps, it is uncoded, and has a bit error rate (BER) of 4% [5]. Using coding to reduce the feedback error rate is infeasible as it increases feedback latency and reduces the user mobility the system can handle.

Feedback errors can cause the antenna selected by the transmitter to be different from the optimum one requested by the receiver. Since the reverse (uncoded) feedback link has a much higher BER than the target BER for the forward (coded) data link, taking the feedback errors into account is crucial for obtaining realistic estimates of the achievable forward link BER.[2] As we will find below, spatial correlation, which depends on the antenna topology and the wireless propagation environment [6], [7] influences the probability and the importance of the feedback errors. Thus, feedback errors and correlation must be jointly taken into account.

The contributions of the paper are the following. For single transmit antenna selection in a spatially correlated data channel and an error-prone feedback channel, and a receiver with an arbitrary number of antennas, we show how the feedback signaling design, which determines the assignment of feedback codewords to the antennas to be used for transmission, affects the average symbol error probability (SEP). We develop a systematic, low-complexity technique to find the optimal signaling assignment, which is intimately coupled with the receiver structure. This is important as the number of signaling assignments grows rapidly as $N_t!$, where N_t is the number of transmit antennas.

To our knowledge, the impact of feedback signaling on the data SEP and its design has not been addressed in the literature. One reason for this is the analytical intractability of this seemingly simple problem. Elegant analytical techniques based on order statistics of independent random variables [3] are not available for correlated channels. Feedback errors further complicate the analysis. However, the simple closed-form analytical approximations developed in this paper overcome this hurdle, at least for finding the optimal signaling.

Feedback non-idealities have been modeled in [8]–[11]. However, [8] is limited to the case of 3 transmit antennas and assumes that when a feedback error occurs, any of the candidate antenna sets is equally likely to be used by the transmitter. Under this assumption, all signaling assignments give exactly the same performance, which is not the case. Furthermore, it assumes that the receiver always knows the antenna used by the transmitter, even if feedback errors occur. Similar assumptions were also made in [9], [10]. While [11] considered feedback delay, it assumed error-free feedback.

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[1]Selection of antenna subsets with more than one antenna can have benefits for data rate and array gain, but its treatment is beyond the scope of this paper.

[2]In this paper, the forward link is from the data transmitter to the receiver, and the reverse link is from the receiver to the data transmitter.

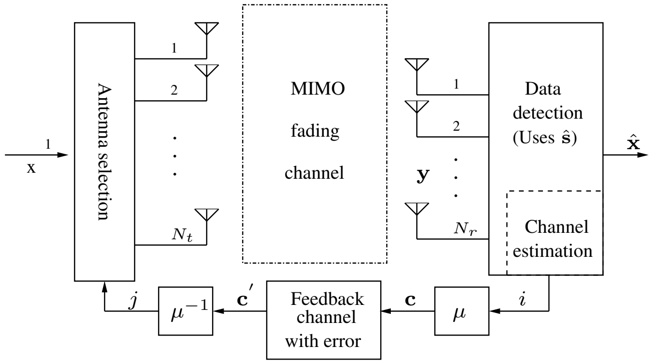

Fig. 1. Single transmit antenna selection with erroneous feedback

The paper is organized as follows. Section II formulates the antenna selection signaling problem. Section III develops the SEP-based performance metrics and the optimization algorithm. Numerical results and conclusions follow in Sections IV and V, respectively.

II. System Model

Figure 1 shows our system that consists of N_t transmit and N_r receive antennas. The received complex signal vector, $\mathbf{y} \triangleq [y_1, y_2, \cdots, y_{N_r}]^T$, can be written as:

$$\mathbf{y} = \mathbf{h}_j x + \mathbf{w}, \tag{1}$$

where x is the QPSK symbol transmitted by the selected antenna, j. In (1), $\mathbf{w} \triangleq [w_1, w_2, \cdots, w_{N_r}]^T$ is unit variance additive white complex Gaussian noise. The complex vector $\mathbf{h}_j$ is the jth column of an $N_r \times N_t$ channel matrix $\mathbf{H}$. The receiver has *a priori* knowledge of $\mathbf{H}$ with the help of a training signal. The signal to noise ratio (SNR) is denoted by γ, where $\gamma \triangleq \mathbb{E}_x[|x|^2]$.

In spatially correlated channels, the forward (data) channel matrix, $\mathbf{H}$, is commonly modeled as [6]

$$\mathbf{H} = \mathbf{R}_r^{1/2} \mathbf{H}_w \mathbf{R}_t^{1/2}, \tag{2}$$

where $\mathbf{R}_t$ and $\mathbf{R}_r$ are the transmit and receive covariance matrices, respectively, and $\mathbf{H}_w$ is a spatially white zero-mean unit variance complex i.i.d. Gaussian matrix. The correlation matrices for a uniform linear array (ULA) and a uniform circular array (UCA) are derived in [6] and [12], respectively. They depend on the angular power spectrum, (in particular, the angular spread, σ_θ and the angle of departure/arrival (AoD/AoA), θ_0) and the wavelength-normalized antenna spacing, Δ. $\mathbf{R}_r$ is assumed to be an identity matrix henceforth.

The symbol $(.)^\dagger$ denotes hermitian transpose, $(.)^T$ the transpose, and $\|.\|$ the norm of a vector. The expectation over the random variable (RV) A given B is denoted by $\mathbb{E}_{A|B}[.]$. The conditional probability of A given B is denoted by $\Pr(A|B)$ if A is a discrete RV, and by $p(A|B)$ if A is a continuous RV.

A. Transmit Antenna Selection

From Fig. 1, the set of N_t antenna indices is $\mathcal{I} \triangleq \{1, 2, \ldots, N_t\}$. The index of the optimal transmit antenna is $i = \arg\max_{k \in \mathcal{I}} \|\mathbf{h}_k\|^2$. The receiver sends the n-bit binary codeword $\mathbf{c}_i \triangleq [c_{i1}, c_{i2}, \cdots, c_{in}]$ to feedback the transmit antenna index to the transmitter. Let $\mathcal{C}$ denote the set of all used n-bit feedback codewords, $\mathcal{C} \triangleq \{\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_{N_t}\}$. For simplicity, N_t is taken to be a power of 2, *i.e.*, $\log_2 N_t$ is an integer and equals n. This is also the *minimum* number of bits required to signal N_t choices. A *signaling assignment* is any bijective mapping $\mu : \mathcal{I} \mapsto \mathcal{C}$ that maps antenna indices to codewords, such that for every $\mathbf{c} \in \mathcal{C}$, there exists an $i \in \mathcal{I}$ such that $\mathbf{c}_i = \mu(i)$, and $\mu(i) \neq \mu(j)$ for any $i \neq j$.

B. Feedback Channel Model and its Impact

With a probability ϵ, a feedback bit changes from 0 to 1, and vice versa. This model is consistent with the model used in [8], and is valid as errors in the feedback and forward links are independent. Let i denote the optimum choice of the transmit antenna made by the receiver. The receiver signals the codeword $\mathbf{c}_i = \mu(i)$, which is received by the transmitter as $\mathbf{c}_j$. The transmitter then uses the antenna $j = \mu^{-1}(\mathbf{c}_j)$, which happens with a probability $\Phi(d) = \epsilon^d (1-\epsilon)^{n-d}$, where d is the Hamming distance between $\mathbf{c}_j$ and $\mathbf{c}_i$. Thus, different Hamming distances lead to different error probabilities. Intuitively, the performance degradation can be reduced if the signaling assignment ensures that most probable feedback error patterns cause the transmitter to use an antenna that is more correlated with the optimal (receiver's) choice.

In this paper, we consider signaling assignments for two receivers: the ideal *perfect selection verification receiver*, which (through additional side information) always knows the antenna used by the transmitter, and *no-selection verification receiver*, which ignores feedback errors and assumes that the transmit antenna used is the one whose index it fed back.

III. Signaling Assignment Optimization

Let $\mathcal{M}_{N_t}$ denote the set of all the signaling assignments (bijective mappings between two sets of cardinality N_t). Then the optimal signaling assignment, μ^*, for a given SNR, γ, is given as:

$$\mu^*(\gamma) = \arg \min_{\mu \in \mathcal{M}_{N_t}} P_e(\mu; \gamma), \tag{3}$$

where $P_e(\mu;\gamma)$ denotes the average symbol error probability (SEP) for the signaling assignment μ at SNR γ. As we will see in Sec. IV, the optimum signaling is independent of the SNR, but it is intimately coupled to selection verification used.

Let the output of the detector be denoted by $\hat{x}$. For the two considered receivers, the average SEP for a signaling assignment, μ, is given by:

$$P_e(\mu;\gamma) = \sum_{i,j \in \mathcal{I}} \mathbb{E}_{x|i,j}\left[\Pr(\hat{x} \neq x | i,j)\right] \Pr(j \,|\, i) \Pr(i), \tag{4}$$

where $\Pr(i)$ is the probability that i is the optimal transmit antenna. Since x is QPSK modulated and the constellation symbols are equi-probable, we have $\mathbb{E}_{x|i,j}\left[\Pr(\hat{x} \neq x|i,j)\right] = \Pr(\hat{x} \neq x|i,j)$. $\Pr(j \,|\, i)$ depends on the feedback error rate, ϵ, and the signaling assignment, μ, as follows:

$$\Pr(j \,|\, i) = \Phi(d(\mathbf{c}_j, \mathbf{c}_i)) = \epsilon^{d(\mathbf{c}_j,\mathbf{c}_i)} (1-\epsilon)^{(n - d(\mathbf{c}_j,\mathbf{c}_i))}. \tag{5}$$

Here, $\mathbf{c}_j = \mu(j)$, $\mathbf{c}_i = \mu(i)$, and $d(\mathbf{c}_j, \mathbf{c}_i)$ denotes the Hamming distance between the two codewords $\mathbf{c}_j$ and $\mathbf{c}_i$. In the presence of spatial correlation, $\Pr(i)$ is not the same for all antennas, $i \in \mathcal{I}$, for the ULA and the UCA. However, for moderate spatial correlations, the variation is minor enough to justify the approximation $\Pr(i) \approx \frac{1}{N_t}$.

The average SEP given i and j, $\mathbb{E}_{x|i,j}\left[\Pr(\hat{x} \neq x|i,j)\right]$, depends on the modulation constellation, the receiver and the channel statistics. The combination of spatial correlation and order statistics makes it difficult to derive general closed-form expressions for the above expectation. Evaluating it numerically or using Monte Carlo simulations, for each and every signaling assignment, is also not practical. We therefore develop easily computable closed-form approximations as a function of the first and second moments of the channel state.

A. Data SEP Metric for Perfect Selection Verification

With perfect selection verification, the receiver knows – from a genie or side information – that the transmitter used index j. Therefore, the decision statistic, given *a priori* knowledge of $\mathbf{H}$, thus becomes

$$\hat{y} = \|\mathbf{h}_j\|^2 x + \mathbf{h}_j^\dagger \mathbf{w}. \quad (6)$$

For QPSK modulation, the SEP, given $\mathbf{h}_j$, approximately equals $2Q\left(\sqrt{\gamma\|\mathbf{h}_j\|^2/2}\right)$. Therefore,

$$\begin{aligned}\Pr(\hat{x} \neq x|i,j) &= \mathbb{E}_{\mathbf{h}_j|i,j}\left[2Q\left(\sqrt{\frac{\gamma}{2}\|\mathbf{h}_j\|^2}\right)\right], \\ &\approx 2Q\left(\sqrt{\frac{\gamma}{2}\mathbb{E}_{\mathbf{h}_j|i,j}\left[\|\mathbf{h}_j\|^2\right]}\right). \quad (7)\end{aligned}$$

In the above equations, we interchanged the expectation operator and the Q function. From Jensen's inequality, the resulting expression is a lower bound on the average SEP.

The correlation, r_{ij}, between $\mathbf{h}_j$ and $\mathbf{h}_i$ is the $(i,j)^{\text{th}}$ element of the matrix $\mathbf{R}_t$. Therefore, $\mathbf{h}_j = r_{ij}\mathbf{h}_i + \sqrt{1-|r_{ij}|^2}\mathbf{n}$, where $\mathbf{n}$ is independent of $\mathbf{h}_j$ and $\mathbf{h}_i$, and its elements are i.i.d. zero-mean unit-variance complex Gaussian RVs. Therefore, $\mathbb{E}_{\mathbf{h}_j|i,j}\left[\|\mathbf{h}_j\|^2\right] = |r_{ij}|^2\mathbb{E}_{\mathbf{h}_i|i}\left[\|\mathbf{h}_i\|^2\right] + (1-|r_{ij}|^2)N_r$. Using the approximation $Q(a) \approx \exp(-a^2/2)$, for $a > 0$, we get

$$\Pr(\hat{x} \neq x|i,j) \approx 2\exp\left(-\beta_{\text{ver}}(\gamma)|r_{ij}|^2\right)\exp\left(-\frac{N_r\gamma}{4}\right), \quad (8)$$

where $\beta_{\text{ver}}(\gamma)$ denotes $\frac{\gamma}{4}(\mathbb{E}_{\mathbf{h}_i|i}\left[\|\mathbf{h}_i\|^2\right] - N_r)$, which is independent of μ. Note that $\beta_{\text{ver}}(\gamma) > 0$ because $\|\mathbf{h}_i\|^2$ is the maximum of the column norms of $\mathbf{H}$.

Substituting into (4) the expressions for $\Pr(\hat{x} \neq x|i,j)$ from (8) and for $\Pr(j\,|\,i)$ from (5), we get

$$P_e(\mu;\gamma) \approx \alpha_{\text{ver}} \sum_{i,j\in\mathcal{I}} \exp\left(-\beta_{\text{ver}}(\gamma)|r_{ij}|^2\right)\left(\frac{\epsilon}{1-\epsilon}\right)^{d(\mu(i),\mu(j))},$$

where $\alpha_{\text{ver}} = \frac{2}{N_t}\exp\left(-\frac{\gamma N_r}{4}\right)(1-\epsilon)^n$ is a common term that does not depend on μ and can be dropped. Hence, we can define the metric for the signaling assignment, μ, for perfect selection verification, $M_{\text{ver}}(\mu;\gamma)$, as

$$M_{\text{ver}}(\mu;\gamma) \triangleq \sum_{i,j\in\mathcal{I}} \exp\left(-\beta_{\text{ver}}(\gamma)|r_{ij}|^2\right)\left(\frac{\epsilon}{1-\epsilon}\right)^{d(\mu(i),\mu(j))}. \quad (9)$$

B. Data SEP Metric for No-Selection Verification

A no-selection verification receiver ignores feedback errors and assumes that the transmitter used the same antenna i, which the receiver fed back. Given a priori knowledge of $\mathbf{H}$, the decision statistic for this receiver is

$$\hat{y} = \mathbf{h}_i^\dagger \mathbf{h}_j x + \mathbf{h}_i^\dagger \mathbf{w}. \quad (10)$$

Therefore when x is QPSK modulated, we have

$$\Pr(\hat{x} \neq x|i,j) = \mathbb{E}_{\mathbf{h}_i,\mathbf{h}_j|i,j}\left[\Pr(\hat{x} \neq x|\mathbf{h}_i,\mathbf{h}_j,i,j)\right].$$

Define ϕ as the phase of the complex number $\mathbf{h}_i^\dagger\mathbf{h}_j$. For small ϕ, the following approximation is accurate:

$$\Pr(\hat{x} \neq x|\mathbf{h}_j,\mathbf{h}_i,i,j) \approx 2Q\left(\sqrt{\frac{\gamma}{4}}\frac{|\mathbf{h}_i^\dagger\mathbf{h}_j|}{\|\mathbf{h}_i\|}\cos(\phi)\right). \quad (11)$$

Writing $\mathbf{h}_j$ in terms of $\mathbf{h}_i$, as we did in Sec. III-A, we get $|\mathbf{h}_i^\dagger\mathbf{h}_j|\cos(\phi) = \|\mathbf{h}_i\|^2\text{Re}\left\{r_{ij}\right\} + \text{Re}\left\{\sqrt{1-|r_{ij}|^2}\mathbf{h}_i^\dagger\mathbf{n}\right\}$. Swapping the expectation operator and the Q function, as before, and using the fact that $\mathbf{n}$ is a zero mean vector uncorrelated with $\mathbf{h}_i$, leads to $\Pr(\hat{x} \neq x|i,j) \approx 2Q\left(\beta_{\text{no-ver}}(\gamma)\text{Re}\left\{r_{ij}\right\}\right)$, where $\beta_{\text{no-ver}}(\gamma) = \sqrt{\frac{\gamma}{4}}\mathbb{E}_{\mathbf{h}_j|i,j}\left[\|\mathbf{h}_i\|\right]$ is independent of μ. As before, the SEP approximation is

$$\begin{aligned}&P_e(\mu;\gamma) \\ &\approx \alpha_{\text{no-ver}} \sum_{i,j\in\mathcal{I}} Q(\beta_{\text{no-ver}}(\gamma)\text{Re}\left\{r_{ij}\right\})\left(\frac{\epsilon}{1-\epsilon}\right)^{d(\mu(j),\mu(i))}, \quad (12)\end{aligned}$$

where $\alpha_{\text{no-ver}} = \frac{2}{N_t}(1-\epsilon)^n$ is a common term does not depend on μ and can be dropped. Therefore, the corresponding metric for no-selection verification is

$$\begin{aligned}&M_{\text{no-ver}}(\mu;\gamma) \\ &\triangleq \sum_{i,j\in\mathcal{I}} Q\left(\beta_{\text{no-ver}}(\gamma)\text{Re}\left\{r_{ij}\right\}\right)\left(\frac{\epsilon}{1-\epsilon}\right)^{d(\mu(j),\mu(i))}. \quad (13)\end{aligned}$$

For the metrics in (9) and (13), the following lemma shows that the optimal signaling assignments do not need to be recalculated if a key system parameter changes. The proof is omitted due to space constraints, and is available in [13].

Lemma *1:* For small feedback bit error probabilities, $\epsilon \ll 1$, the optimal signaling assignments for perfect and no-selection verification are independent of ϵ.

C. Verification of Approximate Metrics: An Example

Figure 2 is a scatter plot of the simulated average SEP, $P_e(\gamma;\mu)$ and the metric for perfect selection verification, $M_{\text{ver}}(\mu;\gamma)$, which is defined in (9), at an SNR of 6 dB. The figure plots 800 different assignments, out of a total of 40320 possible signaling assignments, for $N_t = 8$ and $N_r = 1$. The

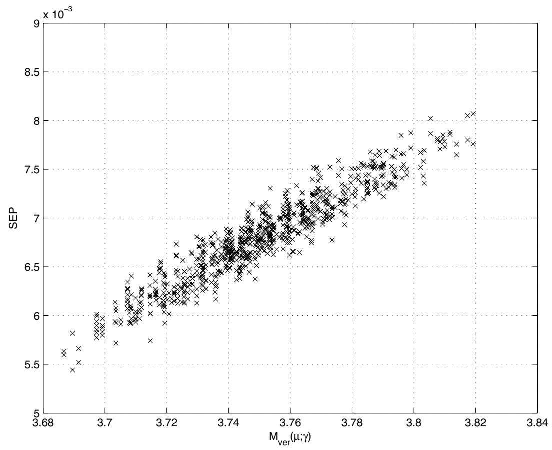

Fig. 2. Monotonic relationship between $M_{\text{ver}}(\mu;\gamma)$ and average data SEP for different signaling assignments μ (ULA, $\gamma = 6$ dB, $N_t = 8$, $N_r = 1$).

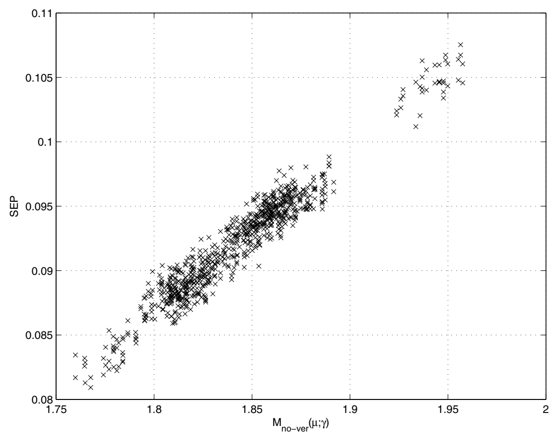

Fig. 3. Monotonic relationship between $M_{\text{no-ver}}(\mu;\gamma)$ and average data SEP for different signaling assignments μ (ULA, $\gamma = 6$ dB, $N_t = 8$, $N_r = 1$).

SNR dependent term, $\beta_{\text{ver}}(\gamma)$, is set to unity. The monotonic relationship between the metric and the average SEP is evident from the plot. The approximations made in the derivation of the metric $M_{\text{ver}}(\mu;\gamma)$ and simulation noise cause some scatter in the plot, which leads to a small uncertainty about the exact SEP value. Note, however, that the region comprising lower values of both $P_e(\mu;\gamma)$ and $M_{\text{ver}}(\mu;\gamma)$ is of primary interest for optimization purposes.

Figure 3 is a scatter plot of the simulated average SEP and the metric for no-selection verification, $M_{\text{no-ver}}(\mu;\gamma)$, which is defined in (13). As before, $\beta_{\text{no-ver}}(\gamma)$ is set to unity. We again see that the all-important monotonic relationship holds. Additional simulations with different sets of system parameters show that the monotonic relationship holds regardless of the value of $\beta_{\text{ver}}(\gamma)$ and $\beta_{\text{no-ver}}(\gamma)$. Therefore, we shall set them to 1 henceforth.

D. Signaling Design: Optimization Algorithm

Given the closed-form approximations for average SEP in III-A and III-B, for perfect selection verification and no-selection verification, respectively, we now find the corresponding optimal signaling assignments that minimize the respective metrics defined in (9) and (13). Each signaling assignment is nothing but a different permutation of the codewords because it determines which codeword is mapped to each transmit antenna index. The number of signaling assignments for N_t codewords is $N_t!$, which is a very large number even for moderate values of N_t.[3]

We use the Binary Switching Algorithm (BSA) for this purpose. It was invented in early source coding literature on vector quantization over noisy channels [14], [15]. Given an initial assignment, the BSA iteratively switches two codewords assigned to two transmit antenna indices such that the total cost decreases in each step. The total cost, $M(\mu;\gamma)$, is defined as $M(\mu;\gamma) \triangleq M_{\text{no-ver}}(\mu;\gamma)$ for no-selection verification and $M(\mu;\gamma) \triangleq M_{\text{ver}}(\mu;\gamma)$ for perfect selection verification. Each switch results in a new (and better) signaling assignment. The algorithm stops when the cost cannot be reduced further by any switch. The reader is referred to [14] for a detailed description of BSA.

To determine which two codewords to swap in BSA, a cost is assigned to each antenna index choice; the total cost is the sum of the costs of all the choices. From the SEP metric in (9) for perfect selection verification, the following cost must be

TABLE I

OPTIMAL SIGNALING ASSIGNMENTS FOR ULA AND UCA FOR DIFFERENT RECEIVER VERIFICATION DESIGNS

Topology	N_t	Selection verification	Optimal Signaling assignment
ULA	16	None	1 7 9 8 10 16 2 15 6 11 5 12 13 4 14 3
	16	Perfect	12 21 16 14 6 8 4 2 1 3 11 9 13 15 7 5
	8	None	8 1 6 3 2 7 4 5
	8	Perfect	8 4 2 6 5 1 3 7
UCA	8	None	3 1 5 2 4 7 8 6
	8	Perfect	4 7 6 2 5 8 3 1

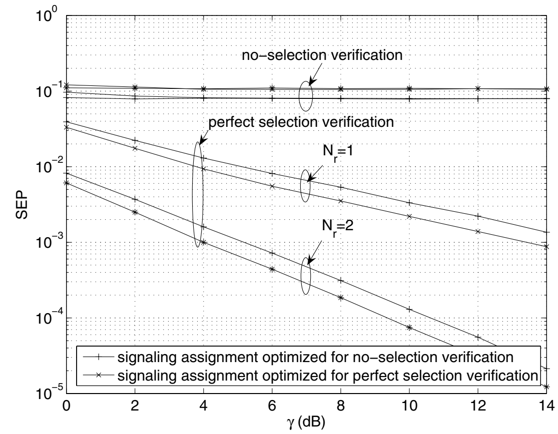

Fig. 4. Average SEP for different N_r when optimal signaling assignments are used with their respective receivers. Also shown is the adverse swapped case when the signaling assignment optimized for perfect verification is used with a no-selection verification receiver, and vice versa (ULA, $N_t = 8$, $L_t = 1$).

[3] The search space size can be reduced to $N_t!/2$ assignments because swapping 0s and 1s in the feedback codewords results in the same performance.

assigned to each choice, $i \in \mathcal{I}$:

$$\hat{M}_i(\mu;\gamma) = \sum_{j\in\mathcal{I}} \exp\left(-\beta(\gamma)|r_{ij}|^2\right)\left(\frac{\epsilon}{1-\epsilon}\right)^{d(\mu(i),\mu(j))}. \quad (14)$$

A similar cost is defined for no-selection verification as well. Clearly, we have $M(\mu;\gamma) = \sum_{i\in\mathcal{I}} \hat{M}_i(\mu;\gamma)$. To speed up convergence, the antenna indices are sorted in decreasing order of their respective costs, and the indices with higher costs are considered first for switching. The algorithm is started with several randomly chosen initial signaling assignments, and the assignment with the lowest total cost is finally chosen. The complexity of BSA is of the order of N_t^3. The complexity can be reduced to $N_t^2 \log_2(N_t)$ for $\epsilon \ll 1$, *i.e.*, when only single feedback bit errors are very likely [13].

IV. Numerical Results

In the numerical results that follow, the feedback BER is $\epsilon = 4\%$. For a ULA, the wavelength-normalized spacing is $\Delta = 0.5$, the angle spread is $\sigma_\theta = 30^\circ$, and the mean AoD is $\theta_0 = 30^\circ$. For a UCA, the wavelength-normalized radius is 4.0, the angle spread for a Laplacian distribution is $\sigma_\theta = 10^\circ$, and the mean AoD is $\theta_0 = 30^\circ$.

Table I lists the optimal signaling assignments found using Sec. III for $N_t = 8$ and 16 for both the ULA and the UCA.[4] The total number of possible signaling assignments is 40320 for $N_t = 8$ and 2.0923×10^{13} for $N_t = 16$, which is extremely large. In the table, the optimal signaling assignment for perfect selection verification with $N_t = 8$ is "8 4 2 6 5 1 3 7", which means that the receiver uses the codeword 111 to signal transmit antenna 1, 011 to signal transmit antenna 2, and so on.[5] From Table I, the optimal signaling assignment for perfect selection verification is also a Gray mapping, which makes intuitive sense as the nearest neighbor has the largest correlation. However, not all of the possible Gray mappings are optimal. For no-selection verification, the optimum assignment is not a Gray mapping. For a UCA, the Gray mapping may not even exist.

Figure 4 compares the SEP performance of the optimal signaling assignments. It can be seen that no-selection verification exhibits an error floor that is of the order of $n\epsilon$, which is unacceptably large for $\epsilon = 4\%$. However, perfect selection verification does not suffer from such a floor. The optimal signaling assignments lower the error floor for no-selection verification and improve the SNR by 1.5 to 2.0 dB for perfect selection verification. Not shown in the figure are the SEP curves from numerical simulations of many randomly generated signaling assignments. They all lie between the ones plotted in the figure. Figure 4 also varies the number of receive antennas, N_r, and shows that the optimal signaling assignment is independent of the number of receive antennas in the system. Increasing N_r substantially reduces the SEP for perfect selection verification, but does not reduce the error floor for no-selection verification. Both Table I and Fig. 4 show that the optimal signaling assignments for perfect selection verification and no-selection verification are different.

[4]The BSA was run for 100 randomly chosen initial signaling assignments.
[5]The antennas in a UCA are indexed in the anti-clockwise direction.

V. Conclusions

We showed that feedback signaling design, which maps transmit antenna index choices to codewords to be fed back to the data transmitter, is important when the feedback channel is error-prone. We derived approximate closed-form metrics for the average data symbol error probability, which hold for an arbitrary number of receive antennas and are sufficient to find the optimal signaling assignments quickly and systematically using the binary switching algorithm. The optimal signaling assignments depend on the transmit antenna topology and the type of selection verification at the receiver, but not on the number of receive antennas. SNR gains of 1 to 2 dB for perfect selection verification, and a lower error floor for no-selection verification, were achieved without introducing any additional redundancy. As we saw, the issue of selection verification at the receiver and mechanisms to enable it is very important. This is treated in detail in [13]. Future work includes generalizing the metrics to other modulation schemes and extending the formulation to the case in which subsets with more than one transmit antenna are selected.

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